17. LECTURE 17

Objectives

☐ Review Partial Derivatives

For Lecture 17, the following exercises were done in class.

17.1. **Partial Derivatives.**

(1) Find f_x and f_y for

$$f(x,y) = xy^2 - x^3y.$$

To find f_x, we treat x as a variable and y as a constant. Therefore,

$$\boxed{f_x(x,y) = y^2 - 3x^2y}$$

By the same logic,

$$\boxed{f_y(x,y) = 2xy - x^3}$$

(2) Find f_x and f_y for

$$f(x,y) = \ln(x + \sqrt{x^2+y^2}).$$

To find f_x, we treat x as a variable and y as a constant. Here, we will need to use the chain rule twice. The "outside" function is the natural log function. The middle function is x plus the square root function. Therefore,

$$f_x(x,y) = \frac{1}{x+\sqrt{x^2+y^2}} \cdot (1 + \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x) = \boxed{\frac{1 + x(x^2+y^2)^{-\frac{1}{2}}}{x+\sqrt{x^2+y^2}}}$$

The approach for f_y is the same, except x is treated as a constant.

$$f_y(x,y) = \frac{1}{x+\sqrt{x^2+y^2}} \cdot (0 + \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y) = \boxed{\frac{y(x^2+y^2)^{-\frac{1}{2}}}{x+\sqrt{x^2+y^2}}}$$

(3) Find f_x, f_y, and f_z for

$$f(x, y, z) = xz - 5x^2y^3z^4.$$

$$\boxed{f_x(x, y) = z - 10xy^3z^4}$$

$$\boxed{f_y(x, y) = 0 - 15x^2y^2z^4}$$

$$\boxed{f_z(x, y) = x - 20x^2y^3z^3}$$

(4) Find f_{xxx} and f_{xyx} for

$$f(x, y) = x^4y^2 - x^3y.$$

$$f_x(x, y) = 4x^3y^2 - 3x^2y$$

$$f_{xx}(x, y) = 12x^2y^2 - 6xy$$

$$f_{xxx}(x, y) = \boxed{24xy^2 - 6y}$$

$$f_{xyx}(x, y) = f_{xxy}(x, y) = \boxed{24x^2y - 6x}$$

(5) Find z_{uvw} for

$$z = u\sqrt{v-w}.$$

$$\begin{aligned} z_u &= \sqrt{v-w} \\ z_{uv} &= \frac{1}{2}(v-w)^{-\frac{1}{2}} \cdot 1 \\ z_{uvw} &= -\frac{1}{4}(v-w)^{-\frac{3}{2}} \cdot (-1) = \boxed{\frac{1}{4}(v-w)^{-\frac{3}{2}}} \end{aligned}$$

(6) Find w_{zyx} and w_{xxy} for

$$w = \frac{x}{y+2z}.$$

$$\begin{aligned} w_x &= \frac{1}{y+2z} \\ w_{xy} &= (-1)(y+2z)^{-2} \cdot 1 \\ w_{zyx} = w_{xyz} &= (-1)(-2)(y+2z)^{-3} \cdot 2 = \boxed{4(y+2z)^{-3}} \\ w_{xxy} = w_{xyx} &= \boxed{0} \end{aligned}$$

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