

Section 6.5 Trigonometric Functions of General Angles

OBJECTIVE 1: Understanding the Four Families of Special Angles

In Section 6.4 we defined the six trigonometric functions of an angle θ where θ was an **acute angle** of a right triangle. In this section, we will extend the definitions of the six trigonometric functions to include *all* angles for which each function is defined. We start by introducing four groups, or families, of special angles.

The Quadrantal Family of Angles

Angles belonging to the quadrantal family must be coterminal with $0, \frac{\pi}{2}, \pi,$ or $\frac{3\pi}{2}$.

The $\frac{\pi}{3}$ Family of Angles

Angles belonging to the $\frac{\pi}{3}$ family must be coterminal with $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3},$ or $\frac{5\pi}{3}$.

The $\frac{\pi}{6}$ Family of Angles

Angles belonging to the $\frac{\pi}{6}$ family must be coterminal with $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$.

The $\frac{\pi}{4}$ Family of Angles

Angles belonging to the $\frac{\pi}{4}$ family must be coterminal with $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$.

OBJECTIVE 2: Understanding the Definitions of the Trigonometric Functions of General Angles

The right triangle definition of the trigonometric functions only apply to **acute angles**. We now want to define the trigonometric functions of *general angles*. The term general angle is used here to indicate that these angles are not restricted in size and can be either positive angles, negative angles, or zero.

In order to develop the definitions of trigonometric functions of general angles, let $P(x, y)$ be a point lying on the terminal side of an acute angle θ in standard position and let $r > 0$ represent the distance from the **origin** to point P . By the **distance formula**, $r = \sqrt{x^2 + y^2}$. We use the right triangle definition of the trigonometric functions to write the trigonometric ratios in terms of $x, y,$ and r .

DO THIS IN CLASS:

a. Draw a point P in QII and then draw a triangle by dropping a perpendicular segment from P to the negative x -axis. Label the angle θ (the angle in standard position (an obtuse angle), the reference angle θ_r (an acute angle), label the sides of the triangle, x , y , and label the hypotenuse r . (More about the reference angle in Objective 5.)

b. Now write the six trigonometric functions of θ , by using the triangle formed (using x , y , and r).

tan

6.5.14 The point (2, -8) lies on the terminal side of an angle θ . Find the exact value of the six trigonometric functions of θ .

OBJECTIVE 3: Finding the Values of the Trigonometric Functions of Quadrantal Angles

If θ is an angle belonging to the **quadrantal** family, then the terminal side of the angle lies along an axis. Any point lying on the terminal side of an angle coterminal to 0 radians (0°) or π radians (180°) has a y -coordinate of 0.

Any point lying on the terminal side of an angle coterminal to $\frac{\pi}{2}$ radians (90°) or $\frac{3\pi}{2}$ radians (270°) has an x -coordinate of 0.

CAUTION: Since division by zero is undefined, it is important to note that for the quadrantal angles, when x is zero (at $\theta = \underline{\hspace{2cm}}$), any trig function that has x in the denominator will be undefined and when y is zero (at $\theta = \underline{\hspace{2cm}}$), any trig function that has y in the denominator will be undefined.

Write the trig functions that are undefined for $\theta = 0$ and $\theta = \pi$.

Write the trig functions that are undefined for $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$.

Memorizing a table of trig values of quadrantal angles is not advised. Instead, draw graphs of the angles (see Figure 46 in the text)

6.5.SbS-17. Let $\theta = -\frac{5\pi}{2}$

a. Where does the terminal side of angle θ lie?

- b. Give the coordinates of the ordered pair located on the terminal side of θ that correspond to $r = 1$. (Write an ordered pair.)
- c. Using the values of x , y , and r , what is the general angle definition of $\sin\theta$?
- d. Find the exact value of $\sin\theta$ without using a calculator.

OBJECTIVE 4: Understanding the Signs of the Trigonometric functions

To determine the value of trigonometric functions of an angle θ we MUST know the quadrant in which the terminal side of the angle lies.

NOTE: The value of r is always positive.

Draw graphs to determine the trigonometric functions of an angle. The signs will follow from the values of x and y .

Do this in class:

Draw four separate graphs showing general θ in QI, QII, QIII, and QIV respectively. Label the sides x and y and the angle θ . Note that x is _____ (pos or neg) and note that y is _____ (pos or neg). For θ in each of the four quadrants write the trig functions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ in terms of x , y , and r , and in terms of + and —, determining by division rules where the function is positive (+) or negative (—).

QI

QII

QIII

QIV

6.5.30 Let θ be an angle in standard position. Name the quadrant in which θ lies if $\tan\theta > 0$ and $\csc\theta > 0$.

6.5.34 Find the value of $\csc\theta$ if $\sec\theta = -\frac{5}{2}$ and $\tan\theta > 0$.

OBJECTIVE 5: Determining Reference Angles

Definition: The **reference angle** θ_r , is the positive acute angle associated with a given angle θ . The reference angle is formed by the “nearest” x -axis and the terminal side of θ .

Do this in class: Draw 4 separate graphs of angles θ with terminal side in QI, QII, QIII and QIV, respectively. Label θ and θ_r

QI

QII

QIII

QIV

6.5.SbS-44 Given $\theta = -229^\circ$:

What is the angle of least positive measure coterminal with θ ? $\theta_c =$ _____

In what quadrant does the terminal side of θ_c lie? _____

What is the reference angle? $\theta_r =$ _____

OBJECTIVE 6: Evaluating Trigonometric Functions of Angles Belonging to the

$\frac{\pi}{6}$, $\frac{\pi}{3}$ or $\frac{\pi}{4}$ Families.

6.5.SbS-57 Given $\theta = -\frac{11\pi}{3}$:

In what quadrant does the terminal side of θ lie? _____

Is the sine function positive or negative in this quadrant? _____

Determine the reference angle? $\theta_r =$ _____

The expression $\sin(-\frac{11\pi}{3})$ is equivalent to _____.

Find the exact value of $\sin(-\frac{11\pi}{3})$: _____