

Section 4.3 The Graphs of Polynomial Functions

Objective 1: Understanding the Definition of a Polynomial Function

Definition Polynomial Function

The function $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$ is a polynomial function of degree n where n is a nonnegative integer. The numbers $a_0, a_1, a_2, \dots, a_n$ are called the **coefficients** of the polynomial function. The number a_n is called the **leading coefficient** and a_0 is called the **constant coefficient**.

4.3.1, 2, and 4

Determine if the given function is a polynomial function. If it is, then identify the degree, the leading coefficient, and the constant coefficient.

1. $f(x) = \frac{1}{3}x^7 - 8x^3 + \sqrt{5}x - 8$ 2. $g(x) = \frac{8x^3 - 5x + 3}{7x^2}$ 4. $f(x) = \frac{3x - 5x^3}{11}$

a) $f(x) =$ _____

b) $f(x) =$ _____

Degree _____

Degree _____

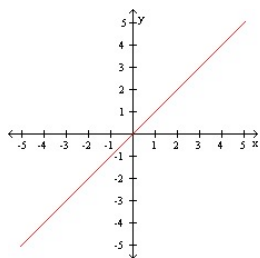
Leading Coefficient _____

Leading Coefficient _____

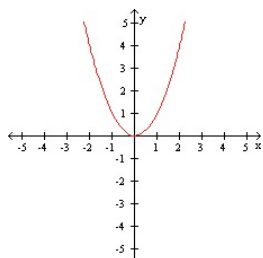
Constant Coefficient _____

Constant Coefficient _____

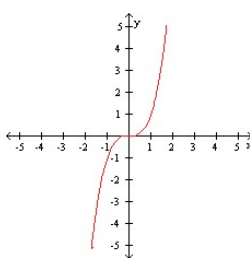
Objective 2: Sketching the Graphs of Power Functions



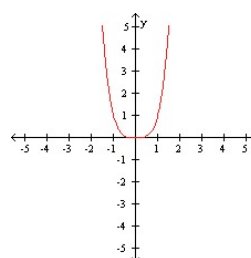
(a) $f(x) = x$



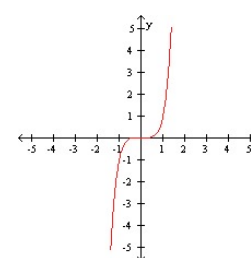
(b) $f(x) = x^2$



(c) $f(x) = x^3$



(d) $f(x) = x^4$



(e) $f(x) = x^5$

4.3.10 and 15

Use the associated power function and transformations to sketch the following functions.

10. $f(x) = \frac{1}{2}(x-1)^4$

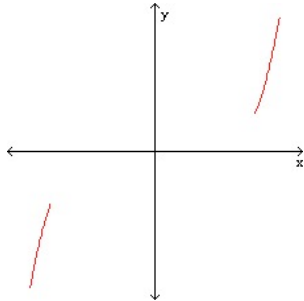
15. $f(x) = -(x-1)^5 + 2$

Objective 3: Determining the End Behavior of Polynomial Functions

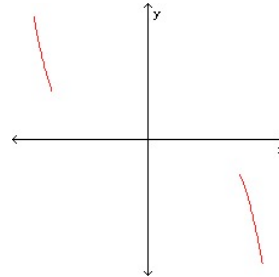
Process for Determining the End Behavior of a Polynomial Function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0.$$

If the degree n is odd, the graph has opposite left-hand and right-hand end behavior, that is, the graph “starts” and “finishes” in **opposite** directions.

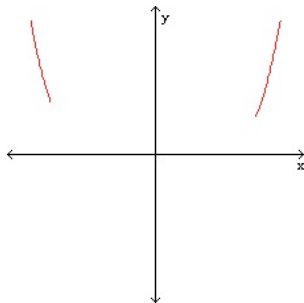


$a_n > 0$, odd degree

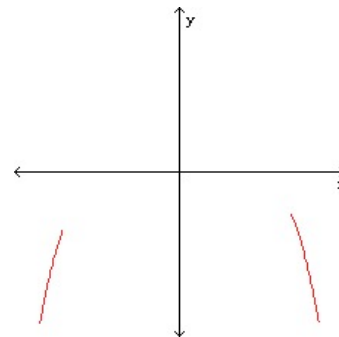


$a_n < 0$, odd degree

If the degree n is even, the graph has the same left-hand and right-hand end behavior, that is, the graph “starts” and “finishes” in the **same** direction.



$a_n > 0$, even degree

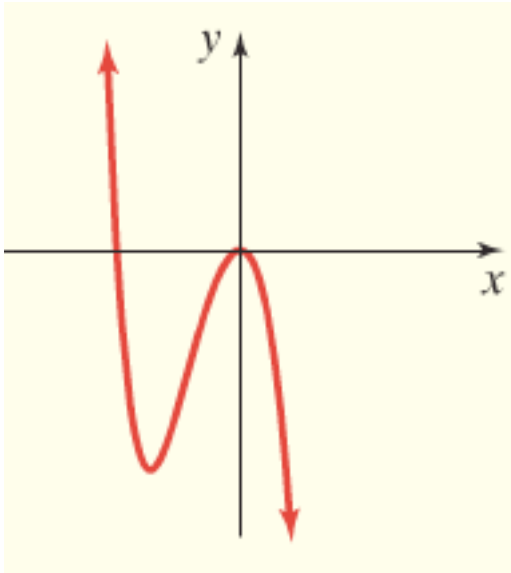


$a_n < 0$, even degree

4.3.16 and 21

Use the end behavior of the graph of the given polynomial function to answer the following:

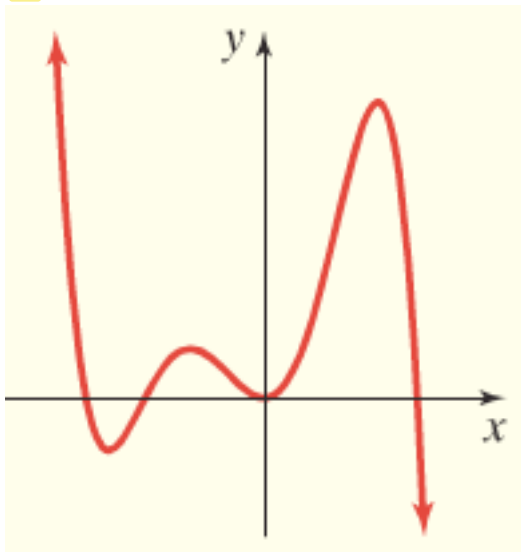
16.



a) The degree is _____ (even or odd).

b) The leading coefficient is _____ (positive or negative).

21.



a) The degree is _____ (even or odd).

b) The leading coefficient is _____ (positive or negative).

Objective 4: Determining the Intercepts of a Polynomial Function

The number $x = c$ is called a **zero** of a function f if $f(c) = 0$. If c is a real number, then c is an x -intercept. Therefore, to find the x -intercepts of a polynomial function $y = f(x)$, we must find the real solutions of the equation $f(x) = 0$.

4.3.23

Find the intercepts of the polynomial function $f(x) = x^3 + 3x^2 - 16x - 48$.

The y -intercept is $y =$ _____.

The x -intercept(s) is/are $x =$ _____.

4.3.25

Find the intercepts of the polynomial function $f(x) = (x^2 - 9)(2 - x)$.

The y -intercept is $y =$ _____.

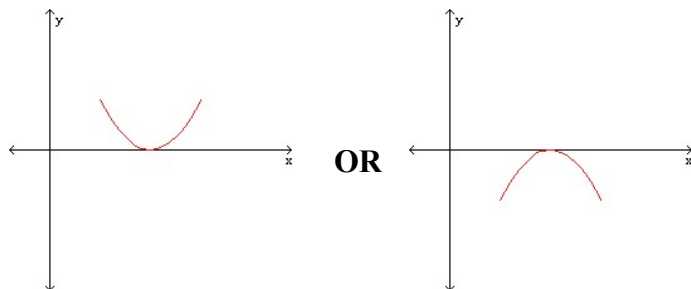
The x -intercept(s) is/are $x =$ _____.

Objective 5: Determining the Real Zeros of Polynomial Functions and Their Multiplicities

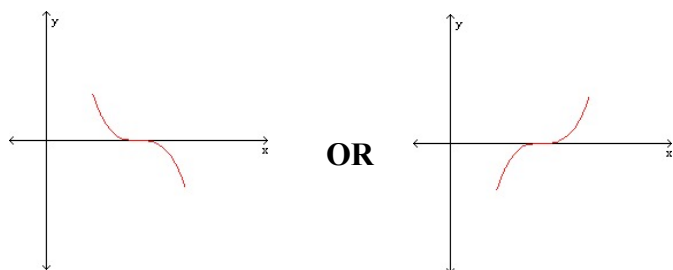
The Shape of the Graph of a Polynomial Function Near a Zero of Multiplicity k .

Suppose c is a real zero of a polynomial function f of multiplicity k , that is, $(x - c)^k$ is a factor of f . Then the shape of the graph of f near $x = c$ is as follows:

If $k > 1$ is even, then the graph **touches** the x -axis at $x = c$.



If $k > 1$ is odd, then the graph **crosses** the x -axis at $x = c$.



4.3.30

Determine the real zeros and their multiplicities of $f(x) = -5(x - 2)(x^2 - 4)^3$.

a) The real zeros of the polynomial are $x =$ _____. (Use a comma to separate answers as needed. Type an exact answer, using radicals as needed.)

b) The multiplicity of the zero located farthest left on the x -axis is _____.

The multiplicity of the zero located farthest right on the x -axis is _____.

The graph _____ the x -axis at the leftmost zero. (touches or crosses)

The graph _____ the x -axis at the rightmost zero. (touches or crosses)

Objective 6: Sketching the Graph of a Polynomial Function

Four-Step Process for Sketching the Graph of a Polynomial Function

1. Determine the end behavior.
2. Plot the y -intercept $f(0) = a_0$.
3. Completely factor f to find all real zeros and their multiplicities*.
4. Choose a test value between each real zero and sketch the graph.

* This is the most difficult step and will be discussed in further detail in the subsequent sections of this chapter.

4.3.36

Sketch the polynomial function $f(x) = x^2(x + 2)(x - 2)$ using the four-step process.

The left-hand behavior starts _____ and the right-hand behavior ends _____.

The y -intercept is _____.

The real zeros of the polynomial are $x =$ _____.

The multiplicity of the zero located farthest left on the x -axis is _____.

The multiplicity of the zero located between the leftmost and rightmost zeros is _____.

The multiplicity of the zero located farthest right on the x -axis is _____.

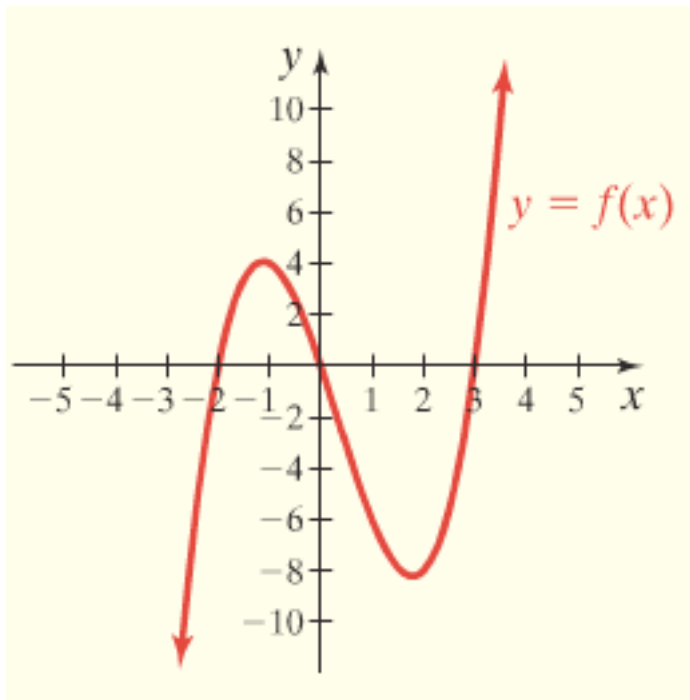
What is the value of the test point at $x =$ _____ ? $y =$ _____

Sketch the graph.

Objective 7: Determining a Possible Equation of a Polynomial Function Given its Graph

4.3.45

Analyze the graph to address the following about the polynomial function it represents.



a) Is the degree even or odd? _____

b) Is the leading coefficient positive or negative? _____

c) The value of the constant coefficient is _____.

d) The leftmost real zero is $x =$ _____, which has (even/odd) _____ multiplicity.

The second real zero from the left is $x =$ _____, which has _____ multiplicity.

The rightmost real zero is $x =$ _____, which has _____ multiplicity.

e) Select a possible function that could be represented by this graph.