Polynomials are expressions such as $x^4 + 3x^2 - 2x + 4$. Polynomial equations can often be solved by factoring. Equations involving radicals can be solved either by substitution or by raising both sides of the equation to a power to remove the radicals.

- Solving by factoring
- Equations involving radicals
- Equations of quadratic type

Solving by Factoring

Find all real solutions of the equation.

1. $x^4 + 3x^3 + 2x^2 = 0$

2. $x^4 + 3x^3 - 2x^2 - 6x = 0$

3. $\frac{1}{x+3} + \frac{x}{x+4} = \frac{x^2-4}{x^2+7x+12}$

Quadratic-Type Equations

4. $(x+4)^2 + 13(x+4) + 36 = 0$

5. $\left(\frac{x}{x-3}\right)^2 - \frac{6x}{x-3} + 8 = 0$

6. $3x^4 + 2x^8 - 6 = 0$

7. $x + x^{1/2} - 6 = 0$

8. $x^8 + 15x^4 - 16 = 0$

9. $x^{3/2} + 3x^{1/2} - 10x^{-1/2} = 0$

10. $x^6 - 9x^4 - 4x^2 + 36$

Radical Equations

11. $2x + \sqrt{x+1} = 8$

12. $\sqrt[3]{4x^2 - 4x} = x$

13. $\sqrt{x + \sqrt{x+2}} = 2$

14. A students debt D (in dollars) can be modeled by the formula

$$D(t) = 10t + 36\sqrt{t} + 196,$$

where t is time (in weeks) since moving to State College. How many weeks will it take for the student's debt to reach $500?

Inequalities are preserved when quantities are added or subtracted from both sides. Multiplying or dividing by a negative reverses the direction of the inequality. Polynomial or rational inequalities are solved by getting 0 on one side of the inequality and then factoring the other side to break up the numberline into test intervals.

- Linear inequalities
- Polynomial inequalities
- Rational inequalities

Linear inequalities

Solve each inequality. Write the solution using interval notation.

1. $3x - 5 \geq 7$

2. $\dfrac{-3x + 4}{2} \leq 2$

3. $3 \leq -2x + 7 \leq 9$

Polynomial Inequalities

4. $(x + 3)(x - 4) < 0$

5. $x^2 \leq -x + 2$

6. $x^3 - 4x \geq 0$

7. $x^5 < x^3$

Rational Inequalities

8. $\dfrac{x+4}{x-2} \geq 0$

9. $\dfrac{x+4}{x-2} \geq 1$

10. $\dfrac{3}{x-1} - \dfrac{4}{x} \geq 1$

Applications

11. A telephone company offers two long-distance plans.
Plan A: $25 per month and $.05 per minute
Plan B: $5 per month and $.12 per minute
For how many minutes of long-distance calls would plan B be financially advantageous?

The ***absolute value*** of a number is the distance from that number to 0. Thus,

$$|x| = c \quad \text{if and only if} \quad x = \pm c,$$
$$|x| < c \quad \text{if and only if} \quad -c < x < c,$$
$$|x| > c \quad \text{if and only if} \quad x < -c \text{ or } x > c.$$

- Absolute value equations
- Absolute value inequalities

Absolute Value Equations

Solve each equation.

1. $|2x - 1| = 5$

2. $|3x + 1| = 6$

3. $2|x + 3| - 4 = 6$

4. $|x + 2| = |2x - 4|$

Absolute Value Inequalities

Write the solution to each inequality in interval form.

5. $|3x+4| \leq 7$

6. $|2x+1| > 7$

7. $\frac{1}{3}|x+4| - 3 \geq 1$

8. $2 \leq |x-1| \leq 5$

9. Write an inequality that describes the set of all numbers that are at least 3 units away from 5.