Worksheet: Implicit Differentiation and Normal Lines

Section 1, 107 Ag Sc & Ind Bldg,

TR 9:05 AM - 9:55 AM

Before we begin, let's list some facts you will use.

- Remember that whenever we write $F(x, y, z)$, we mean the function which has all variables on the same side! For example, if $z = 2xy + y^2$ then we write

$$F(x, y, z) = 2xy + y^2 - z$$

- **Implicit differentiation** follows the formula

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

- Notice that the variables flip and the sign changes.
- Recall that a **line** in three-dimensional space can be described by a vector equation

$$\vec{r}(t) = \langle a + v_1 t, b + v_2 t, c + v_3 t \rangle$$

where $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is the slope vector and (a, b, c) is a point that the line passes through.

- The line travels in the *same* direction at the slope vector.

Please answer the following questions

Implicit Differentiation

1. Find $\dfrac{\partial z}{\partial x}$ for the function

$$xyz + z^2 = x^2$$

2. Find $\dfrac{\partial z}{\partial y}$ for the function

$$xyz + yxz^2 = yx^2$$

3. Find $\dfrac{\partial r}{\partial t}$ for the function

$$rt + s^2 - rs = st^2$$

Normal Lines

1. What vector is perpendicular to $z = f(x, y)$ at a point (x, y, z)? Hint: Think about the vector used to define the tangent plane.

2. For the function $f(x, y) = x^2 + y^2$, find the vector perpendicular to the surface at $(1, 1, 2)$.

3. Use that vector to define a line perpendicular to the surface (the normal line).

4. Use the same technique to define the normal line at the point $(3, 1, 4)$ for the following function

$$z^2 + xy = 7$$