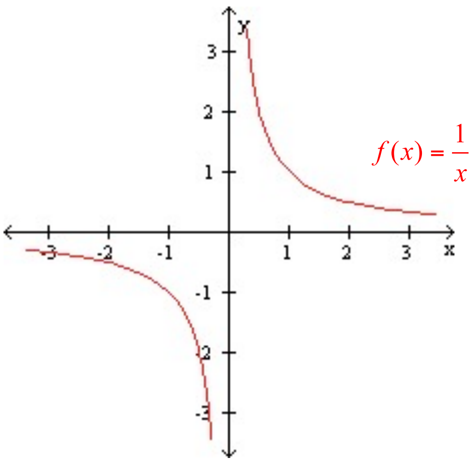
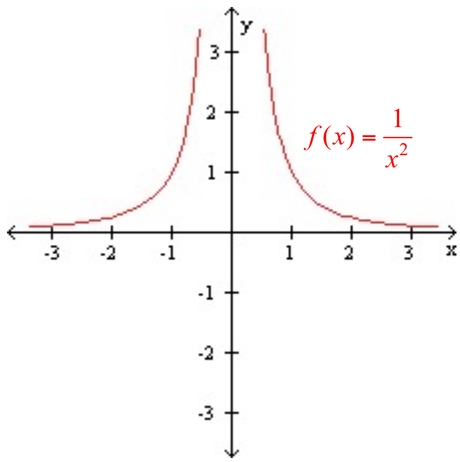
Objective 4: Using Transformations to Sketch the Graphs of Rational Functions

The graphs of $f(x)=\frac{1}{x}$ and $f(x)=\frac{1}{x^2}$





Properties of the graph of $f(x)=\frac{1}{x}$

Domain: $(-\infty,0)\cup(0,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$

No intercepts

Vertical Asymptote: $x=0$

Horizontal Asymptote: $y=0$

Odd function $f(-x)=-f(x)$

The graph is symmetric about the origin.

Properties of the graph of $f(x)=\frac{1}{x^2}$

Domain: $(-\infty,0)\cup(0,\infty)$

Range: $(0,\infty)$

No intercepts

Vertical Asymptote: $x=0$

Horizontal Asymptote: $y=0$

Even function $f(x)=f(-x)$

The graph is symmetric about the y-axis.

4.6.21 and 26

Use transformations of $f(x)=\frac{1}{x}$ or $f(x)=\frac{1}{x^2}$ to sketch the rational function. Label all intercepts and find the equations of all asymptotes.

21. $f(x)=\frac{1}{x}-2$

26. $f(x)=\frac{1}{(x+3)^2}-2$

Objective 5: Sketching Rational Functions Having Removable Discontinuities

A rational function $f(x) = \dfrac{g(x)}{h(x)}$ may sometimes have a "hole" in its graph. In calculus, these "holes" are called **removable discontinuities.** Removable discontinuities occur when $g(x)$ and $h(x)$ share a common factor. It is possible to have a rational function with a common factor in the numerator and denominator whose graph still looks like the graph of a rational function but has a "hole" in it.

4.6.35 Identify the coordinates of all removable discontinuities and sketch the graph of the rational function.

$f(x) = \dfrac{x^2 - 9}{x - 3}$

Objective 7: Sketching Rational Functions

Steps for Graphing Rational Functions of the Form $f(x) = \dfrac{g(x)}{h(x)}$

1. Find the domain.

2. If $g(x)$ and $h(x)$ have common factors, cancel all common factors determining the x-coordinates of any removable discontinuities and rewrite f in lowest terms.

3. Check for symmetry.
   If $f(-x) = -f(x)$, then the graph of $f(x)$ is *odd* and thus symmetric about the origin.
   If $f(x) = f(-x)$, then the graph of $f(x)$ is *even* and thus symmetric about the y-axis.

4. Find the y-intercept by evaluating $f(0)$.

5. Find the x-intercepts by finding the zeros of the numerator of f, being careful to use the new numerator if a common factor has been removed.

6. Find the vertical asymptotes by finding the zeros of the denominator of f, being careful to use the new denominator if a common factor has been removed. Use test values to determine the behavior of the graph on each side of the vertical asymptotes.

7. Determine if the graph has any horizontal asymptotes.

8. Plot points, choosing values of x between each intercept and choosing values of x on either side of the all vertical asymptotes.

9. Complete the sketch.

4.6.43 and 45

Follow the nine-step graphing strategy to sketch the graph of the rational function.

43. $f(x) = \frac{2x+4}{x-1}$

45. $f(x) = \frac{x^2-1}{x^2+1}$