# Math 231 Midterm Prep

**1.** Let $\mathbf{v} = \langle\, 1, 1, 2\,\rangle$ and $\mathbf{w} = \langle\, -2, 3, 1\,\rangle$.

**a.** Find the unit vector in the same direction as $\mathbf{v}$.

**b.** Find the dot product $\mathbf{v} \cdot \mathbf{w}$.

**c.** Find the cross product $\mathbf{v} \times \mathbf{w}$.

**d.** Find the vector projection of $\mathbf{v}$ onto $\mathbf{w}$.

**2.** What is the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$ in terms of $|\,\mathbf{a}\,|$, $|\,\mathbf{b}\,|$, and $\theta$ (the angle between the vectors)?

**3.** **a.** Given two vectors, how can you tell if they are parallel?

**b.** Given two lines, how can you tell if they are parallel?

**c.** Given two planes, how can you tell if they are parallel?

**d.** Given a vector and a line, how can you tell if they are parallel?

**e.** Given a vector and a plane, how can you tell if they are parallel?

**f.** Given a line and a plane, how can you tell if they are parallel?

**4.** **a.** Given two vectors, how can you tell if they are orthogonal?

**b.** Given two lines, how can you tell if they are perpendicular?

**c.** Given two planes, how can you tell if they are orthogonal?

**d.** Given a vector and a line, how can you tell if they are perpendicular?

**e.** Given a vector and a plane, how can you tell if they are normal?

**f.** Given a line and a plane, how can you tell if they are normal?

**5.** Consider the lines with parametric equations

$$\begin{aligned} \ell_1 &: \quad x = t \qquad, \; y = 2t - 2, \; z = t + 10 \\ \ell_2 &: \quad x = 1 - s, \; y = -2s \quad, \; z = 7 - 3s \end{aligned}$$

**a.** Find the point of intersection of $\ell_1$ and $\ell_2$.

**b.** Find a vector normal to the plane containing $\ell_1$ and $\ell_2$.

**c.** Write an equation for the plane containing $\ell_1$ and $\ell_2$ in "standard form" (that is, $ax + by + cz + d = 0$).

**6.** **a.** Sketch and identify the surface $x^2 + y^2 - (z+1)^2 = 0$.

**b.** Sketch and identify the surface $z = 2$.

**c.** Sketch the **region** in space bounded by $x^2 + y^2 - (z+1)^2 = 0$ and $z = 2$.

**7.** **a.** Sketch and identify the surface $x^2 + y^2 + z^2 = 4$.

**b.** Sketch and identify the surface $z = \sqrt{4 - x^2 - y^2}$.

**8.** **a.** Sketch and identify the surface $y^2 + z^2 = 9$.

**b.** Sketch and identify the surface $x + y = 1$.

**c.** Sketch both $y^2 + z^2 = 9$ and $x + y = 1$ and mark their curve of intersection.

**d.** Give a vector function that traces out the intersection of $y^2 + z^2 = 9$ and $x + y = 1$.

**9.** Consider the ellipse $x^2 + \dfrac{y^2}{4} = 1$ in the $xy$-plane.

**a.** Give a vector function in 2D that traces out the ellipse **counter-clockwise**.

**b.** Give a vector function in 2D that traces out the ellipse **clockwise**.

**10.** Consider the helix $\mathbf{r}(t) = \langle\, 12\cos t, 5t, 12\sin t\,\rangle$.

**a.** Calculate the unit tangent vector $\mathbf{T}$ at the the point $(-12, 5\pi, 0)$.

**b.** Calculate the unit normal vector $\mathbf{N}$ at the the point $(-12, 5\pi, 0)$.

**c.** Calculate the unit binormal vector $\mathbf{B}$ at the the point $(-12, 5\pi, 0)$.

**d.** Calculate the curvature at the point $(-12, 5\pi, 0)$.

**e.** Calculate the distance travelled along the curve from $(12, 0, 0)$ to $(-12, 5\pi, 0)$.

**f.** Reparametrize $\mathbf{r}(t)$ by arclength starting from $(12, 0, 0)$.

**11.** A particle moves along the path $\mathbf{r}(t) = \left\langle \sqrt{t+1}, t-5, t^2 \right\rangle$. At what point is the velocity of this particle parallel to vector $\langle\, 1, 4, 24\,\rangle$?

**12.** Find the position vector $\mathbf{r}(t)$ for a particle that starts at the origin and has velocity vector $\mathbf{v}(t) = \langle\, t, 4, 1-2t\,\rangle$.

**13.** A particle starts at the origin and has velocity $\mathbf{v}(t) = \langle\, t, 3t^2, 2t - 1\,\rangle$. Which of the following is a point through which the point travels?

(a) $(2, 8, 2)$

(b) $(1, 3, 1)$

(c) $(1, 0, 2)$

(d) $(2, 6, 1)$

(e) $(2, 3, 2)$

**14.** Which of the following statements is NOT true?

(a) At any point on a curve, the curvature $\kappa$ is a scalar that is greater than or equal to 0.

(b) Two vectors are orthogonal if and only if their dot product is zero.

(c) The equation $x^2 - y + z^2 = 1$ describes an elliptic paraboloid.

(d) The equation $ax + by + cz = 0$ describes a plane that passes through the origin.

(e) If two lines are not parallel, they intersect at exactly one point.

**15.** Suppose $|\,\mathbf{a}\,| = 6$, $|\,\mathbf{b}\,| = \sqrt{3}$, and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/6$. Which of the following is the value of $\mathbf{a} \cdot \mathbf{b}$?

(a) $-1$

(b) $3\sqrt{3}$

(c) $0$

(d) $9$

(e) Not enough information.

(f) None of the above.

**16.** Which of the following planes is normal to the line $x = t + 1,\ y = 3t - 1,\ z = 2t - 3$?

(a) $x - 3y - 2z = 0$

(b) $x - y - 3z + 2 = 0$

(c) $2x + 6y + 4z - 1 = 0$

(d) $2x + 6y + z = 0$

(e) $2x + y - 3z + 3 = 0$

**17.** What is the vector projection of $\mathbf{v} = \langle 5, -1/2, 0 \rangle$ onto $\mathbf{w} = \langle 8, -1, 4 \rangle$?

(a) $\langle 4, -1/2, 2 \rangle$

(b) $\langle 16, -2, 4 \rangle$

(c) $\langle 10, -1, 0 \rangle$

(d) $\langle -8, 1, -4 \rangle$

(e) $\frac{1}{101} \langle 810, 81, 0 \rangle$

(f) None of the above.

**18.** Which of the following pairs of vectors are orthogonal?

| | | |
|---|---|---|
| (a) $\langle 23, 4, 1 \rangle$ | and | $\langle 0, 1, -3 \rangle$ |
| (b) $\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}$ | and | $19\,\mathbf{i} + \mathbf{j} + 7\,\mathbf{k}$ |
| (c) $\left\langle \frac{1}{2}, 3, -1 \right\rangle$ | and | $\langle -18, 4, 1 \rangle$ |
| (d) $-6\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$ | and | $\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}$ |
| (e) $\langle 3, -5, 2 \rangle$ | and | $\langle -1, 3, 2 \rangle$ |
| (f) $2\,\mathbf{i} - \mathbf{j} + 12\,\mathbf{k}$ | and | $-4\,\mathbf{i} + 3\,\mathbf{j}$ |