Objectives

- ☐ I understand that the tragedy of the commons is a result of two or more actors maximize while using a shared resource.
- ☐ I understand that there are methods to counteract the tragedy of the commons. Some of these methods mean changing the game, others restrict what players can do.

Last time, we discussed some of the basic ideas of game theory. In particular, we saw how two individuals who share a resource can maximize their strategy but end up with a sub-optimal solution. We also saw that if we change the off-diagonal entries, we can get different equilibrium points. The example of this arose in the prisoner's dilemma. In the original set-up, Alice and Bob picked sub-optimal solutions:

Complete Pay-Off Matrix

	Bob Confesses	Bob Remain Silent
Alice Confesses	5 yrs \ 5 yrs	0 yrs \ 20 yrs
Alice Remains Silent	20 yrs \ 0 yrs	1 yr \ 1 yr

In the alternate set-up, Alice and Bob likely picked the optimal solution assuming there was no spite between them or they were not risk averse.

Complete Pay-Off Matrix

	Bob Confesses	Bob Remain Silent
Alice Confesses	5 yrs \ 5 yrs	2 yrs \ 20 yrs
Alice Remains Silent	20 yrs \ 2 yrs	1 yr \ 1 yr

Today, we explore use these concepts to understand the tragedy of the commons as well as how different approaches work.

Since Bob and Alice are serving time in jail, let's consider two cow herder's in India: Ali and Buvana. Suppose they both have access to a plot of land which is not owned by anyone. Furthermore, suppose the interaction between the land and the cows is the following:

With n cows on the land, each cow will have enough grass to produce $10 - \dfrac{n}{2}$ gallons of milk per day

What is the optimum number of cows to have on this land? This is an optimization problem in one variable, where the total milk produced is $M(n) = n(10 - n/2)$, or the number of cows times the milk produced per cow. Therefore,

$$M'(n) = 10 - n = 0 \implies n = 10$$

At 10 cows, the maximum amount of milk is produced, which is $M(10) = 10(10 - 5) = 50$ gallons.

Now, suppose that Ali and Buvana each own 5 cows, which graze on this patch of land. What is their pay-off matrix for buying one more cow? How can we figure this out?

- Well, If they each keep 5 cows, they see no increase in the amount of milk they produce. They each produce $5(10 - 10/2) = 25$ gallons of milk. (**Notice** that we did the number of cows each person had multiplied by the *total* number of cows on the land!)
- Suppose only Ali buys an extra cow, giving him a total of 6. Then there are 11 cows grazing on this land: Ali's 6 cows and Buvana's 5 cows. Ali gets $6(10 - 11/2) = 27$ gallons while Buvana gets $5(10 - 11/2) = 22.5$ gallons.
- If we suppose that Buvana buys and extra cow and Ali does not, then Buvana is getting 27 gallons of milk while Ali gets 22.5 gallons.
- If they both purchase a cow, then there are 12 cows on the land. Both Buvana and Ali each get $6(10 - 12/2) = 24$.

Let's organize this into a pay-off matrix.

Complete Pay-Off Matrix

	Buvana keeps 5	Buvana adds 1 more
Ali keeps 5	25 gal \ 25 gal	22.5 gal \ 27 gal
Ali adds 1 more	27 gal \ 22.5 gal	24 gal \ 24 gal

We see that there is a dominant strategy for both Ali and Buvana: to buy one more cow! So both Ali and Buvana will grow beyond what is optimum. How far above the optimum will they go? Some may argue that people will just take forever. In truth, at some point both Buvana and Ali will stop growing their herd. It won't be beneficial for them.

To show that this is possible, let's suppose they both have 8 cows. Then the total number of cows on the land is 16.

- If both keep 8 cows, then each herder gets $8(10 - 16/2) = 8(2) = 16$ gallons of milk per day.
- If one keeps 8 cows and the other grows to 9, then the herder with 8 cows gets $8(10 - 17/2) = 12$ gallons of milk per day while the herder with 9 cows gets $8(10 - 17/2) = 13.5$ gallons of milk per day.
- If they both increase to 9 cows, the each herder gets $9(10 - 18/2) = 9$ gallons of milk per day.

Therefore the pay-off matrix is the following:

Complete Pay-Off Matrix

	Buvana keeps 8	Buvana adds 1 more
Ali keeps 8	16 gal \ 16 gal	12 gal \ 13.5 gal
Ali adds 1 more	13.5 gal \ 12 gal	9 gal \ 9 gal

Here, the dominant strategy for both Ali and Buvana is to stay at 8 cows. They are both better off with 8 regardless of what the other does.

This tells us that at some point, the two will stop growing. Unfortunately, that point will be beyond the social optimum. The more herders, the further that points is from the social optimum.

So, how does a society deal with this problem? There are four methods:

(1) **Privatization.**

Privatization amounts to selling the public good to one person (or a cooperative of people). This person is financially invested in maintaining this resource.

In the example we discussed in lecture, this amounts to selling the land to some person. This person will either house their own cows on it, or allow access to Ali and Buvana. If it is the later, that access will come with restrictions in order to maintain the property (think of how lease agreements work).

Essentially, this technique reduces the players to 1.

Pros: Effective and cheap. *Cons:* Limited in application. Works well for land, but not for a river, the atmosphere, or a school or fish.

(2) **Cap and Trade.**

Cap and trade entails some regulatory body determining the appropriate amount of usage of the public resource and administering permits for that use.

In the example we discussed in lecture, this amounts to distributing 5 cow permits to Ali and 5 cow permits to Buvana. Suppose Ali uses 5 of his permits and Buvana uses only 4. Her fifth permit could be used to buy a 5th cow or she could sell the permit to Ali. The two parties would agree on a mutually beneficial price. Below are the figures for the possible outcomes.

	Ali's Production	Buvana's Production
Current	27.5 gallons	22 gallons
Buvana gets 5	25 gallons	25 gallons
Ali gets 6	30 gallons	20 gallons

Based on the numbers above, Buvana will want a price that (1) makes up for the loss of the 2 gallons of milk and (2) considers the potential 3 gallons she could have made. A reasonable price for her to ask for is one equivalent to 5 gallons of milk per day.

At first glance, it may seem like Ali would not pay this price. He's only seeing an increase of 2.5 gallons of milk. However, if Buvana adds an extra cow, we would have seen a loss of 2.5 gallons. So a price equivalent to 5 gallons per day will be agreeable to him as well.

Essentially, this technique places a cap on the total amount of usage. The players can then decide who uses what amounts.

Pros: Once it is set up, the process is taken care of by market forces. *Cons:* It is difficult to determine the proper number of permits required. Also, some resources will be needed to enforce that players abide by permits.

(3) **(Revenue Neutral) Taxation**

Taxation by itself can help the issue, but is often has too strong an effect. For example, suppose there is a 1 gallon per cow per day tax and suppose both Ali and Buvana have 4 cows.

- If both have four cows, they both produce $4(10 - 8/2) = 24$ gallons of milk. After taxes, they produce 20 gallons each.

- If one has four cows and the other has five, the one with four produces $4(10-9/2) = 22$ gallons and the one with five produces $5(10-9/2) = 27.5$. After taxes, these amounts drop to 18 and 22.5.
- If both have five cows, they both produce $5(10-10/2) = 25 gallons$. After taxes, this amount is 20 gallons.

When we draw the payoff matrix, we see that both Ali and Buvana will want to keep their herds at 4 cows each. This is *below* the social optimum of 5.

Complete Pay-Off Matrix

	Buvana keeps 4	Buvana adds 1 more
Ali keeps 4	24 gal \ 24 gal	18 gal \ 22.5 gal
Ali adds 1 more	22.5 gal \ 18 gal	20 gal \ 20 gal

From an environmental standpoint, taxation works. From an economics standpoint, it leads to some inefficiencies. An alternative is to choose a revenue neutral tax. Such taxes work by taxing everyone and redistributing the collected tax in equal amounts to everyone. This creates an incentive for staying small. For example, suppose there is a 4 gallon tax per cow per day.

- If both have four cows, they both produce $4(10-8/2) = 24$ gallons of milk. The taxing agency collects 16 gallons from each one (totally 32 gallons), splits what they collected and gives that amount back to Ali and Buvana. So they both end up with 24 gallons of milk.
- If one has four cows and the other has five, the one with four produces $4(10-9/2) = 22$ gallons and the one with five produces $5(10-9/2) = 27.5$. The taxing agency collects 16 from the person with 4 cows and 20 from the person with 5 cows. They have a total of 36 gallons of milk. So they given each person 18 gallons of milk back. The person with 4 cows has 24 gallons, and the person with 5 cows has 25.5 gallons.
- If both have five cows, they both produce $5(10-10/2) = 25$ gallons. Each has 20 gallons collected, and then 20 gallons returned.

The resulting pay-off matrix shows that both Ali and Buvana will not stay at 4 cows.

Complete Pay-Off Matrix

	Buvana keeps 4	Buvana adds 1 more
Ali keeps 4	24 gal \ 24 gal	24 gal \ 25.5 gal
Ali adds 1 more	25.5 gal \ 24 gal	25 gal \ 25 gal

So there won't be the inefficiencies we saw before. And if we do the math for each having 5 cows, we see that both Ali and Buvana have a dominant strategy to keep 5 cows.

Complete Pay-Off Matrix

	Buvana keeps 5	Buvana adds 1 more
Ali keeps 5	25 gal \ 25 gal	25 gal \ 24.5 gal
Ali adds 1 more	25 gal \ 24.5 gal	24 gal \ 24 gal

Essentially, this method changes the off-diagonal entries by ensuring larger players pay greater costs.

Pros: Diminishing returns are felt more strongly by the players when they grow. *Cons:* Setting this up would be difficult and would require outside funding to run.

(4) **Laws and Enforcement** This entails creating a law that limits the usage of a public resource. In this case, the government of Ali and Buvana would step in and say no herder can have more than 5 cows. If, however, more herders show up, the law would need to be updated.

Essentially, this method dictates how players play the game.

Pros: Works well and can be applied to a variety of settings. *Cons:* It is expensive to run and requires constant updates as the market changes.

Summary of Ideas: Lecture 28

- The tragedy of the commons can be explained using competitive games.
- There are four methods of dealing with the tragedy of the commons. Each has it advantages and disadvantages.