

Objectives

- I understand that when competing forces try to optimize, it can sometimes lead to a sub-optimal solution for everyone.
- I can write out a **pay-off matrix** for competing parties and determine what is a **dominant** strategy.
- I can find an **equilibrium** of a pay-off matrix.
- I understand what the **tragedy of the commons** means.

Optimization, as we have come to understand it in this class, has been about finding maximums and minimums of a function. A function, as we saw much earlier in this course, is just a way to model reality based on our observations.

Up to this point, we have learned that we can observe a system, describe it with a function, and find the inputs that yield some optimal output (a max or min). But what happens when you have two distinct systems? Under what conditions can that optimize? What does it mean for them to cooperate versus compete?

Let's consider a scenario you likely know well: *group work*.⁹ Suppose you and a partner have a group presentation and an exam coming up. You only have time to prepare for one.

- If both of you prepare for the presentation, you'll both get 100% on the presentation and an 80% on the exam. Your average will be a 90%.
- If both of you study for the exam, you'll both get 92% on the exam and 84% on the presentation. Your average will be 88%.
- If one of you studies for the exam while the other prepares for the presentation, then...
 - both of you will receive a 92% on the presentation,
 - the one who studied for the exam will get a 92% (their average will be 92%),
 - the one who did not study for the exam will get an 80% (their average will be 86%).

What do you do? You want some kind of **dominant strategy**; some way to determine how best to respond regardless of what your partner does. To find this dominant strategy, let's write out *your pay-off matrix* (which is just a table) to help us consider your outcomes.

Your Pay-Off Matrix

	You work on Presentation	You study for exam
Partner works on presentation	90%	92%
Partner studies for exam	86%	88%

You don't know what your partner will do. If your partner works on the presentation, you are better off studying for the exam (90% versus 92%). If your partner studies for the exam, you are better off studying for the exam (86% versus 88%). So regardless of what your partner does, you are better off studying for the exam. Therefore, your dominant strategy is to study for the exam.

Unfortunately, the same is true for your partner!

⁹This scenario was taken from the book *Networks, Crowds, and Markets*.

Your Partner's Pay-Off Matrix

	You work on Presentation	You study for exam
Partner works on presentation	90%	86%
Partner studies for exam	92%	88%

Just like you, your partner doesn't know how you will spend your time. If you study for the presentation, they are better off studying for the exam (90% versus 92%). If you study for the exam, they are better off studying for the exam (86% versus 88%).

Weirdly enough, when you and your partner choose a dominant strategy, you'll both end up with 88%. If, however, you both collaborated and worked on the project, you would have ended up with a 90% grade. This fact is made clear when we combine the pay-off matrices of you and your partner:

Complete Pay-Off Matrix

	You work on Presentation	You study for exam
Partner works on presentation	90% \ 90%	86% \ 92%
Partner studies for exam	92% \ 86%	88% \ 88%

Because both of you will study for the exam, the outcome of you both getting 88% is called an **equilibrium point**.

Example 27.1: (Prisoner's Dilemma) Alice and Bob are two criminals are caught committing a crime. With the current evidence on each of them, they are looking at a one-year sentence... assuming they remain silent! If they both confess, then they both get 5 years in prison. If only one of them confesses, then the confessor goes free and the one who remained silent gets 20 years in prison. Do Alice and Bob have dominant strategies? What is the equilibrium?

Solution 27.2: Let's write the complete pay-off matrix.

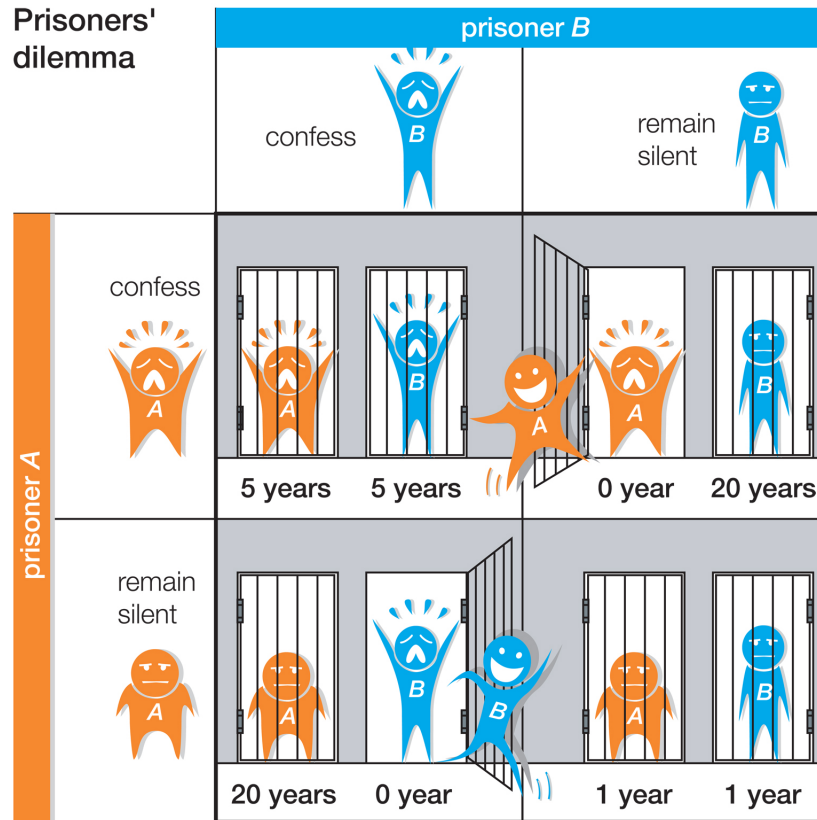
Complete Pay-Off Matrix

	Bob Confesses	Bob Remain Silent
Alice Confesses	5 yrs \ 5 yrs	0 yrs \ 20 yrs
Alice Remains Silent	20 yrs \ 0 yrs	1 yr \ 1 yr

What happens? If Bob confesses, Alice is better off confessing as well (5 years versus 20 years). If Bob does not confess, Alice is still better off confessing (0 years versus 1 year). So Alice will choose to confess.

The same is true for Bob! If Alice confesses, Bob is better off confessing (5 years versus 20 years). If Alice does not confess, Bob is still better off confessing (0 years versus 1 year). So Bob will choose to confess as well!

Both Alice and Bob have the same dominant strategy: they both confess. As a result, the equilibrium is that both serve 5 years.



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It's not always true that the equilibrium will be suboptimal. If we changed the example above by a little bit, the problem works differently.

Example 27.3: (Prisoner's Dilemma) Alice and Bob are two criminals are caught committing a crime. With the current evidence on each of them, they are looking at a one-year sentence... assuming they remain silent! If they both confess, then they both get 5 years in prison. If only one of them confesses, then the confessor gets 2 years in jail and the one who remained silent gets 20 years in prison. Do Alice and Bob have dominant strategies? What is the equilibrium?

Solution 27.4: The only difference with this example is that confessing (when your partner does not) means you will get a 2 year sentence. Let's write the complete pay-off matrix.

Complete Pay-Off Matrix

	Bob Confesses	Bob Remain Silent
Alice Confesses	5 yrs \ 5 yrs	2 yrs \ 20 yrs
Alice Remains Silent	20 yrs \ 2 yrs	1 yr \ 1 yr

What happens now? If Bob confesses, Alice is better off confessing as well (5 years versus 20 years). If Bob does not confess, Alice is still better off remaining silent (2 years versus 1 year). So Alice will pick her strategy based on what she thinks Bob will do.

The same is true for Bob! If Alice confesses, Bob is better off confessing (5 years versus 20 years). If Alice does not confess, Bob is better off remaining silent (2 years versus 1 year). So Bob will pick his strategy based on what he thinks Alice will do.

So, neither Alice nor Bob have a dominant strategy. Neither can guarantee a good outcome that is independent of what the other chooses.

Even though there is not dominant strategy, there are *two* equilibrium points: either they both confess or they both remain silent!

Many social problems arise from competing forces trying to maximize, which in many cases can make everyone worse off. A prime example of this is known as the **Tragedy of the Commons**.

The tragedy of the commons comes from a very real situation discussed by William Forster Lloyd in 1832.¹⁰ Commons are lands that are unowned and, therefore, accessible to everyone. Herders used these lands for cow grazing. Unfortunately, herders brought too many cows (and other grazing animals) resulting in the degradation of the land from overgrazing.

Although the concept was originally used to discuss lands accessible to anyone, the concept extends to a variety of settings, including: fisheries, wild game, air and water (as a thing to pollute), or fresh water (as a resource we take).

So what causes this phenomenon and how does it relate to competition? It turns out to be very similar to what we've just discussed. We will talk more about this next time.

Summary of Ideas: Lecture 27

- When competing forces try to optimize, it can result in sub-optimal solutions depending on the incentives in place.
- A pay-off matrix is a table that organizes the strategies of players and the outcomes of those strategies.
- An equilibrium solution is any solution where both players can settle without having an incentive to change their strategy.
- The tragedy of the commons is a phenomenon where a share resource is used beyond an optimal level.

¹⁰This is a link to Lloyd's lectures on the Checks to Population.