

Section 5.4 Properties of Logarithms

Objective 1: Using the Product Rule, Quotient Rule and Power Rule for Logarithms

Properties of Logarithms

If $b > 0$, $b \neq 1$, u and v represent positive numbers, and r is any real number, then

$$\log_b uv = \log_b u + \log_b v \quad \text{Product Rule for Logarithms}$$

$$\log_b \frac{u}{v} = \log_b u - \log_b v \quad \text{Quotient Rule for Logarithms}$$

$$\log_b u^r = r \log_b u \quad \text{Power Rule for Logarithms}$$



Warning!

$\log_b(u + v)$ is NOT equivalent to $\log_b u + \log_b v$.

$\log_b(u - v)$ is NOT equivalent to $\log_b u - \log_b v$.

$\frac{\log_b u}{\log_b v}$ is NOT equivalent to $\log_b u - \log_b v$.

$(\log_b u)^r$ is NOT equivalent to $r \log_b u$.

5.4.4, 5, 7, and 9

Use the properties of logarithms to expand the expression. Wherever possible, evaluate the expression.

4. $\log_3(27w)$

5. $\ln 5e^2$

7. $\log 100P$

9. $\log_{\sqrt{2}} 8x$

Objective 2: Expanding and Condensing Logarithmic Expressions

5.4.13

Use the properties of logarithms to expand the expression. Wherever possible, evaluate the expression.

$$\log \frac{\sqrt{x}}{10y^3}$$

5.4.27

Use the properties of logarithms to rewrite the expression as a single logarithm. Wherever possible, evaluate the expression.

$$\log_5(x-2) + \log_5(x+2)$$

Objective 3: Solving Logarithmic Equations Using the Logarithm Property of Equality

The Logarithm Property of Equality

If a logarithmic equation can be written in the form $\log_b u = \log_b v$, then $u = v$.

Furthermore, if $u = v$, then $\log_b u = \log_b v$.

5.4.34

Use the properties of logarithms and the logarithm property of equality to solve the equation.

$$\ln 5 + \ln x = \ln 7 + \ln(3x-2)$$

Objective 4: Using the Change of Base Formula

Change of Base Formula

For any positive base $b \neq 1$ and for any positive real number u , then

$$\log_b u = \frac{\log_a u}{\log_a b}$$

where a is any positive number such that $a \neq 1$.

5.4.37 First, *guess* the value based on your knowledge of logarithms, then use the change of base formula and a calculator to approximate the following expressions. Do not round until the final answer. Then round to four decimal places as needed.

$$\log_4 51$$

5.4.46

Solve the equation and simplify the answer.

$$\log_{1/3} x = \log_3 20$$