HW 42 Solutions

Note: There are *many* ways to show these identities.

~~[illegible]~~

Quotient Identies

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

2. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

Reciprocal Identies

1. $\csc\theta = \frac{1}{\sin\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\cot\theta = \frac{1}{\tan\theta}$
4. $\sin\theta = \frac{1}{\csc\theta}$
5. $\cos\theta = \frac{1}{\sec\theta}$
6. $\tan\theta = \frac{1}{\cot\theta}$

17 Extra

Pythagoriean Identies

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \cot^2\theta = \csc^2\theta$
3. $\tan^2\theta + 1 = \sec^2\theta$

Odd Properties

1. $\sin(-\theta) = -\sin(\theta)$
2. $\csc(-\theta) = -\csc(\theta)$
3. $\tan(-\theta) = -\tan(\theta)$
4. $\cot(-\theta) = -\cot(\theta)$

Even Properties

1. $\cos(-\theta) = \cos(\theta)$
2. $\sec(-\theta) = \sec(\theta)$

8.) $1 + \cot^2(-\theta) = \csc^2\theta$

$$1 + \cot^2(-\theta) = 1 + (-\cot(\theta))^2 = 1 + \cot^2(\theta) = \csc^2\theta \quad ✓$$

11.) $\left(3\cos(\theta) - 4\sin(\theta)\right)^2 + \left(4\cos(\theta) + 3\sin(\theta)\right)^2 = 25$

FOIL: $(3\cos\theta - 4\sin\theta)^2 + (4\cos(\theta) + 3\sin\theta)^2$

$$= 9\cos^2\theta + 16\sin^2\theta \cancel{- 12\cos\theta\sin\theta} + 16\cos^2\theta + 9\sin^2\theta \cancel{+ 12\cos\theta\sin\theta}$$

$$= 9(\cos^2\theta + \sin^2\theta) + 16(\sin^2\theta + \cos^2\theta) = 9 + 16 = 25 \quad ✓$$

15.) $\cos\theta \tan\theta \csc\theta = 1$

$\cos\theta \tan\theta \csc\theta = \cancel{\cos\theta} \cdot \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{1}{\cancel{\sin\theta}} = 1$ ✓

18.) $\frac{\csc\theta}{\sin\theta} - \cos(-\theta)\sec(-\theta) = \cot^2\theta$

even

$\frac{\csc\theta}{\sin\theta} - \cos(-\theta)\sec(-\theta) = \frac{1}{\sin\theta} \cdot \frac{1}{\sin\theta} - \cos(\theta)\sec(\theta)$

$= \frac{1}{\sin^2\theta} - \cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}}$

$= \csc^2\theta - 1 = \cot^2\theta$ ✓

19.) $\sec x + \tan^2 x \sec x = \sec^3 x$

$\sec x + \tan^2 x \sec x = \sec x\left(1 + \tan^2 x\right) = \sec x\left(\sec^2 x\right)$

$= \sec^3 x$

22.) $\frac{6\csc^2\theta - 7\csc\theta - 3}{1 + 3\csc\theta} = 2\csc\theta - 3$

Factor: $6x^2 - 7x - 3$

$\frac{6\csc^2\theta - 7\csc\theta - 3}{1 + 3\csc\theta} = \frac{\cancel{(3\csc\theta + 1)}(2\csc\theta - 3)}{\cancel{1 + 3\csc\theta}}$

$= \boxed{2\csc\theta - 3}$ ✓

24) $\frac{1 + 2\sec\theta}{\sec\theta} = 2 + \cos\theta$

$\frac{1 + 2\sec\theta}{\sec\theta} = \frac{1 + 2\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}} \cdot \frac{\cos\theta}{\cos\theta} = \frac{\cos\theta + 2}{1} = \cos\theta + 2$ ✓

26.) $\dfrac{\tan\alpha + \cot\alpha}{\tan\alpha} - \dfrac{\cot\alpha + \tan\alpha}{\cot\alpha} = \cot^2\alpha - \tan^2\alpha$

Note: $\tan\alpha \cdot \cot\alpha = 1$ (why?)

$$\frac{\tan\alpha + \cot\alpha}{\tan\alpha} - \frac{\cot\alpha + \tan\alpha}{\cot\alpha} = \frac{\tan\alpha + \cot\alpha}{\tan\alpha}\cdot\frac{\cot\alpha}{\cot\alpha} - \frac{\cot\alpha + \tan\alpha}{\cot\alpha}\cdot\frac{\tan\alpha}{\tan\alpha}$$

$$= \frac{1 + \cot^2\alpha}{1} - \frac{1 + \tan^2\alpha}{1}$$

$$= \cancel{1} + \cot^2\alpha - (\cancel{1} + \tan^2\alpha)$$

$$= \cot^2\alpha - \tan^2\alpha \checkmark$$

30.) $\dfrac{\sec\beta}{\sin\beta} - \dfrac{\sin\beta}{\sec\beta} = \dfrac{\tan^2\beta + \cos^2\beta}{\tan\beta}$

$$\frac{\sec\beta}{\sin\beta} - \frac{\sin\beta}{\sec\beta} =$$

$$= \frac{\sec^2\beta - \sin^2\beta}{\sin\beta\,\sec\beta} \longrightarrow \sin\beta\cdot\frac{1}{\cos\beta} = \tan\beta$$

$$= \frac{\sec^2\beta - \sin^2\beta}{\tan\beta}$$

pythagorean identities

$$= \frac{(\tan^2\beta + 1) - (1 - \cos^2\beta)}{\tan\beta}$$

$$= \frac{\tan^2\beta + \cos^2\beta}{\tan\beta} \checkmark$$

32.) $\dfrac{\sec t - 1}{\tan t} = \dfrac{\tan t}{\sec t + 1}$

$$\frac{\tan t}{\sec t + 1}\cdot\frac{\sec t - 1}{\sec t - 1} = \frac{\tan t(\sec t - 1)}{\sec^2 t - 1} = \frac{\cancel{\tan t}(\sec t - 1)}{\tan^2 t} = \frac{\sec t - 1}{\tan t} \checkmark$$

35.) $\tan(-x)\cos(x) = -\sin x$

$$\tan(-x)\cos(x) = -\tan(x)\cos(x) = -\frac{\sin(x)}{\cancel{\cos(x)}}\cancel{\cos(x)} = -\sin(x)$$ ✓

39.) $$\frac{\cos^2 t + 3\cos t - 10}{\cos t + 5} = \frac{1-2\sec t}{\sec t}$$

factor: $x^2+3x-10$

$$\frac{\cos^2 t + 3\cos t - 10}{\cos t + 5} = \frac{\cancel{(\cos t + 5)}(\cos t - 2)}{\cancel{\cos t + 5}} = \cos t - 2$$

$$= \frac{1}{\sec t} - 2 = \frac{1}{\sec t} - \frac{2\sec t}{\sec t} = \frac{1-2\sec t}{\sec t}$$

40.) $$1 + \frac{1-\cot^2 x}{1+\cot^2 x} = 2\sin^2 x$$

Note: $1+\cot^2 x = \csc^2 x$

and, $1-\cot^2 x = 1-(\csc^2 x - 1) = 2 - \csc^2 x$

$$1 + \frac{2-\csc^2 x}{\csc^2 x} = \frac{\csc^2 x}{\csc^2 x} + \frac{2-\csc^2 x}{\csc^2 x}$$

$$= \frac{\cancel{\csc^2 x} + 2 - \cancel{\csc^2 x}}{\csc^2 x} = \frac{2}{\csc^2 x}$$

$$= 2\sin^2 x$$ ✓