Section 8.1 Trigonometric Identities

This is the "games" portion of Trigonometry. Correctly using algebra rules along with fundamental identities allows you to solve mathematical puzzles. Have Fun!

OBJECTIVE 1: Reviewing the Fundamental Identities

Look back at Section 6.4 if necessary to write the Fundamental Properties below:

The Quotient Identities

1. 2.

The Reciprocal Identities

1. 2.

3. 4.

5. 6.

The Pythagorean Identities

1. 2. 3.

The Odd Properties

1. 2.

3. 4.

The Even Properties

1. 2.

OBJECTIVE 2: Substituting Known Identities to Verify an Identity

Verifying trigonometric identities is the process of showing that the expression on one side of the identity can be simplified to be the exact expression that appears on the other side. We typically start with what appears to be the more complicated side of the identity and attempt to use known identities to transform that side into the trigonometric expression that appears on the other side of the identity. There is no one correct approach.

HINT 1: Watch for squared trigonometric expressions because they may indicate the use of a Pythagorean Identity.

EXAMPLES. Verify each identity.

8.1.8 $1+\cot^2(-\theta)=\csc^2\theta$

8.1.11 $(3\cos\theta-4\sin\theta)^2+(4\cos\theta+3\sin\theta)^2=25$

OBJECTIVE 3: Changing to Sines and Cosines to Verify an Identity

When we have simple forms of multiple trigonometric function on both side of the identity, it is often efficient first to rewrite all of the functions on one side of the identity in terms of sines and cosines and then simplify.

EXAMPLES. Verify each identity.

8.1.15 $\cos\theta\tan\theta\csc\theta=1$

8.1.18 $\dfrac{\csc\theta}{\sin\theta}-\cos(-\theta)\sec(-\theta)=\cot^2\theta$

OBJECTIVE 4: Factoring to Verify an Identity

Algebra rules for factoring may be used for trigonometric expressions. You should comfortable factoring trinomials of the form ax^2+bx+c. It is also worthwhile reminding yourself of the following special factoring formulas (write them below):

Difference of Two Squares:

Perfect Square Formulas (2):

Sum of Two Cubes:

Difference of Two Cubes:

EXAMPLES. Verify each identity.

8.1.19 $\sec x + \tan^2 x \sec x = \sec^3 x$

8.1.22 $\dfrac{6\csc^2\theta - 7\csc\theta - 3}{1 + 3\csc\theta} = 2\csc\theta - 3$

OBJECTIVE 5: Separating a Single Quotient into Multiple Quotients to Verify an Identity

When one side of a trigonometric identity is a quotient of the form $\dfrac{A+B}{C}$ where C is a single trigonometric expression, then it is often advantageous to begin by separating the quotient into multiple quotients. We do this using the algebra property $\dfrac{A+B}{C} = \dfrac{A}{C} + \dfrac{B}{C}$.

EXAMPLES. Verify each identity.

8.1.24 $\dfrac{1 + 2\sec\theta}{\sec\theta} = 2 + \cos\theta$

8.1.26 $\dfrac{\tan\alpha + \cot\alpha}{\tan\alpha} - \dfrac{\cot\alpha + \tan\alpha}{\cot\alpha} = \cot^2\alpha - \tan^2\alpha$

OBJECTIVE 6: Combining Fractional Expressions to Verify an Identity

When two or more fractional expressions appear on one side of an identity and the other side contains only one term, it may be useful to begin by combining all fractions into a single quotient using a common denominator, substituting if necessary, and then simplifying.

EXAMPLES. Verify each identity.

8.1.30 $\frac{\sec\beta}{\sin\beta} - \frac{\sin\beta}{\sec\beta} = \frac{\tan^2\beta + \cos^2\beta}{\tan\beta}$

OBJECTIVE 7: Multiplying by Conjugates to Verify an Identity

Given an expression of the form $A + B$, we define its conjugate as the expression $A - B$. When verifying an identity, if the numerator or denominator of one of the expressions is of the form $A + B$, try first multiplying the numerator and denominator of the expression by $\frac{A-B}{A-B} = 1$

8.1.32 $\frac{\sec t - 1}{\tan t} = \frac{\tan t}{\sec t + 1}$

OBJECTIVE 8: Summarizing the Techniques for Verifying Identities

The techniques in Objectives 1-8 give you strategies to use when verifying identities.

Try some:

8.1.35 $\tan(-x)\cos x = -\sin x$

8.1.39 $\frac{\cos^2 t + 3\cos t - 10}{\cos t + 5} = \frac{1 - 2\sec t}{\sec t}$

8.1.40 $1 + \frac{1 - \cot^2 x}{1 + \cot^2 x} = 2\sin^2 x$