

HW 3 E/O Solutions

1. $x^4 + 3x^3 + 2x^2 = 0$

$$x^2(x^2 + 3x + 2) = 0$$

$$x^2(x+1)(x+2) = 0$$

$$\boxed{x = 0, -1, -2}$$

$x = 0, -1, -2$

$(p, p = x)$

3. $\frac{1}{x+3} + \frac{x}{x+4} = \frac{x^2-4}{x^2+7x+12}$

$$\frac{x+4 + x(x+3)}{(x+4)(x+3)} = \frac{(x+2)(x-2)}{(x+4)(x+3)}$$

$$\frac{x^2 + 4x + 4}{(x+4)(x+3)} = \frac{(x+2)(x-2)}{(x+4)(x+3)}$$

$$\boxed{(x+2)^2} = (x+2)(x-2) \quad \text{where } x \neq -4, -3$$

If $x = -2$, both sides are zero. If $x \neq -2$, then

$$(x+2) = (x-2)$$

$$x = x - 4$$

$$0 = -4 \quad \text{Not possible.}$$

$$\boxed{x = -2}$$

Alternatively, we can see here that there is no number that is equal to itself minus 4.

4: $(x+4)^2 + 13(x+4) + 36 = 0$

$$u = x+4$$

$$u^2 + 13u + 36 = 0$$

$$(u+4)(u+9) = 0$$

$$u = -4, -9$$

$$x+4 = -4 \quad | \quad x+4 = -9$$

$$\boxed{x = -8}$$

$$\boxed{x = -13}$$

2. $x^4 + 3x^3 - 2x^2 - 6x = 0$

$$x^3(x+3) - 2x(x+3) = 0$$

$$(x^3 - 2x)(x+3) = 0$$

$$x(x^2 - 2)(x+3) = 0$$

$$x(x - \sqrt{2})(x + \sqrt{2})(x+3) = 0$$

$$\boxed{x = 0, \sqrt{2}, -\sqrt{2}, -3}$$

$(\frac{x^2-2}{x}) \cdot (x+3) = 0$

$0 = 0 - 2 + 3x - 2$

$0 = 0 - 2 + 3x - 2$

$0 = 0 - 2 + 3x - 2$

5: $\left(\frac{x}{x-3}\right)^2 - \frac{6x}{x-3} + 8 = 0$

$$u = \frac{x}{x-3}$$

$$u^2 - 6u + 8 = 0$$

$$(u-2)(u-4) = 0$$

$$u = 2, 4$$

$$x = 2(x-3) \quad | \quad x = 4(x-3)$$

$$-x = -6$$

$$\boxed{x = 6}$$

$$-3x = -12$$

$$\boxed{x = 4}$$

$$6. 3x^4 + 2x^8 - 6 = 0$$

$$0 = x^4 \cdot 3x^0 + 2x^8 + 0x^4$$

$$0 = 3u + 2u^2 - 6 = 0 \quad (u = x^4)$$

$$u = \frac{-3 \pm \sqrt{9 + 48}}{4}$$

$$u = \left(\frac{-3}{4} + \frac{\sqrt{57}}{4}, \frac{-3}{4} - \frac{\sqrt{57}}{4} \right)$$

$$x^4 = \frac{-3}{4} + \frac{\sqrt{57}}{4}, \frac{-3}{4} - \frac{\sqrt{57}}{4}$$

$$x = \left(\frac{-3}{4} + \frac{\sqrt{57}}{4} \right)^{1/4}, \left(\frac{-3}{4} - \frac{\sqrt{57}}{4} \right)^{1/4}$$

$$7. x + x^{1/2} - 6 = 0$$

$$0 = u = x^{1/2} \cdot x^2 + 1 \cdot x$$

$$0 = u^2 + u - 6 = 0$$

$$0 = (u+3)(u-2) = 0$$

$$u = -3, 2$$

$$x^{1/2} = -3, 2$$

$$x = 9, 4$$

$$\text{Check: } 4 + 2 - 6 = 0 \checkmark$$

$9 + 3 - 6 \neq 0$ unless we take $x^{1/2}$ to be negative.

$$8. x^8 + 15x^4 - 16 = 0$$

$$u = x^4$$

$$u^2 + 15u - 16 = 0$$

$$(u+16)(u-1) = 0$$

$$u = -16, 1$$

$$x^4 = -16, 1$$

$$x^4 + 16 = 0$$

$$(x^2 + 4i)(x^2 - 4i) = 0$$

$$x = \frac{\pm \sqrt{-16i}}{2} = \pm 2\sqrt{-i}$$

$$x = \pm 2\sqrt{-i}$$

Not real

$$x^4 - 1 = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$$(x-1)(x+1)(x+i)(x-i) = 0$$

$$x = 1, -1, i, -i$$

only real solutions: $x = 1, -1$

$$0 = (p-u)(q-u)$$

$$p, q = u$$

$$(p-x) = x$$

$$p = 2x$$

$$p = x^2$$

$$(q-x) = x$$

$$q = 2x$$

$$q = x^2$$

$$9. x^{3/2} + 3x^{1/2} - 10x^{-1/2} = 0 \quad (\text{mult. by } x^{1/2})$$

$$(x^2 + 3x - 10) = 0 \quad \text{where } x \neq 0$$

$$(x+5)(x-2) = 0 \quad x \neq 0$$

$$x = -5, 2 \quad x \neq 0$$

$$10. (x^6 - 9x^4 - 4x^2 + 36) = 0$$

$$p = x^2 = x^4(x^2 - 9) - 4(x^2 - 9) = 0$$

$$p = (x^4 - 4)(x^2 - 9) = 0$$

$$(x^2 + 2)(x^2 - 2)(x + 3)(x - 3) = 0$$

$$(x + \sqrt{2}i)(x - \sqrt{2}i)(x + \sqrt{2})(x - \sqrt{2})(x + 3)(x - 3) = 0$$

real solutions

$$x = \pm \sqrt{2}, \pm 3$$

$$0 = (p+u)(p-u)$$

$$p = p = u$$

$$p = p + x$$

$$p = x$$

$$p = p + x$$

$$p = x$$

252
 -1 -252
 -2 -126
 -3 -84
 -4 -63
 -6 -42
 -7 -36
 -9 -28
 -12 -21

11. $2x + \sqrt{x+1} = 8$

$\sqrt{x+1} = 8 - 2x$

$x+1 = (8-2x)^2$

$x+1 = 64 - 32x + 4x^2$

$0 = 63 - 33x + 4x^2$

$s+t = -33$

$st = 252$

$0 = 4x^2 - 12x - 21x + 63$

$0 = 4x(x-3) - 21(x-3)$

$0 = (4x-21)(x-3)$

$0 = 2(2x-1)(x-3)$

$x = \frac{1}{2}, 3$

12. $4x^2 - 4x = x^3$

$0 = 4x - 4x^2 + x^3$

$0 = x(4 - 4x + x^2)$

$0 = x(x-2)(x-2)$

$x = 0, 2$

13. $\sqrt{x+\sqrt{x+2}} = 2$

$x+\sqrt{x+2} = 4 \quad x \neq 0$

$\sqrt{x+2} = 4-x$

$x+2 = 16-8x+x^2$

$0 = 14-9x+x^2$

$0 = (x-2)(x-7)$

$x = 2, 7$

14. $500 = 10t + 36\sqrt{t} + 196$

$0 = 10t + 36\sqrt{t} - 304$

$u = t^{1/2}$

$0 = 10u + 36u - 304$

$u = \frac{-36 \pm \sqrt{1296 + 12160}}{20}$

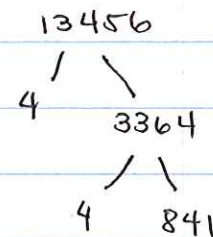
$u = \frac{-36 \pm \sqrt{13456}}{20}$

$u = \frac{-9 \pm \sqrt{841}}{5}$

$t = \left(\frac{-9}{5} + \frac{\sqrt{841}}{5}\right)^2, \left(\frac{-9}{5} - \frac{\sqrt{841}}{5}\right)^2$

$t = 16$

57.76



36
 36

 276
 1080

 1296

$$\begin{aligned}
 15. \quad & x^2 = x^2 - 4x^2 \quad \cdot 1 \\
 & x^2 + 4x^2 - x^2 = 0 \\
 & (4x^2 + x^2 - x^2) \cdot x = 0 \\
 & (4-1)(x-1) \cdot x = 0 \\
 & \boxed{x = 0, 1, 3}
 \end{aligned}$$

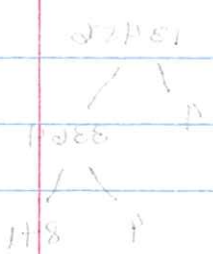
$$\begin{aligned}
 13. \quad & x^2 = \sqrt{x^2 + 4} \quad \cdot 1 \\
 & x^2 - 4 = \sqrt{x^2 + 4} \\
 & (x^2 - 4)^2 = x^2 + 4 \\
 & x^4 - 8x^2 + 16 = x^2 + 4 \\
 & x^4 - 9x^2 + 12 = 0 \\
 & (x^2 - 3)(x^2 - 4) = 0 \\
 & \boxed{x = 2, 3}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & x^2 = \sqrt{x^2 + 1} \quad \cdot 1 \\
 & x^2 - 1 = \sqrt{x^2 + 1} \\
 & (x^2 - 1)^2 = x^2 + 1 \\
 & x^4 - 2x^2 + 1 = x^2 + 1 \\
 & x^4 - 3x^2 = 0 \\
 & x^2(x^2 - 3) = 0 \\
 & x^2 = 0 \quad \text{or} \quad x^2 - 3 = 0 \\
 & x = 0 \quad \text{or} \quad x = \pm\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & x^2 = \sqrt{x^2 + 1} \quad \cdot 1 \\
 & x^2 - 1 = \sqrt{x^2 + 1} \\
 & (x^2 - 1)^2 = x^2 + 1 \\
 & x^4 - 2x^2 + 1 = x^2 + 1 \\
 & x^4 - 3x^2 = 0 \\
 & x^2(x^2 - 3) = 0 \\
 & \boxed{x = 0, \pm\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & 200 = 10x + 30\sqrt{x} + 10x \\
 & 0 = 10x + 30\sqrt{x} - 200 \\
 & x = 10
 \end{aligned}$$

$$\begin{aligned}
 & 0 = 10x + 30\sqrt{x} - 200 \\
 & x = 10 \pm \sqrt{100 + 1200} \\
 & x = 10 \pm 35 \\
 & x = 45 \quad \text{or} \quad x = -25
 \end{aligned}$$



$$\begin{aligned}
 & f(x) = \frac{1}{2} \left(\sqrt{x^2 + 1} + x \right) \\
 & f'(x) = \frac{1}{2} \left(\frac{x}{\sqrt{x^2 + 1}} + 1 \right) \\
 & f''(x) = \frac{1}{2} \left(\frac{1}{\sqrt{x^2 + 1}} - \frac{x^2}{(x^2 + 1)^{3/2}} \right)
 \end{aligned}$$