A functions is a special relation between variable quantities. A correct understanding of the language and uses of functions is essential for success in this course and all courses to come.

- Definition of a Function
- Domain and Range

Definition of a Function

A ***function*** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B. The set A is the ***domain*** of f (i.e. the input of the function), and the set $\{f(x) \mid x \in A\}$ is the ***range*** of f (i.e. the output of the function).

1. Let $f(x) = |x|$. Evaluate $f(-2)$ and $f(1)$. Find the domain and range of f.

2. Let $f(x) = (x-1)^2$. Evaluate $f(-2)$, $f(1)$, $f(2a)$, and $f(x^3)$. Find the domain and range of f.

3. Let $g(x) = \dfrac{x}{|x|}$. Evaluate $g(-2)$, $g(1)$, $g(2a)$, and $g(x^2)$. Find the domain and range of g.

4. Find the domain of $f(x) = \dfrac{\sqrt{x^2 - 5x + 6}}{x}$.

Section 3.2 Properties of a Function's Graph

Objective 1: Determining the Intercepts of a Function

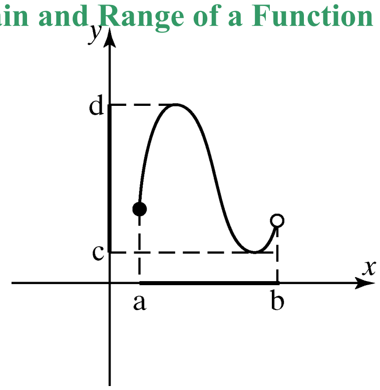
An **intercept** of a function is a point on the graph of a function where the graph either crosses or touches a coordinate axis. There are two types of intercepts:
1) The ***y*-intercept**, which is the *y*-coordinate of the point where the graph crosses or touches the *y*-axis.
2) The ***x*-intercepts**, which are the *x*-coordinates of the points where the graph crosses or touches the *x*-axis.

The *y*-intercept:
A function can have **at most** one *y*-intercept. The *y*-intercept exists if $x = 0$ is in the domain of the function. The *y*-intercept can be found by evaluating $f(0)$.

The *x*-intercept(s):
A function may have several (even infinitely many) *x*-intercepts. The *x*-intercepts, also called **real zeros,** can be found by finding all *real solutions* to the equation $f(x) = 0$. Although a function may have several zeros, only the real zeros are *x*-intercepts.

Objective 2: Determining the Domain and Range of a Function from its Graph

The domain is the interval $[a,b)$ while the range is the interval $\left[c,d\right]$.

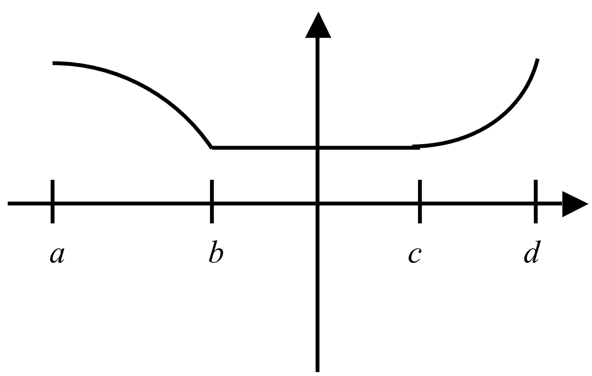


Objective 3: Determining Where a Function is Increasing, Decreasing or Constant

The graph of *f* rises from left to right on the interval in which *f* is increasing.

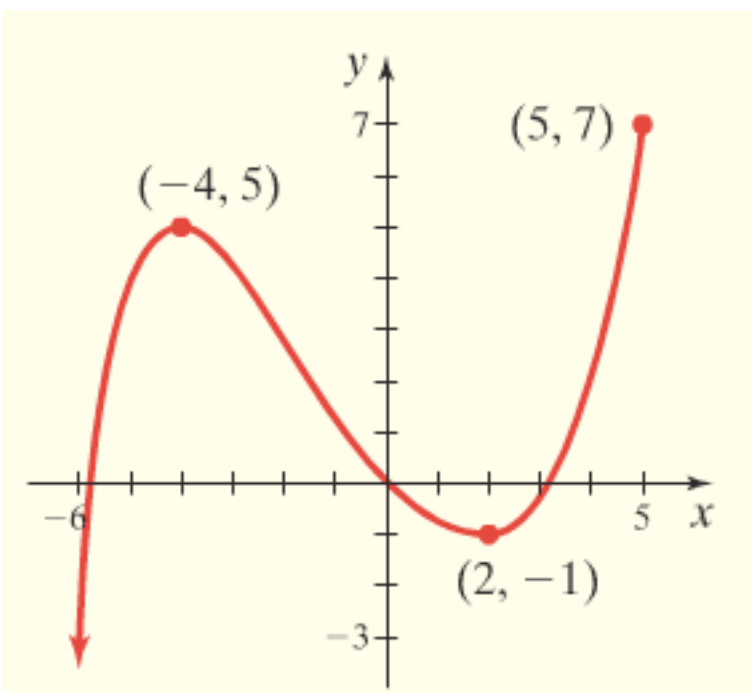
The graph of *f* falls from left to right on the interval in which *f* is decreasing.

A graph is constant on an open interval if the values of $f(x)$ do not change as x gets larger on the interval. In this case, the graph is a horizontal line on the interval.



The function is increasing on the interval (c, d).
The function is decreasing on the interval (a, b).
The function is constant on the interval (b, c).

3.2.13
Determine the interval(s) for which the function is (a) increasing, (b) decreasing, and (c) constant. Type your answer in interval notation. Use a comma to separate answers as needed.

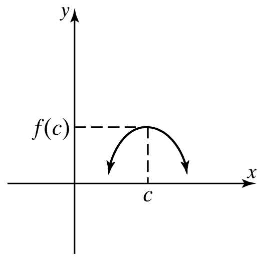


Objective 4: Determining Relative Maximum and Relative Minimum Values of a Function

When a function changes from increasing to decreasing at a point $(c, f(c))$, then f is said to have a **relative maximum** at $x = c$. The **relative maximum value** is $f(c)$.

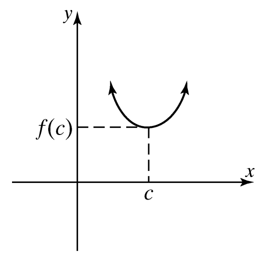
Similarly, when a function changes from decreasing to increasing at a point $(c, f(c))$, then f is said to have a **relative minimum** at $x = c$. The **relative minimum value** is $f(c)$.

(a)



The relative maximum occurs at $x = c$, the relative maximum value is $f(c)$.

(b)



The relative minimum occurs at $x = c$, the relative minimum value is $f(c)$.

The word "relative" indicates that the function obtains a maximum or minimum value relative to some open interval. It is not necessarily the maximum (or minimum) value of the function on the entire domain.

CAUTION A relative maximum cannot occur at an endpoint and must occur in an open interval. This applies to a relative minimum as well.

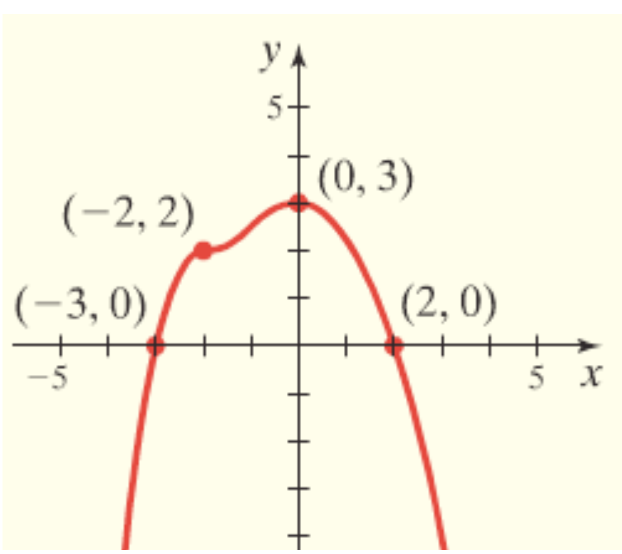
3.2.18
Use the graph to find the following information:

Find the number(s) for which the function obtains a relative minimum.

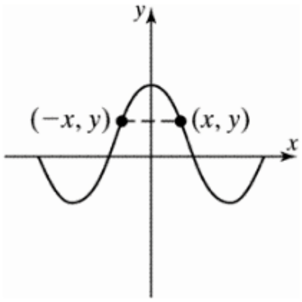
Find the relative minimum value.

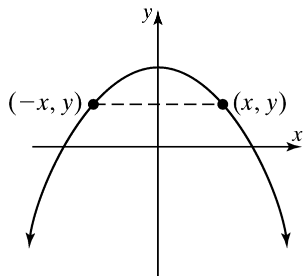
Find the number(s) for which the function obtains a relative maximum.

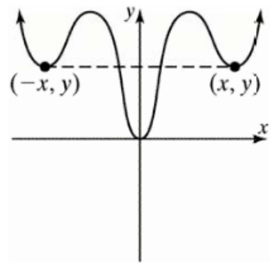
Find the relative maximum value.



Objective 5: Determining if a Function is Even, Odd or Neither

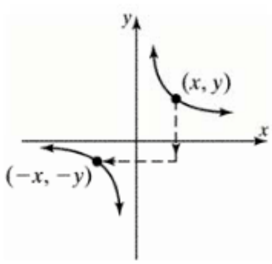


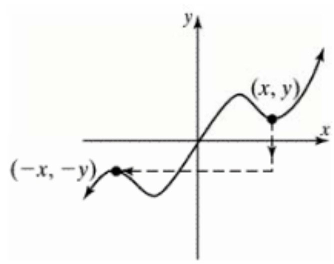


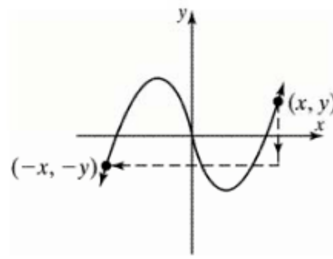


Definition	**Even Functions**

A function f is **even** if for every x in the domain, $f(x) = f(-x)$. Even functions are symmetric about the y-axis. For each point (x, y) on the graph, the point $(-x, y)$ is also on the graph.





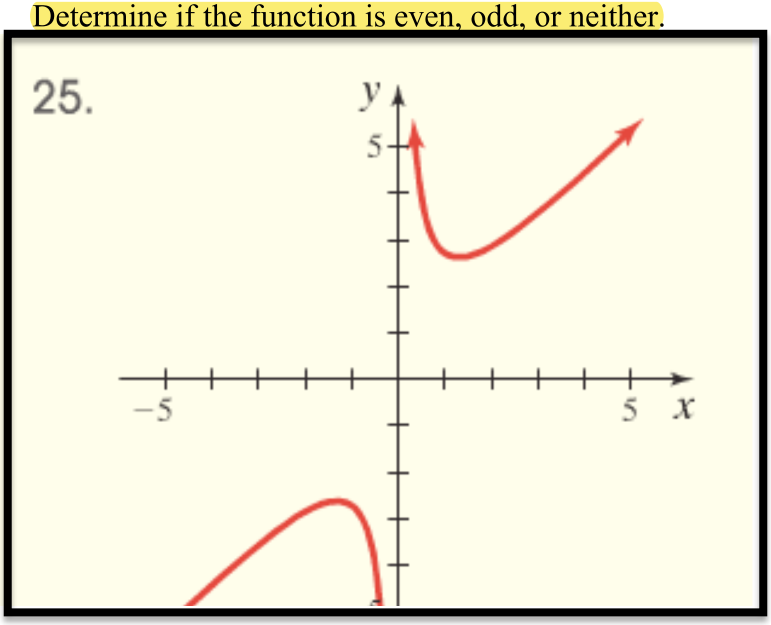


Definition	**Odd Functions**

A function f is **odd** if for every x in the domain, $-f(x) = f(-x)$. Odd functions are symmetric about the origin. For each point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

3.2. 25, 27, 29

Determine if the function is even, odd, or neither.



27. $g(x) = x^5 - x^2$

29. $F(x) = \dfrac{2x^2 - 4}{x}$

Objective 6: Determining Information about a Function from a Graph

3.2.31
Use the graph to find the following information:

a. What is the domain, in interval notation?

b. What is the range, in interval notation?

c. On what intervals is the function increasing, decreasing, or constant?

d. For what value(s) of x does f obtain a relative minimum? What are the relative minima?

e. For what value(s) of x does f obtain a relative maximum? What are the relative maxima?

f. What are the real zeros of f?

g. What is the y-intercept?

h. For what values of x is $f(x)$ less than or equal to 0?

i. For how many values of x is $f(x)$ = -3?

j. What is the value of f (2)?