1. Lecture 1

Objectives

- ☐ I can distinguish between measurements which are stocks (quantities) versus measurements which are flows (rates).
- ☐ I understand that a function is a mathematical description of a relationship between two or more measurements.
- ☐ I understand the difference between an independent and dependent variable.

What does it mean to do mathematics? What exactly are we doing when we do math? These aren't easy questions to answer. Some of the responses I heard in class were:

- Math is the act of solving quantitative problems.
- Math involved analyzing data.
- Math is the logical treatment of variables.
- Math requires critical thinking.

All of these capture an important component of math, especially applied math (mathematics used to characterize our observations in reality).

Definition 1.1

For this class, mathematics is the attempt to describe and understand relationships between two or more measurements.

It is important to note that the subject of mathematics is much broader than this, but the math we cover in this class can be described as above.

There are two key words in our definition that demand our attention: *relationship* and *measurement*.

We can classify measurements as being of two types:

(1) **Stocks**
(2) **Flows**

A **stock** is an an amount of something existing at a particular point in time. Here are some examples of stocks.

Example 1.2: The amount of junk food in pounds in your house right now.

Example 1.3: The amount of money in US dollars your bank account tomorrow morning.

Example 1.4: The number of chipotle burritos in the US right now.

Each of these is an example of a stock. They are accumulated amounts of something (junk food, money, chipotle burritos) at some point in time (right now, tomorrow morning, right now). Because these are measured items, they each have a corresponding unit we use to *quantify* them: pounds, US dollars, chipotle burritos.

A **flow** is a measurement of how a stock changes over time. Let us list some flows associated to our stocks in the above examples.

Example 1.5: The amount of junk food (in pounds) you eat in a week.

Example 1.6: The amount of money (in US dollars) you make in a month.

Example 1.7: The number of chipotle burritos eaten every 5 minutes in the US.

Each of these amounts will tell us how the stocks previously listed will change. Not all stocks describe the same movementment. In examples 1.5 and 1.7, we are looking at **outflows**, flows that shrink the stock. In example 1.6, we are looking at an **inflow**, a flow that increases the stock.

For most stocks, we can find both outflows an inflows associated to them.

Check your Understanding

For examples 1.2 and 1.4, list an inflow. For example 1.3, list an outflow.

Now that we've discussed how we can classify measurements, let's look at the term "relationship." In previous math classes, a relationship between two or more quantities has usually been a **function**. For this class, it is no different. When we discuss a relation between a set of measurements, we will be referring to a function.

When we write a function, we are implying what quantity depends on what other quantity. Let us look at an example to illustrate this point.

Example 1.8: The number of dogs a person has[1] is related to the amount of money they spend on dog food per month.[2] If dog food for one dog costs \$50 per month, how can we describe this relationship?

There are two ways we can write the equation of a function relating these two measurements. The first is

$$y = 50x$$

where x is the number of dogs owned, and y is the amount of money spend on dog food per month.

The second possible equation is

$$x = \frac{y}{50}.$$

Of the two equations, which do you think is better? They are both mathematically equivalent, but the first one is more meaningful. By writing "$y =$," we are implying that y depends on x. That is, x causes changes in y. For this situation, that means the number of dogs we have influences the amount of money we spend on dog food per month.

For the second equation, we wrote "$x =$." So we were implying that the number of dogs you have depends on the amount of money you spend on dog food. It is like saying, "When a person buys dog food, a dog magically appears at home." But doesn't work that way. It's the amount of dogs a person has that influences how much the spend. Not the other way around.

To specify that y depends on the value of x, we call y the **dependent variable**. We therefore call x the **independent variable**.

[1]Notice that this is a stock.

[2] Notice that this is a flow, not related to the stock. It's an outflow of the owners bank account!

This distinction isn't mathematical, per se. It is psychological. When we write a function, we are implying a *direction of influence*. In the equation

$$y = 50x,$$

I am implying that the stock of dogs a person has influences the outflow of money in their bank account.

Remark 1.9: A more rigorous understanding of a function is *a map where every individual input produces only one output*. Requiring that relationships between measurements have this property encourages us to make better models.[3]

For example, if 2 dogs meant we could either spend \$40, \$100, or \$120 on dog food a month (that's three outputs from our one input), then the relationship is **not** a function. Physically, it means we probably need to consider more than just the number of dogs. We could consider dog sizes or dog food brands. If we did, we would need more *independent variables*.

Summary of Ideas: Lecture 1

- For this class, we will think of mathematics as the attempt to characterize a relationship between two or more measurements.
- Measurements are either stocks or flows.
- A **stock** is an an amount of something existing at a particular point in time.
- A **flow** is a measurement of how a stock changes over time.
- A flow can describe an **inflow**, one that grows a stock, or an **outflow**, one that shrinks a stock.
- For this class, functions will describe how two measurements are related to each other.
- One variable will always be described as depending on the other variable(s). We call this variable **dependent**. The remaining variable(s) are called **independent**.

[3] A model is a description of some kind of physical system. The functions that we use are "models," because they help us predict reality.