Hw 46 Solutions:

Section 8.4 The Product-to-sum and Sum-to-Product Formulas

OBJECTIVE 1: Understanding the Product-to-Sum Formulas

There are four product-to-sum formulas. That is, there are four different formulas that can be used to change the product of two trigonometric expressions into the sum or difference of two trigonometric expressions. These formulas are especially useful in calculus. Although the formulas appear to be difficult to memorize, they are actually fairly easy to derive provided that you can recall the sum and difference formulas for sine and cosine.

Product-to-Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \quad (1)$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \quad (2)$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)] \quad (3)$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)] \quad (4)$$

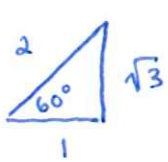
8.4.4 Write the product as the sum or difference containing only sines or cosines.

(3) $\sin\left(\frac{5\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left[\sin\left(\frac{6\theta}{2}\right) + \sin\left(\frac{4\theta}{2}\right)\right]$

$= \boxed{\frac{1}{2}[\sin(3\theta) + \sin(2\theta)]}$

8.4.6 Determine the exact value of the expression without using a calculator.

(1) $(\sin 75°)(\sin 15°) = \frac{1}{2}[\cos(60°) - \cos(90°)] = \frac{1}{2}\left[\frac{1}{2} - 0\right] = \boxed{\frac{1}{4}}$



OBJECTIVE 2: Understanding the Sum-to-Product Formulas

We can use the four product-to-sum formulas to verify each of the four sum-to-product formulas. These formulas are particularly useful when solving trigonometric equations.

Sum-to-Product Formulas

$$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha-\sin\beta=2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

8.4.11 Write the sum $\sin(11\theta)+\sin(9\theta)$ as a product containing only sines and/or cosines.

$$\sin(11\theta)+\sin(9\theta)=2\sin\left(\frac{20\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)$$

$$=\boxed{2\sin(10\theta)\cos(\theta)}$$

8.4.15 Determine the exact value of the expression $\sin 225°+\sin 135°$ without the use of a calculator.

$$\sin(225°)+\sin(135°)=2\sin\left(\frac{360°}{2}\right)\cos\left(\frac{90°}{2}\right)$$

$$=2\sin(180°)\cos(45°)$$

$$=\boxed{0}$$

OBJECTIVE 3: Using the Product-to-Sum and Sum-to-Product Formulas to Verify Identities

If one side of the identity includes a trigonometric expression involving the sum or difference of sine and cosine, then first substitute the appropriate sum-to-product formula. Likewise, if one side of the identity includes the product of sines and/or cosines, then first substitute the appropriate product-to-sum formula. Next, use the strategies we developed in Section 8.1 for verifying identities.

Verify each identity.

8.4.19 $\dfrac{\sin x+\sin y}{\cos x+\cos y}=\tan\dfrac{x+y}{2}$

$$\frac{\sin x+\sin y}{\cos x+\cos y}=\frac{\cancel{2}\sin\left(\frac{x+y}{2}\right)\cancel{\cos\left(\frac{x-y}{2}\right)}}{\cancel{2}\cos\left(\frac{x+y}{2}\right)\cancel{\cos\left(\frac{x-y}{2}\right)}}=\tan\left(\frac{x+y}{2}\right)\ \checkmark$$

8.4.21 $\dfrac{\sin(6\theta)+\sin(8\theta)}{\sin(6\theta)-\sin(8\theta)}=-\dfrac{\tan(7\theta)}{\tan\theta}$