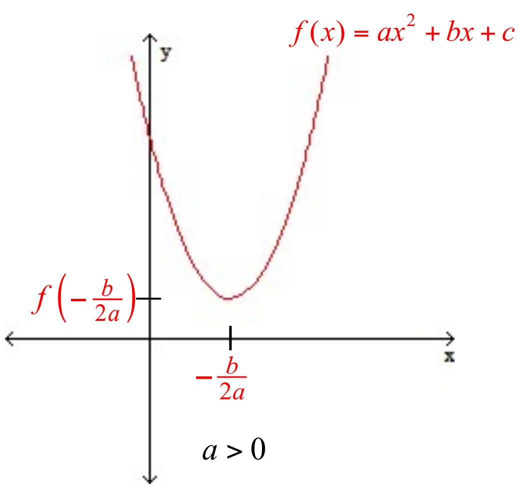
Section 4.2 Applications and Modeling of Quadratic Functions

Objectives

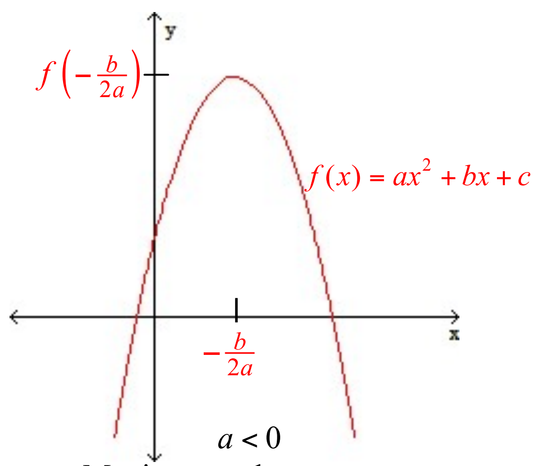
1. **Maximizing Projectile Motion Functions**
2. **Maximizing Functions in Economics**
3. **Maximizing Area Functions**

With application problems involving functions, we are often interested in finding the maximum or minimum value of the function.

Quadratic functions are relatively easy to maximize or minimize because we know a formula for finding the coordinates of the vertex. Recall that if $f(x) = ax^2 + bx + c$, $a \neq 0$, we know that the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. If $a > 0$, the parabola *opens up* and has a minimum value at the vertex. If $a < 0$, the parabola *opens down* and has a maximum value at the vertex.



Minimum value at vertex



Maximum value at vertex

Objective 1: Maximizing Projectile Motion Functions

An object launched, thrown or shot vertically into the air with an initial velocity of v_0 meters/second (m/s) from an initial height of h_0 meters above the ground can be modeled by the function $h(t) = -4.9t^2 + v_0 t + h_0$ where $h(t)$ is the height of the projectile t seconds after its departure*.

*Note that the leading coefficient of the projectile motion model is -4.9. This constant is derived using calculus and the acceleration of gravity on earth, which is 9.8 meters per second per second. If the height was measured in feet, the leading coefficient of the projectile motion model would be -16 derived using calculus and acceleration of gravity on earth, which is 32 feet per second per second.

4.2.3 A toy rocket is shot vertically into the air from a 5-foot tall pad with an initial velocity of 112 feet per second. Suppose the height of the rocket in feet t seconds after being launched can be modeled by the function $h(t) = -16t^2 + v_0 t + h_0$ where v_0 is the initial velocity of the rocket and h_0 is the initial height of the rocket. How long will it take for the rocket to reach its maximum height? What is the maximum height?

Objective 3: Maximizing Area Functions

Suppose you are asked to build a rectangular fence which borders a river but only have a limited number of feet of fencing available. How many ways can you build this fence? Although there are infinitely many ways in which you could build this fence, there is only one way to build this fence in order to maximize the area!

4.2.22 Jim wants to build a rectangular parking lot along a busy street but only has 2,500 feet of fencing available. If no fencing is required along the street, find the maximum area of the parking lot.