Section 7.2 More on Graphs of Sine and Cosine: Phase Shift

OBJECTIVE 1: Sketching Graphs of the Form $y = \sin(x - C)$ and $y = \cos(x - C)$

The third factor that can affect the graph of a sine or cosine curve is known as phase shift. In general the number $\frac{C}{B}$ is known as the **phase shift.**

In Objective 1, $A = 1$ and B=1, so amplitude = 1, period is $P = \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$, and phase shift $= \frac{C}{1} = C$.

The x-coordinates of the quarter points are $C,\ C + \frac{\pi}{2},\ C + \pi,\ C + \frac{3\pi}{2},\ C + 2\pi$.

7.2.2 Determine the amplitude, range, period, and phase shift, then sketch the graph of the function $y = \cos\left(x + \frac{\pi}{2}\right)$.

amplitude = 1

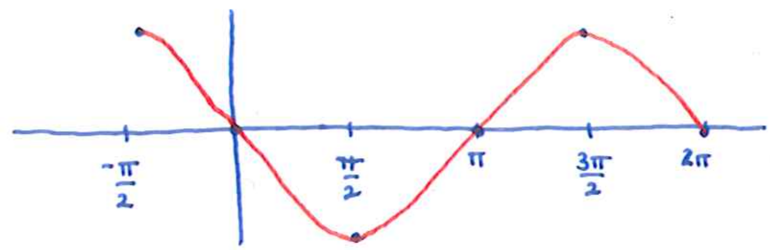
range = $[-1, 1]$

period = 2π

phase shift = $\pi/2$

→ (or $-\pi/2$)

I will accept both



OBJECTIVE 2: Sketching Graphs of the Form $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$

Steps for Sketching Functions of the Form and $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$

Step 1: If $B < 0$, rewrite the function in an equivalent form such that $B > 0$. Use the odd property of the sine function or the even property of the cosine function. **It is often helpful to factor out B when $B \neq 1$ such that**

$$y = A\sin(Bx - C) = A\sin\left(B\left(x - \frac{C}{B}\right)\right) \text{ or } y = A\cos(Bx - C) = A\cos\left(B\left(x - \frac{C}{B}\right)\right)$$

In the factored form, the amplitude, period, and phase shift are more apparent.

Step 2: The amplitude is $|A|$. The range is $[-|A|, |A|]$.

Step 3: The period is $P = \frac{2\pi}{B}$.

Step 4: The phase shift is $\frac{C}{B}$.

Step 5: The x-coordinate of the first quarter point is $\frac{C}{B}$. The x-coordinate of the last quarter point is $\frac{C}{B} + P$. An interval for one complete cycle is $\left[\frac{C}{B}, \frac{C}{B} + P\right]$. Subdivide this interval into 4 equal subintervals of length $P \div 4$ by starting with $\frac{C}{B}$ and adding $(P \div 4)$ to the x-coordinate of each successive quarter point.

Step 6: Multiply the y-coordinates of the quarter points of $y = \sin x$ or $y = \cos x$ by A to determine the y-coordinates of the corresponding quarter points for $y = A\sin(Bx - C)$ and $y = A\cos(Bx - C)$

Step 7: Connect the quarter points to obtain one complete cycle.

In the following exercises, determine the amplitude, range, period, and phase shift and then sketch the graph.

7.2.7 $y = -\sin(2x - \pi)$

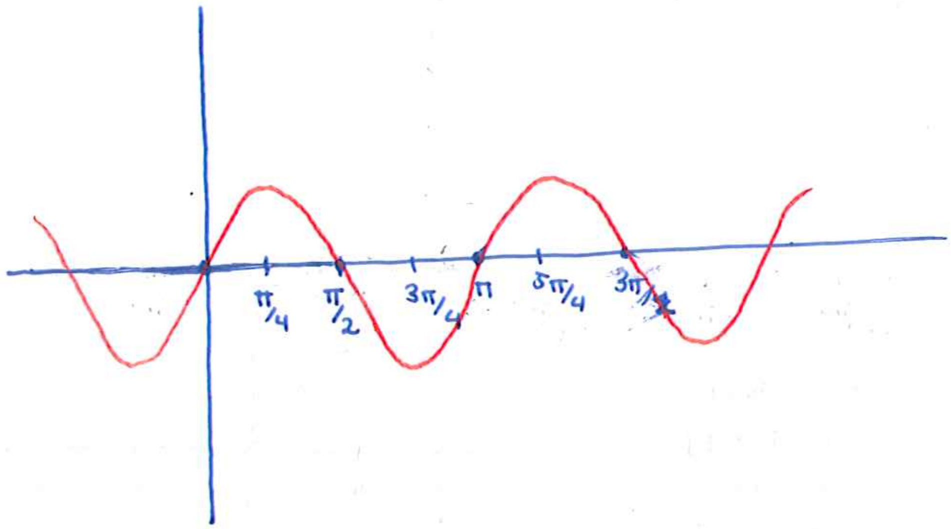
$= -\sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

amplitude = 1

range = $[-1, 1]$

period = $\frac{2\pi}{2} = \pi$

phase shift = $\frac{\pi}{2}$



- $2x - \pi = 0$
  $x = \pi/2$
- $2x - \pi = \pi/2$
  $x = 3\pi/4$
- $2x - \pi = \pi$
  $x = \pi$

7.2.10 $y = 4\cos(-2x - \pi)$

$= 4\cos\left(-2\left(x + \pi/2\right)\right)$

$= 4\cos\left(2\left(x + \pi/2\right)\right)$

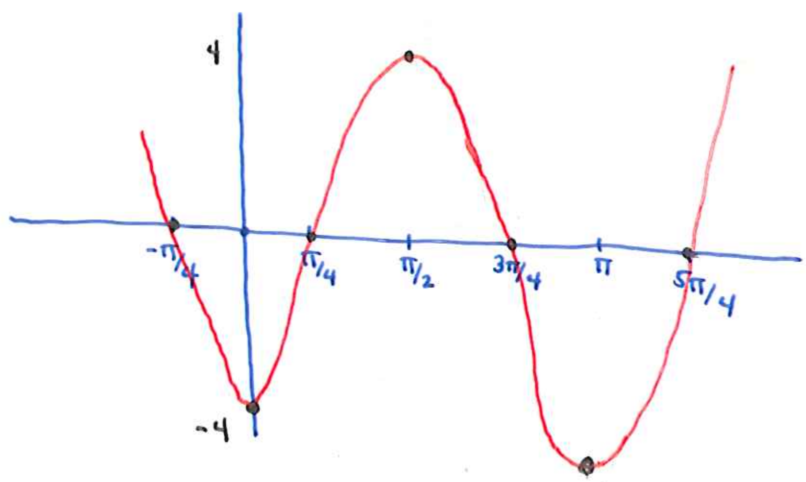
amplitude = 4

range = $[-4, 4]$

period = $\frac{2\pi}{2} = \pi$

phase shift = $\pi/2$

(or $-\pi/2$)



OBJECTIVE 3: Sketching Graphs of the Form $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$

Steps for Sketching Functions of the Form $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$

Step 1: If $B < 0$, rewrite the function in an equivalent form such that $B > 0$. Use the odd property of the sine function or the even property of the cosine function.

It is often helpful to factor out B when $B \neq 1$ such that

$y = A\sin(Bx - C) + D = A\sin\left(B(x - \frac{C}{B})\right) + D$ or $y = A\cos(Bx - C) + D = A\cos\left(B(x - \frac{C}{B})\right) + D$.

In the factored form, the amplitude, period, and phase shift are more apparent.

Step 2: The amplitude is $|A|$. The range is $[-|A| + D, |A| + D]$.

Step 3: The period is $P = \dfrac{2\pi}{B}$.

Step 4: The phase shift is $\dfrac{C}{B}$.

Step 5: The x-coordinate of the first quarter point is $\dfrac{C}{B}$. The x-coordinate of the last quarter point is $\dfrac{C}{B} + P$. An interval for one complete cycle is $\left[\dfrac{C}{B}, \dfrac{C}{B} + P\right]$. Subdivide this interval into 4 equal subintervals of length $P \div 4$ by starting with $\dfrac{C}{B}$ and adding $(P \div 4)$ to the x-coordinate of each successive quarter point.

Step 6: Multiply the y-coordinates of the quarter points of $y = \sin x$ or $y = \cos x$ by A then add D to determine the y-coordinates of the corresponding quarter points for $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$.

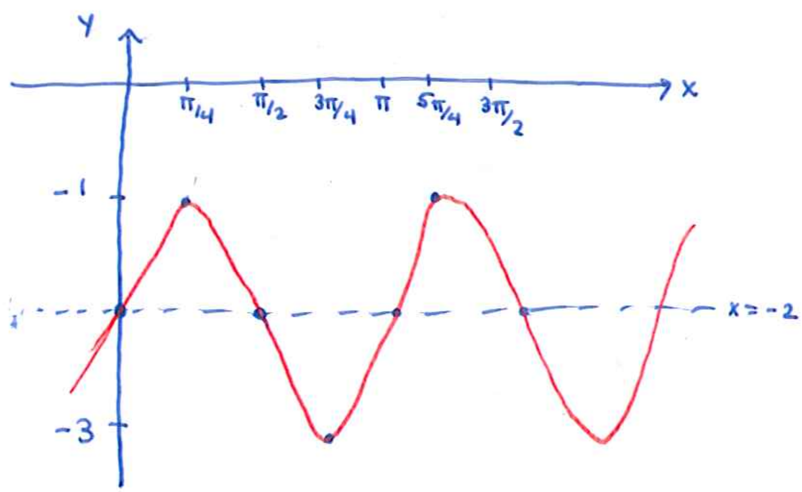
Step 7: Connect the quarter points to obtain one complete cycle.

In the following exercises, determine the amplitude, range, period, and phase shift and then sketch the graph.

7.2.17 $y = -\sin(2x - \pi) - 2$

$= -\sin\left(2\left(x - \frac{\pi}{2}\right)\right) - 2$

- amplitude $= 1$
- range $= [-1-2, 1-2] = [-3, -1]$
- period $= \frac{2\pi}{2} = \pi$
- phase shift $= \pi/2$



7.2.20 $y = 4\cos(-2x-\pi)+3$

$= 4\cos(-2(x+\pi/2))+3$

$= 4\cos(2(x+\pi/2))+3$

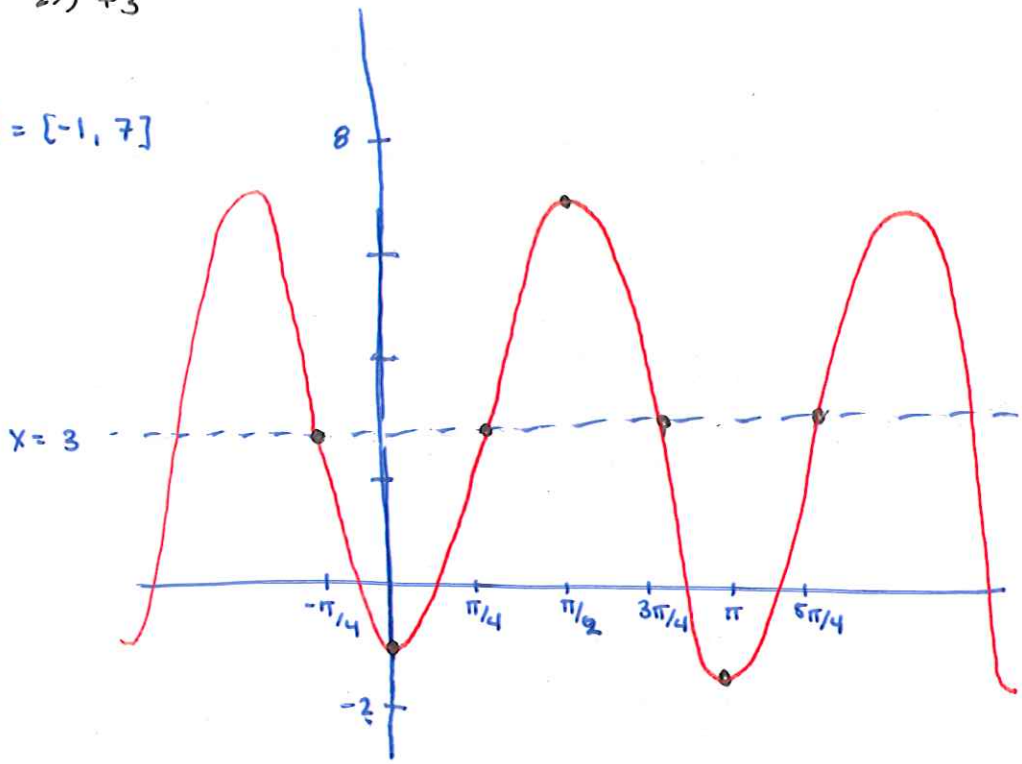
amplitude = 4

range = $[-4+3, 4+3] = [-1, 7]$

period = $\frac{2\pi}{2} = \pi$

phase shift = $\pi/2$

(or $-\pi/2$)



7.2.23 $y = -4\cos\left(-x-\frac{\pi}{3}\right)+3$

$= -4\cos\left(-\left(x+\frac{\pi}{3}\right)\right)+3$

$= -4\cos\left(x+\frac{\pi}{3}\right)+3$

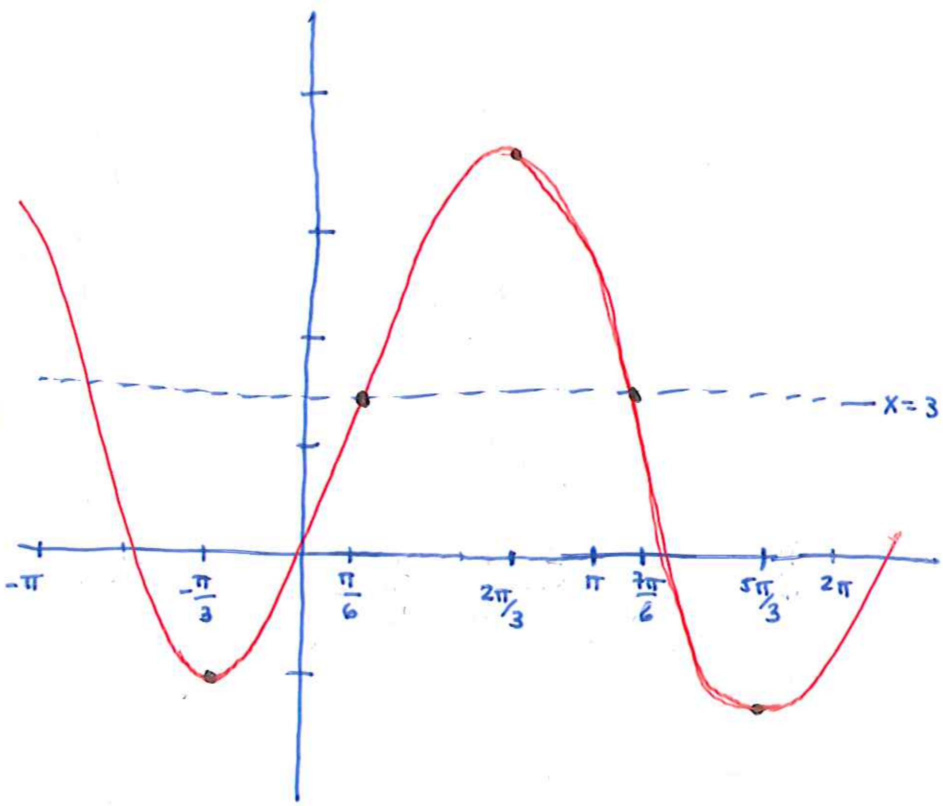
amplitude = 4

range = $[-4+3, 4+3] = [-1, 7]$

period = 2π

phase shift = $\pi/3$

(or $-\pi/3$)



- $x + \pi/3 = \pi/2$
  $x = \pi/6$
- $x + \pi/3 = \pi$
  $x = 2\pi/3$
- $x + \pi/3 = 3\pi/2$
  $x = 7\pi/6$
- $x + \pi/3 = 2\pi$
  $x = 5\pi/3$