Section 6.1 An Introduction to Angles: Degree and Radian Measure

OBJECTIVE 1: Understanding Degree Measure
OBJECTIVE 2: Finding Coterminal Angles Using Degree Measure

A 360° angle is one complete counterclockwise rotation. An angle of 90° is ¼ of one complete counterclockwise revolution. An angle of -45^0 is 1/8 of one complete clockwise revolution.

Draw an angle of 90°. Label the **initial side** and the **terminal side**; show the direction of the angle. Label the quadrants.

Draw an angle of –45°. Label the **initial side** and the **terminal side**; show the direction of the angle. Label the quadrants.

Draw an angle of 120°. Label the **initial side** and the **terminal side**; show the direction of the angle. Label the quadrants.

Draw an angle of –270°. Label the **initial side** and the **terminal side**; show the direction of the angle. Label the quadrants.

Definition: **Coterminal Angles** are angles in standard position having the same terminal side.

Coterminal Angles can be obtained by adding any nonzero integer multiple of 360° to any given angle. Given any angle θ and any nonzero integer *k*, the angles θ and $\theta + k \cdot 360°$ are coterminal angles.

Draw Angles in standard position having degree measure:

45° 405° –315°

If θ is a given, your text uses θ_c to denote the angle of least positive measure coterminal with θ.

6.1.7 Find the angle of least positive measure, θ_c, that is coterminal with $\theta = -622°$.

OBJECTIVE 3: Understanding Radian Measure

Consider the circle $x^2 + y^2 = r^2$ and angle θ. An angle whose vertex is at the center of a circle is called a **central angle**. Every central angle intercepts a portion of the circle called an **intercepted arc**. We typically use the variable s to represent the length of an intercepted arc.

When a central angle has an intercepted arc whose length is equal to the radius of the circle, the central angle is said to have a measurement equal to **1 radian**. $1\ radian \approx 57.3°$.

1. Draw a circle centered at the origin of radius 1.
2. Draw a central angle in standard position of 1 radian.
3. Label the intercepted arc and the angle.
4. Measure with a string the length of the arc and compare it to the length of the radius.

1. Draw a circle centered at the origin of radius 4.
2. Draw a central angle in standard position of 1 radian.
3. Label the intercepted arc and the angle.
4. Measure with a string the length of the arc and compare it to the length of the radius.

Relationship Between Degrees and Radians

$360° = 2\pi$ radians

$180° = \pi$ radians

The abbreviation "rads" may be used to label an angle in radian measure. Typically, this designation is left off when angles measured in radians are written in term of π.

On one set of axes draw angles of the following radian measures: $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$

On one set of axes draw angles of the following radian measures:

$0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{11\pi}{6}, 2\pi$ (HINT: Find a common denominator)

6.1.10 and 6.1.13 Draw each angle in standard position and state the quadrant in which the terminal side of the angle lies or the axis on which the terminal side of the angle lies.

10. 5π

13. $-\frac{13\pi}{6}$

OBJECTIVE 4: Converting Between Degree Measure and Radian Measure

Converting Between Degrees and Radians

To convert degrees to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

To convert radians to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

6.1.16 and 6.1.18 Convert each angle given in degree measure into radians.

16. 15°

18. -540°

6.1.22 and 6.1.26 Convert each angle given in radian measure into degrees.

22. $\frac{3\pi}{5}$ radians 26. 3 radians

OBJECTIVE 5: Finding Coterminal Angles Using Radian Measure

Coterminal Angles can be obtained by adding any nonzero integer multiple of 2π to any given angle. Given any angle θ and any nonzero integer k, the angles θ and $\theta + k \cdot 2\pi$ are coterminal angles.

CAUTION: Do not write $-\frac{21\pi}{4} = \frac{3\pi}{4}$. These angles are coterminal but they are not equal.

6.1.31 Find the angle of least positive measure, θ_c, that is coterminal with $-\frac{11\pi}{12}$. Then find the measure of the negative angle coterminal with $-\frac{11\pi}{12}$ such that the angles lies between -8π and 4π.