Section 3.1 Relations and Functions

Objective 1: Understanding the Definitions of Relations and Functions

Definition **Relation**

A **relation** is a correspondence between two sets *A* and *B* such that each element of set *A* corresponds to one or more elements of set *B*. Set *A* is called the **domain** of the relation and set *B* is called the **range** of the relation.

Definition **Function**

A **function** is a relation such that for each element in the domain, there corresponds *exactly one and only one* element in the range. In other words, a function is a well-defined relation.

The elements of the domain and range are typically listed in ascending order when using set notation.

Objective 2: Determine if Equations Represent Functions

To determine if an equation represents a function, we must show that for any value of in the domain, there is exactly one corresponding value in the range.

Objective 3: Using Function Notation; Evaluating Functions

When an equation is explicitly solved for *y*, we say that "*y* is a function of *x*" or that the variable *y* depends on the variable *x*. Thus, *x* is the independent variable and *y* is the dependent variable.

The symbol $f(x)$ does not mean *f* times *x*. The notation $f(x)$ refers to the value of the function at *x*.

The expression $(-1)^2$ does not equal -1^2.

The expression $\frac{f(x+h)-f(x)}{h}$ is called the **difference quotient** and is very important in calculus.

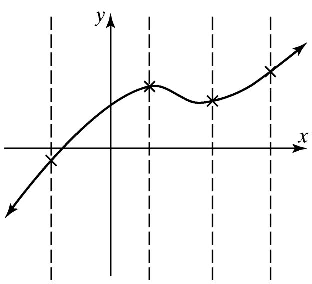
3.1.16

Evaluate the following function at the values *f(x+1), f(x+h), and* $\frac{f(x+h)-f(x)}{h}$.

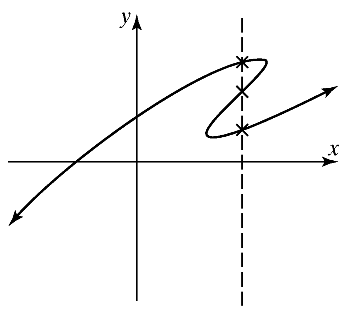
$f(x)=x^2-2x$

Objective 4: Using the Vertical Line Test

The Vertical Line Test	A graph in the Cartesian plane is the graph of a function if and only if no vertical line intersects the graph more than once.



This graph is a function.
(No vertical line intersects the graph more than once).



This graph is not a function.
(The graph does not pass the vertical line test).

Objective 5: Determining the Domain of a Function Given the Equation

The domain of a function $y = f(x)$ is the set of all values of x for which the function is defined.

It is very helpful to classify a function to determine its domain.

Definition **Polynomial Function**

The function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ is a **polynomial function** of degree n where n is a nonnegative integer and $a_0, a_1, a_2, \ldots, a_n$ are real numbers.
The domain of every polynomial function is $(-\infty, \infty)$.

Many functions can have restricted domains.

Definition **Rational Function**

A **rational function** is a function of the form $f(x) = \frac{g(x)}{h(x)}$ where g and h are polynomial functions such that $h(x) \neq 0$.
The domain of a rational function is the set of all real numbers such that $h(x) \neq 0$.

Definition **Root Function**

The function $f(x) = \sqrt[n]{g(x)}$ is a **root function** where n is a positive integer.

1. If n is *even*, the domain is the solution to the inequality $g(x) \geq 0$.
2. If n is *odd*, the domain is the set of all real numbers for which $g(x)$ is defined.

Class of function	Form	Domain
Polynomial Functions	$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$	Domain is $(-\infty, \infty)$
Rational Functions	$f(x) = \dfrac{g(x)}{h(x)}$ where g and $h \neq 0$ are polynomial functions	Domain is all real numbers such that $h(x) \neq 0$
Root Functions	$f(x) = \sqrt[n]{g(x)}$, where $g(x)$ is a function and n is a positive integer	1. If n is even, the domain is the solution to the inequality $g(x) \geq 0$. 2. If n is odd, the domain is the set of all real numbers for which g is defined.

3.1.32, 35, 36

Classify the given function as a polynomial function, rational function, or root function and then find the domain. Write the domain in interval notation. Simplify your answer. Use integers or fractions for any numbers in the expression.

32. $f(t) = \sqrt[3]{t-5}$

35. $g(x) = 2x - 1$

36. $h(x) = \dfrac{x^2 + 4}{x^2 + x - 42}$