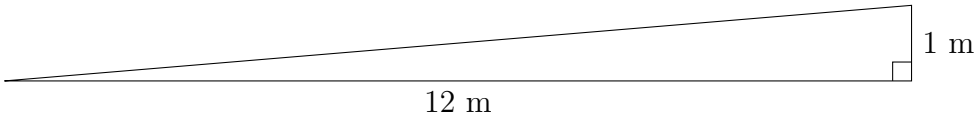
MATH 111: Techniques of Calculus II

Section 1: 110 Osmond Lab, TR 10:10 AM - 11:00 AM
Section 2: 106 Osmond Lab, TR 1:25 PM - 2:15 PM

Due: April 12 @ midnight

Few hiking trails in the US satisfy the Americans with Disabilities Act's (ADA) standards for accessibility. This prevents thousands with limited mobility from enjoying the outdoors. Of those that exist, almost none are on a hill or mountain. The primary reason being the restriction that inclined surfaces cannot exceed a height increase of $\frac{1}{12}$. Put another way, for every 12 meter distance, the heigh cannot increase more than 1 meter.



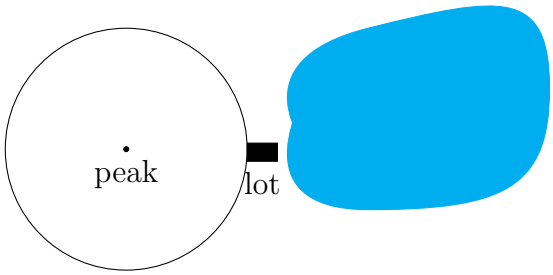
Suppose you have an opportunity to propose an ADA-compliant trail. You've found a hill that neighbors a beautiful fishing lake. At the base of the hill is a parking lot as well, which serves as the perfect starting point for the trail.

Based on the data from surveyors, you determine that the hill can be approximated by the function

$$f(x, y) = 16 - 0.01x^2 - 0.05y^2,$$

which is in meters.

Using this equation (and assuming x describes movements in the east-west direction, y in the north-south direction), the approximate layout is sketched below.



The peak of the hill is $(0, 0, 16)$. The parking lot is at (40, 0, 0).

Here is your task:

Plot a path beginning at the parking lot to the peak by listing a set of points where z changes by no more than 1 meter. Make sure the increase in elevation does not exceed $1/12 \approx 0.083$. Explain

why you know your path satisfies ADA requirements and provide a sketch of the path. A special prize will be given to the student(s) with the shortest path.

Your write up should follow the format of the previous homework assignments. No page limit is required.

Extra Credit: Extra credit will be given to anyone who can accurately describe their path as a function of x and y.

FAQs

What does it mean to plot a path?

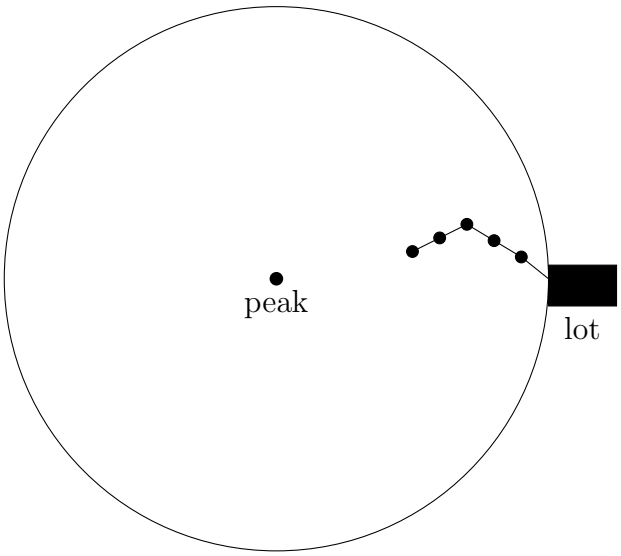
Plotting a path just means listing the points that you travel to. You are assuming that the path between these two points is a straight line. For example, below is a path.

path
(40, 0, 0)
(36, 2, 2.84)
(32, 3, 5.31)
(28, 4, 7.36)
(24, 3, 9.79)
(20, 2, 11.8)

Note: this path will **not** satisfy ADA requirements.

What does it mean to sketch a path?

The path above can be sketched as a series of line segments connecting the points.



In what direction will the elevation change by 1/12?

There are many ways to determine this. This is the "math" question behind the task. I recommend you either think about level curves, distances, and/or derivatives.