Section 6.6 The Unit Circle

OBJECTIVE 1: Understanding the Definition of the Unit Circle

The **Unit Circle** is a circle centered at the origin with a radius of 1 unit whose equation is given by $x^2 + y^2 = 1.$

Do this in class: Draw a graph of the Unit Circle. Label the points of the unit circle that lie on the axes.

NOTE: The points on the axes are the only four points whose coordinates are integers. In order to find missing coordinates, use the equation of the circle $x^2 + y^2 = 1.$ Give coordinates in exact form unless otherwise instructed.

6.6.5 Determine the missing coordinate of a point that lies on the graph of the unit circle, given the quadrant in which it is located.

$\left(\frac{1}{3\sqrt{5}}, y\right)$; Quadrant IV

OBJECTIVE 2: Using Symmetry to Determine Points that Lie on the Unit Circle

The graph of the unit circle is symmetric about both axes and the origin.

For example the point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ lies on the unit circle. Therefore, the points $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, and $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ also lie on the unit circle.

6.6.8 Verify that the given point lies on the graph of the unit circle. Then use symmetry to find three other points that also lie on the graph of the unit circle.

$\left(-\frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right)$

OBJECTIVE 3: Understanding the Unit Circle; Definitions of the Trigonometric Functions

We have defined trigonometric functions in terms of right triangles (6.4) and in terms of general angles (6.5). We can also define trigonometric functions in terms of the unit circle.

Given the formula for arc length (6.2) $s = r\theta$

Use (for the unit circle) $r = 1$ and $\theta = t$

Then the formula becomes $s = t$

IN THE UNIT CIRCLE, the arc length of a sector is exactly equal to the measure of the central angle.

Do this in class: Draw the unit circle and locate a point P (x, y) in QI. Label the point P (x, y), the angle t and the radius 1.

Give the Unit Circle Definitions of the Trigonometric Functions in terms of *t, x,* and *y*.

6.6.9 The point $\left(\frac{1}{7}, -\frac{4\sqrt{3}}{7}\right)$ lies on the graph of the unit circled and corresponds to a real number t. Find the values of the six trigonometric functions of t.

OBJECTIVE 4: Using the Unit Circle to Evaluate Trigonometric Functions at Increments of $\frac{\pi}{2}$

On the unit circle by labeling only the points (x, y) on the x-axis and on the y-axis, all the trigonometric functions of the quadrantal angles can be easily determined.

Do this in class: Draw a graph of the Unit Circle. Label the points of the unit circle that lie on the axes.

6.6.17 Use the unit circle to determine the value of $\sin(-6\pi)$ or state that it is undefined.

OBJECTIVE 5: Using the Unit Circle to Evaluate Trigonometric Functions for increments of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$.

Using triangles with hypotenuse = 1, similar to the special triangles, the values of the trigonometric functions for the special angles can be determined.

You should become familiar with the Unit Circle provided (next set of notes).

NOTE: It is not necessary to MEMORIZE the unit circle, but it IS necessary to be able to quickly determine (by drawing triangles) the values of the coordinates for any point corresponding with a special angle or quadrantal angle or multiple of a special angle or quadrantal angle.

In the following exercises use the unit circle to determine the value of the given expressions or state that it is undefined.

6.6.21 $\cos\left(\frac{3\pi}{4}\right)$

6.6.23 $\csc(315°)$

6.6.27 $\cot\left(-\frac{5\pi}{4}\right)$

6.6.29 $\tan\left(\frac{11\pi}{3}\right)$

6.6.40 $\csc(5\pi)$

6.6.42 $\cos\left(\frac{5\pi}{2}\right)$

6.6.50 $\sec\left(-\frac{\pi}{4}\right)$