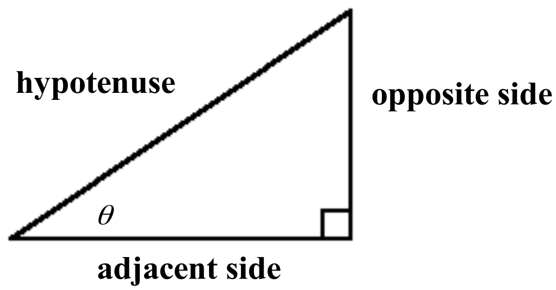
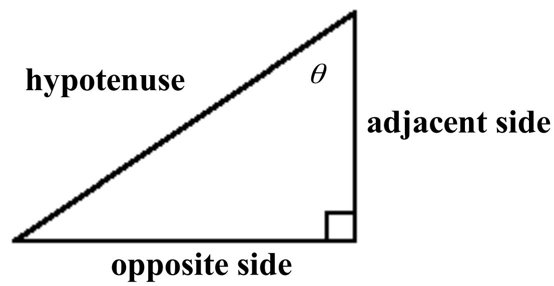
Section 6.4 Right Triangle Trigonometry

OBJECTIVE 1: Understanding the Right Triangle Definitions of the Trigonometric Functions





We will be concerned about the *lengths* of each of these three sides. For simplicity, we will abbreviate the length of the hypotenuse as "*hyp.*" The length of the opposite side will be referred to as *"opp"* and the length of the adjacent side will be denoted as "*adj.*"

Choosing side lengths two at a time, we can create a total of six ratios. These six ratios are

$$\frac{opp}{hyp}, \frac{adj}{hyp}, \frac{opp}{adj}, \frac{hyp}{opp}, \frac{hyp}{adj}, \text{ and } \frac{adj}{opp}.$$

The value of each of these six ratios depends on the measure of the acute angle θ. Thus, the six ratios are functions of the variable θ and are called the **trigonometric functions of the acute angle** θ. For convenience, the six ratios have been given names. Historically, these six trigonometric functions have been named **sine** of theta, **cosine** of theta, **tangent** of theta, **cosecant** of theta, **secant** of theta, and **cotangent** of theta. The six functions are abbreviated as $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$.

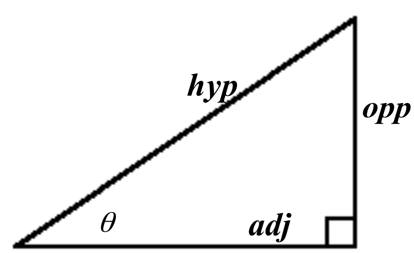
The Right Triangle Definition of the Trigonometric Functions

Given a right triangle with acute angle θ and side lengths of *hyp*, *opp,* and *adj*, the six trigonometric functions of angle θ are defined as follows:

$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$



It is extremely important to memorize the six trigonometric functions. To memorize the ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$, you could use the following silly phrase:

"**S**ome **O**ld **H**orse **C**aught **A**nother **H**orse **T**aking **O**ats **A**way"

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

You might also use the acronym SOHCAHTOA pronounced "So-Kah-Toe-Ah" to help you memorize these functions.

Once you have memorized the ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$, you can easily obtain the ratios of $\csc\theta$, $\sec\theta$, and $\cot\theta$ since the ratios for $\csc\theta$, $\sec\theta$, and $\cot\theta$ are simply the reciprocals of the ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$ respectively.

The value of the six trigonometric functions for a specific acute angle θ will be exactly the same regardless of the size of the triangle (similar triangles).

6.4.10 If θ is an acute angle of a right triangle and if $\sec\theta = \frac{7}{\sqrt{5}}$, then find the values of the remaining five trigonometric functions for angle θ.

OBJECTIVE 2: Using the Special Right Triangles

Check your knowledge:

Draw a $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ $(45°, 45°, 90°)$ triangle with leg length 1.

Draw a $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ $(30°, 60°, 90°)$ triangle with length of the shortest side 1.

Confirm values in Table 1. **NOTE: These values do not have to be rationalized.**

Table 1 The Trigonometric Functions for Acute Angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$

θ	$\frac{\pi}{6}$ $(30°)$	$\frac{\pi}{4}$ $(45°)$	$\frac{\pi}{3}$ $(60°)$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

6.4.18 Use special right triangles to evaluate the expression.

$$\frac{\sin^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{6}}{\sec^2 \frac{\pi}{4}}$$

OBJECTIVE 3: Understanding the fundamental Trigonometric Identities

Trigonometric identities are equalities involving trigonometric expressions that hold true for any angle θ for which all expressions are defined.

The Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Use the Right Triangle Definition of the Trigonometric Functions in Objective 1 to prove one of the Quotient Identities.

The Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Use the Right Triangle Definition of the Trigonometric Functions in Objective 1 to prove one of the Reciprocal Identities.

The Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

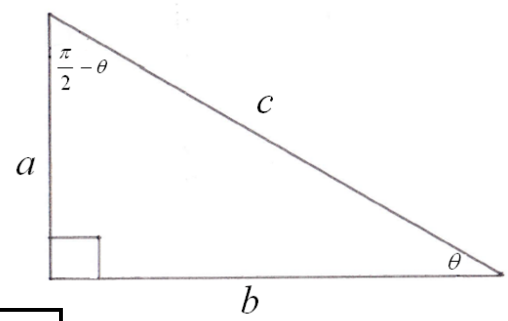
Use the Right Triangle Definition of the Trigonometric Functions in Objective 1 to prove one of the Pythagorean Identities.

6.4.26 Use identities to find the exact value of the trigonometric expression. Assume θ is an acute angle.

$$\tan\frac{5\pi}{12}\left(\frac{1}{\cos^2\frac{5\pi}{12}}-\frac{1}{\cot^2\frac{5\pi}{12}}\right)-\tan\frac{5\pi}{12}$$

OBJECTIVE 4: Understanding Cofunctions

Prove the measures of the acute angles are as shown to the right.



Cofunction Identities

$\sin\theta=\cos\left(\frac{\pi}{2}-\theta\right)$	$\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$
$\tan\theta=\cot\left(\frac{\pi}{2}-\theta\right)$	$\cot\theta=\tan\left(\frac{\pi}{2}-\theta\right)$
$\sec\theta=\csc\left(\frac{\pi}{2}-\theta\right)$	$\csc\theta=\sec\left(\frac{\pi}{2}-\theta\right)$

If angle θ is given in degrees, replace $\frac{\pi}{2}$ with $90°$.

6.4.32 Rewrite the expression $\cos(90°-\theta)\sec\theta$ as one of the six trigonometric functions of acute angle θ.

OBJECTIVE 6: Applications of Right Triangle Trigonometry

Exam questions will use special triangles or will ask students to set up a problem without evaluating to avoid the necessity of calculators.

6.4.40 The angle of elevation to the top of a flagpole is $\frac{3\pi}{10}$ radians from a point on the ground 72 feet away from its base. Find the height of the flagpole. Round to 2 decimal places.

6.4.44 A mine shaft with a circular entrance has been carved into the side of a mountain. From a distance of 100 feet from the base of the mountain, the angle of elevation to the bottom of the circular opening is $27.7°$. The angle of elevation to the top of the opening is $33°$. Determine the diameter of the circular entrance.