E.C. Packet
for T.G. Break

Section 7.3 The Graphs of the Tangent, Cosecant, Secant, and Cotangent Functions.

OBJECTIVE 1: Understanding the Graph of the Tangent Function and its Properties

Recall that $y = \tan x$ can equivalently be written $y = \frac{\sin x}{\cos x}$. Recall also that division by zero is never allowed. Therefore, the domain of $f(x) = \tan x$ is not the set of all real numbers.

Complete the tables below.

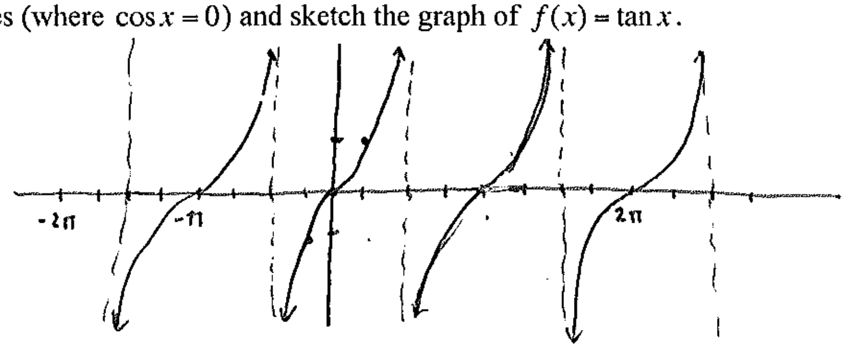
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \tan x$	0	∞	0	$-\infty$	0

x	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
$y = \tan x$	1	-1	1	-1

Using information above and the graph of $f(x) = \tan x$, write the characteristics of the Tangent Function:

- The domain is all #s except $x \neq \frac{n\pi}{2}$ where n is odd
- The range is $(-\infty, \infty)$ (interval notation)
- The function is periodic with a period of $P =$ π.
- The principal cycle (includes 0) of the graph occurs on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$
- The function has infinitely many vertical asymptotes with equations $x = \frac{n\pi}{2}$ where n is odd
- The y-intercept is $(0,0)$
- For each cycle there is one center point. The x-coordinates of the center points are also the x-intercepts or zeros are of the form $n\pi$ where n is an integer
- For each cycle there are two halfway points. The halfway point to the left of the x-intercept has a y-coordinate of -1, and the halfway point to the right of the x-intercept has a y-coordinate of 1
- The function is odd (odd or even or neither), which means $\tan(-x) =$ $-\tan(x)$. The graph is symmetric about the origin.
- The graph of each cycle of $y = \tan x$ is one-to-one (this will take on more meaning in 7.4)

Draw a set of axes. Determine appropriate tic marks for x and y, plot points, dash and label vertical asymptotes (where $\cos x = 0$) and sketch the graph of $f(x) = \tan x$.



7.3.16 $y = 4\tan(-2x+\pi)+1$

$= 4\tan\left(-2\left(x-\frac{\pi}{2}\right)\right)+1$

$= -4\tan\left(2\left(x-\frac{\pi}{2}\right)\right)+1$

period $= \frac{\pi}{2}$

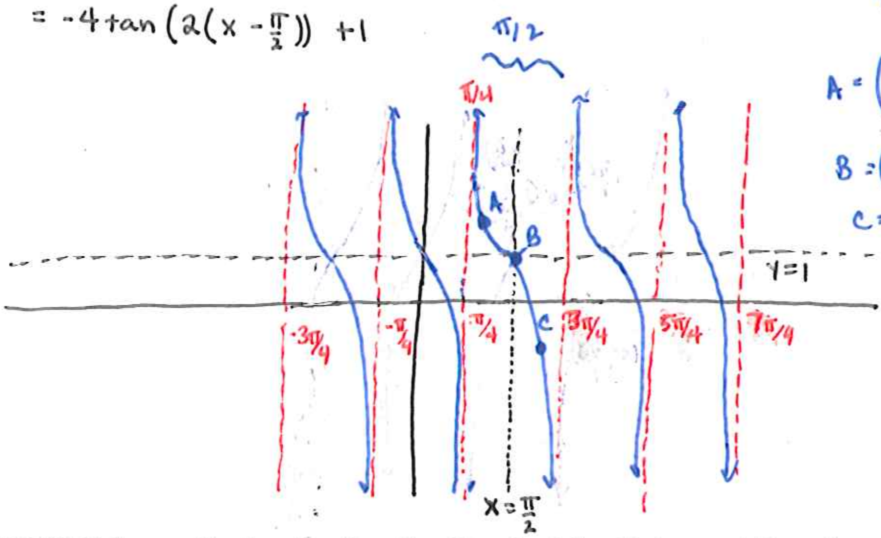
shift $= \frac{\pi}{2}$

$\frac{\pi}{4} - \frac{\pi}{8}$ ↓

$A = \left(\frac{3\pi}{8}, 5\right)$

$B = \left(\frac{\pi}{2}, 1\right)$

$C = \left(\frac{5\pi}{8}, -3\right)$



OBJECTIVE 3: Understanding the Graph of the Cotangent Function and its Properties

Recall that $y = \cot x$ can equivalently be written $y = \frac{\cos x}{\sin x}$. Recall also that division by zero is never allowed. Therefore, the domain of $f(x) = \cot x$ is not the set of all real numbers.

Complete the tables below.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cot x$	DNE	0	DNE	0	DNE

x	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
$y = \cot x$	1	-1	1	-1

Using information above and the graph of $f(x) = \cot x$, write the characteristics of the Cotangent Function:

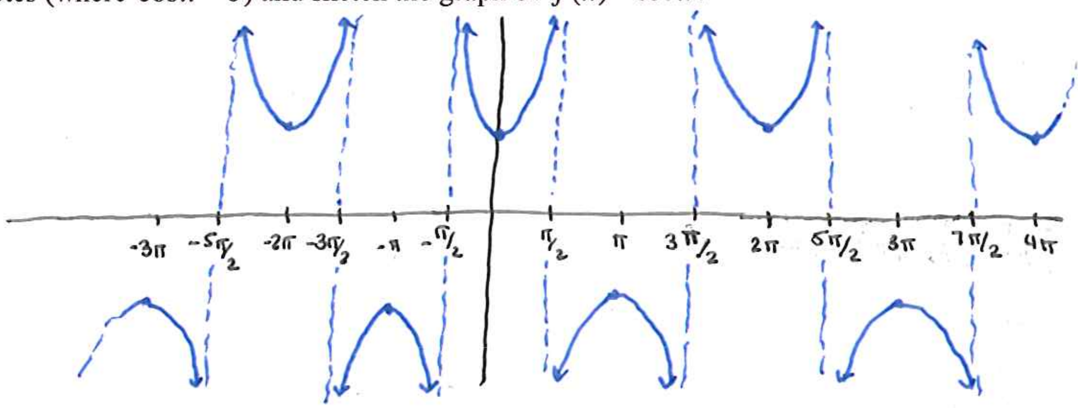
- The domain is $x \neq \frac{n\pi}{2}$ where n is odd
- The range is $(-\infty, \infty)$ (interval notation)
- The function is periodic with a period of $P =$ π.
- The principal cycle (includes 0) of the graph occurs on the interval $(0, \pi)$
- The function has infinitely many vertical asymptotes with equations $n\pi$

Rewrite $y = \sec x$ as $y = \dfrac{1}{\cos x}$ and use knowledge of the graph of the cosine function to graph the secant function.

For $f(x) = \sec x$,

- The domain is $x \neq \frac{n\pi}{2}$
- The range is $(-\infty, \infty)$ (interval notation)
- The function is periodic with a period of $P =$ 2π.
- The function has infinitely many vertical asymptotes with equations $\frac{n\pi}{2}$ where n is odd
- The function obtains a relative maximum at $n\pi$ where n is an even #
  - The relative maximum value is -1
- The function obtains a relative minimum at $n\pi$ where n is an odd #.
  - The relative minimum value is 1
- The function is even (odd or even or neither), which means $\sec(-x) =$ $\sec(x)$. The graph is symmetric about y-axis.

Draw a set of axes. Determine appropriate tic marks for x and y, plot points, dash and label vertical asymptotes (where $\cos x = 0$) and sketch the graph of $f(x) = \sec x$.



To sketch functions of the form $y = A\csc(Bx - C) + D$ and $y = A\sec(Bx - C) + D$, first sketch the graph of the corresponding reciprocal function.

Steps for Sketching Functions of the Form $y = A\csc(Bx - C) + D$ and $y = A\sec(Bx - C) + D$

Step 1: Lightly sketch at least two cycles of the corresponding reciprocal function using the process outlined in Section 7.2. If $D \neq 0$, lightly sketch two reciprocal functions, one with $D = 0$ and one with $D \neq 0$.

Step 2: Sketch the vertical asymptotes. The vertical asymptotes will correspond to the x-intercepts of the reciprocal function $y = A\sin(Bx - C)$ or $y = A\cos(Bx - C)$.

Step 3: Plot all maximum and minimum points on the graph of $y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$.

Step 4: Draw smooth curves through each point from Step 3, making sure to approach the vertical asymptotes.

In the examples below, determine the equations of the vertical asymptotes and all relative maximum and relative minimum points of two cycles of each function and then sketch its graph.

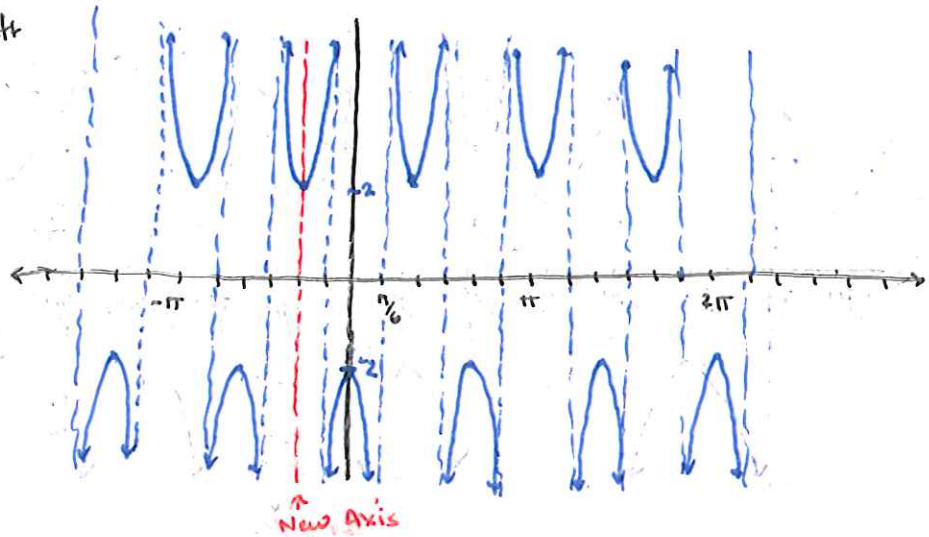
When I graded these, all I cared about was the graph.

7.3.45 $y = 2\sec(3x + \pi)$

$= 2\sec(3(x + \pi/3))$

Phase Shift $= \pi/3$ to the left

half period $= \pi/3$



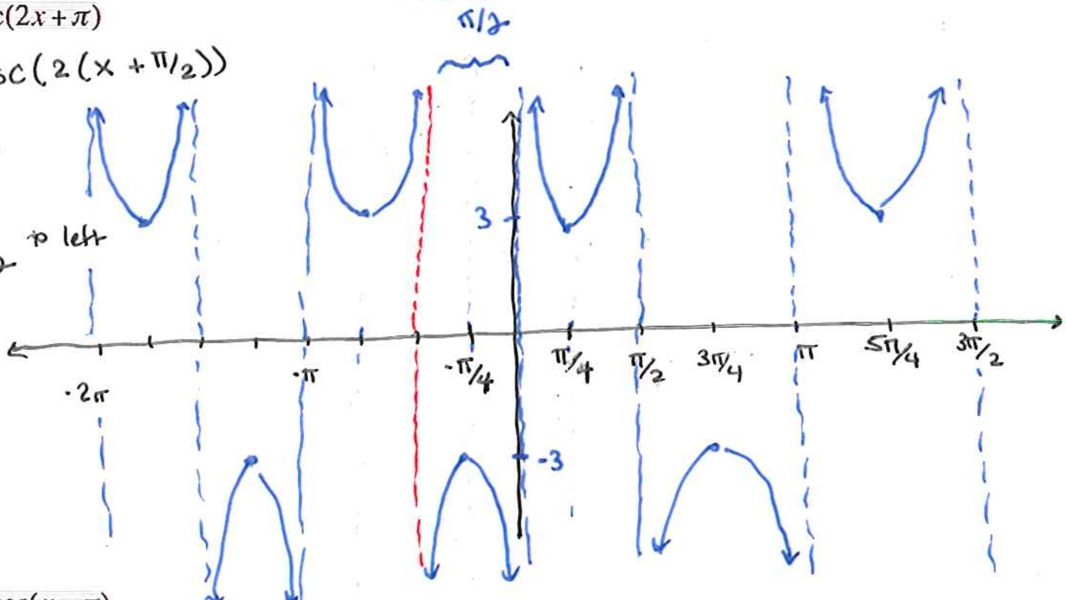
7.3.46 $y = -3\csc(2x + \pi)$

$= -3\csc(2(x + \pi/2))$

$-3 \Rightarrow$ flipped

half period $= \pi/2$

P. Shift $= \pi/2$ to left



7.3.49 $y = 5 - 2\sec(x - \pi)$

$= -2\sec(x - \pi) + 5$

half period $= \pi$

P. Shift $= \pi$ to the right