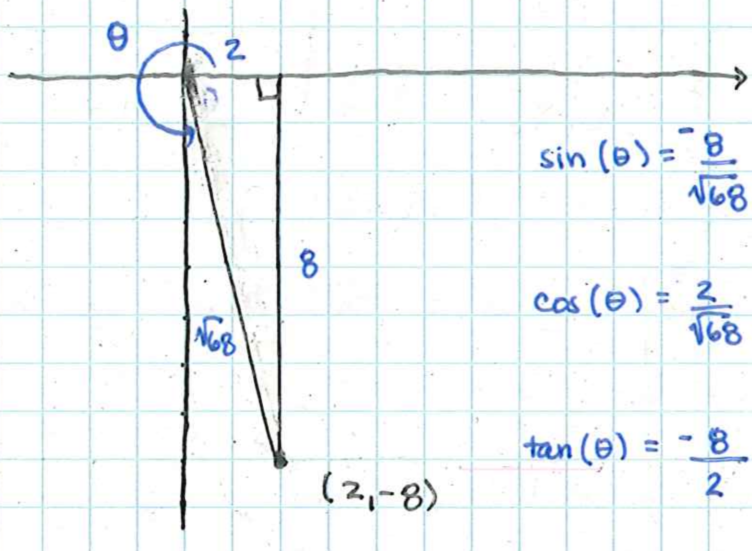
Hw 32 Solutions

6.5.14 The point $(2,-8)$ lies on the terminal side of an angle θ. Find the exact value of the six trig functions of θ.



$\sin(\theta) = \frac{-8}{\sqrt{68}} = \frac{-4}{\sqrt{17}}$ $\csc(\theta) = \frac{-\sqrt{17}}{4}$

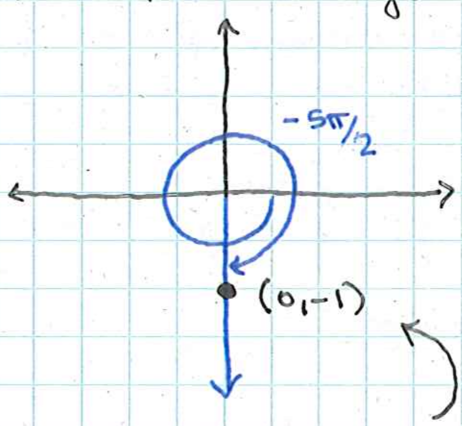
$\cos(\theta) = \frac{2}{\sqrt{68}} = \frac{1}{\sqrt{17}}$ $\sec(\theta) = \sqrt{17}$

$\tan(\theta) = \frac{-8}{2} = -4$ $\cot(\theta) = \frac{-1}{4}$

6.5. SbS - 17. Let $\theta = \frac{-5\pi}{2}$

a. Where does the terminal side of angle θ lie?

$\frac{-5\pi}{2} + \frac{4\pi}{2} = \frac{-\pi}{2}$



It lies along the negative portion of the y-axis.

b. Give the coordinates of the ordered pair location on the terminal side of θ that corresponds to $r = 1$.

If the radius is 1, it needs to be 1 away from the origin. So the only option is $(0,-1)$

c. Using the values of $x, y,$ and r, what is the general angle definition of $\sin\theta$?

on the unit circle, $x = \cos\theta$ and $y = \sin\theta$.

If $r \neq 1$, then we want
$$x^2 + y^2 = r^2$$

since $\cos^2\theta + \sin^2\theta = 1$ compare

we can multiply by r!

$$r^2\cos^2\theta + r^2\sin^2\theta = r^2$$

Then $x = r\cos\theta$ and $y = r\sin\theta$

So $\boxed{\sin\theta = \frac{y}{r}}$ (This is opposite, y, over hypotenuse, r)

d. Find the exact value of $\sin\theta$ w/o using a calculator.

$$\sin\left(\frac{5\pi}{2}\right) = \frac{y}{r} = \frac{-1}{1} = -1$$

6.5.30 Let θ be an angle in standard position. Name the quadrant in which θ lies if $\underline{\tan\theta > 0}$ and $\underline{\csc\theta > 0}$

$0 < \tan\theta = \frac{\sin\theta}{\cos\theta}$ either both are positive or both are negative

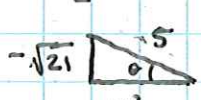
$\csc\theta = \frac{1}{\sin\theta}$ positive

Therefore, both $\sin\theta$ and $\cos\theta$ are <u>positive</u>. Hence θ lies in Quadrant I.

6.5.34 Find the value of $\csc\theta$ if $\sec\theta = -\frac{5}{2}$ and $\tan\theta > 0$

$0 < \tan\theta = \frac{\sin\theta}{\cos\theta}$ Both negative or both positive.

$-\frac{5}{2} = \sec\theta = \frac{1}{\cos\theta}$ Negative. So $\csc\theta = \frac{1}{\sin\theta} < 0$.



$$\boxed{\csc\theta = -\frac{5}{\sqrt{21}}}$$

6.5.SbS-44 Given $\theta = -229^\circ$

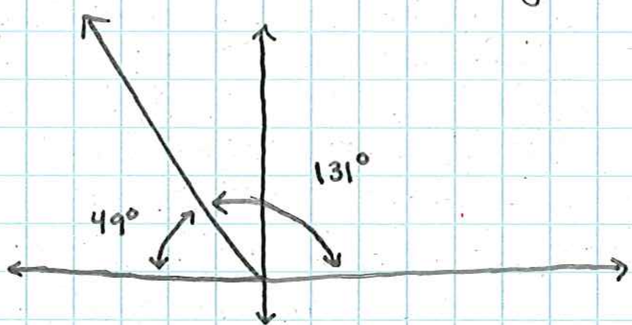
- What is the angle of least positive measure coterminal with θ?

$-229^\circ + 360^\circ = \boxed{131^\circ}$ $\theta_c = 131^\circ$

- What quadrant does the terminal side of θ_c lie?

Since $180^\circ > 131^\circ > 90^\circ$, it's in Quadrant II.

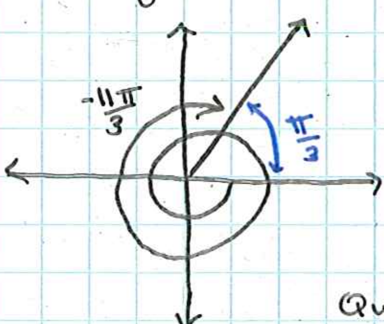
- What is the reference angle?



The reference angle is the smallest angle near the x-axis.
So it's $\boxed{49^\circ}$

6.5.SbS-57 Given $\theta = \frac{-11\pi}{3}$

- In what quadrant is the terminal side of θ?



$\frac{-11\pi}{3} + \frac{6\pi}{3} = \frac{-5\pi}{3}$

$\frac{-5\pi}{3} + \frac{6\pi}{3} = \frac{\pi}{3}$

Quadrant I

- Is the sine function positive or negative in this quadrant?

$\boxed{\text{positive}}$

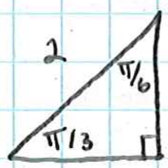
- Determine the reference angle?

$\frac{\pi}{3}$ (smallest positive angle nearest x-axis.)

- The expression $\sin\left(-\frac{11\pi}{3}\right)$ is equivalent to

$$\sin\left(\frac{\pi}{3}\right)$$

- Find the exact value of $\sin\left(-\frac{11\pi}{3}\right)$:



$$\boxed{\sin\left(\frac{\cancel{-}11\pi}{3}\right) = \frac{\sqrt{3}}{2}}$$