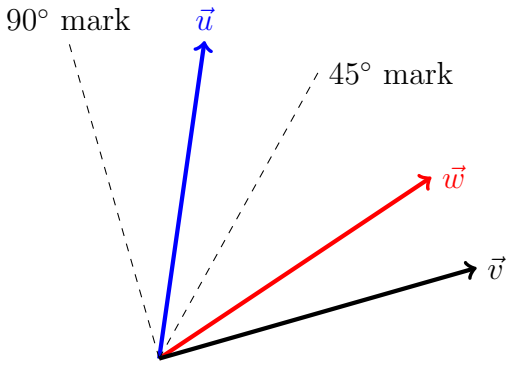
MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

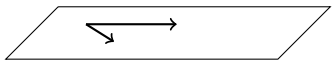
Homework 5: Due Tuesday, Sept 17

1. The following problem is written to walk you through my discussions on determining if two vectors are more perpendicular or more parallel. Please do not use a calculator for this problem.

   Let's first clarify what it means to be "more perpendicular" versus "more parallel." If the angle between two vectors is between 45° and 90°, we say they are more perpendicular. In the picture below, $\vec{u}$ is more perpendicular than parallel to vector $\vec{v}$. We say $\vec{w}$ is more parallel than perpendicular, because the angle between $\vec{w}$ and $\vec{v}$ is less than 45° degrees.



   Remember that any two vectors exist in a flat plane. When we are looking at three dimensional vectors, we're thinking about the angle *inside* that plane.



   Let $\vec{v} = \langle 1, 0, 1 \rangle$ and $\vec{x} = \langle 2, 2, 0 \rangle$.

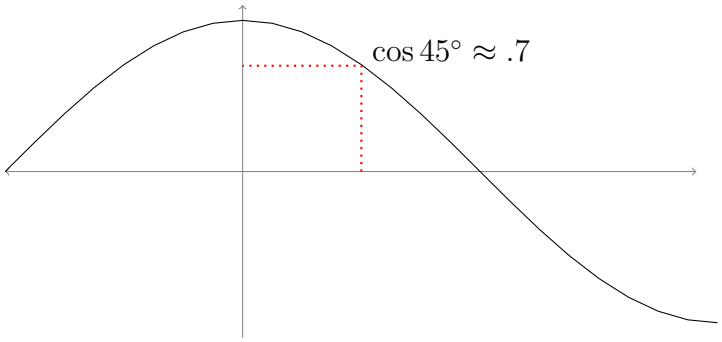
   (a) Find $\vec{v} \cdot \vec{x}$.

   (b) Find $|\vec{v}|$ and $|\vec{x}|$

   (c) Calculate $\cos\theta$. Remember that

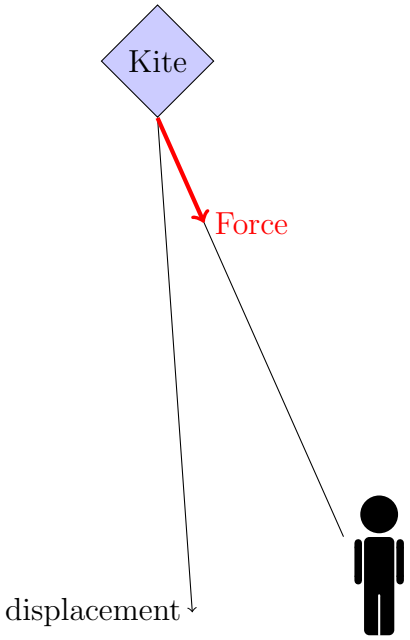
$$\cos\theta = \frac{\vec{v} \cdot \vec{x}}{|\vec{v}||\vec{x}|}$$

   (d) Determine if the two vectors are more perpendicular or more parallel. Use the graph of $\cos\theta$ to help you figure this out.



2. When a force, $\vec{F}$, acts upon an object to cause a displacement of that object, we say that **work** has been done on it. In mathematical terms, work is a measurement of how much these two vectors, force and displacement, move together. It is a scalar value. Answer the following questions about work.

   (a) A kite is being pulled in by a child during a windy day. The displacement of the kite can be described by the vector $\langle 4, 2, -5 \rangle$ while the tension on the rope is described by $\langle 1, 1, -1 \rangle$. What is the work done by the child?



   (b) A tow truck drags a stalled car along a road. The chain makes an angle of 30° with the road and the tension in the chain is 1500N. Draw a picture of this, then find the work done by the truck in pulling the car 1 km.

3. Recall that $\vec{i}, \vec{j}$ and $\vec{k}$ are vectors of length 1 that go in the x, y, and z directions, respectively. Specifically,

$$\vec{i} = \langle 1, 0, 0 \rangle$$
$$\vec{j} = \langle 0, 1, 0 \rangle$$
$$\vec{k} = \langle 0, 0, 1 \rangle$$

   (a) Using the descriptions of $\vec{i}, \vec{j}$ and $\vec{k}$ above, show why

$$2\vec{i} + 3\vec{j} - \vec{k} = \langle 2, 3, -1 \rangle$$

   using vector addition.

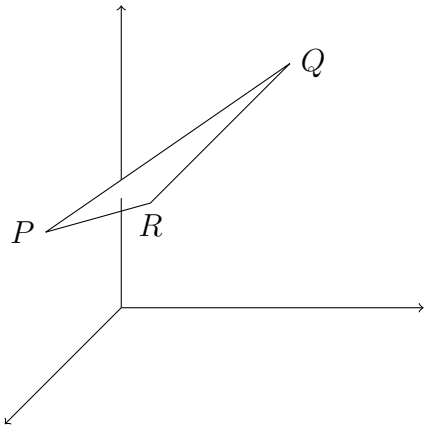
   (b) Show that $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$

   (c) Show that $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$

   (d) Show that $\vec{i} \times \vec{j} = \vec{k}$. Given your work, predict what $\vec{j} \times \vec{k}$ and $\vec{i} \times \vec{k}$ will be.

   (e) Find a *unit* vector that is orthogonal to vectors $\vec{v} = \vec{i} + \vec{j}$ and $\vec{w} = \vec{i} + \vec{j}$

4. Use cross products to find the area of the triangle formed by $P(0, -1, 1)$, $Q(2, 3, 4)$, and $R(-1, 0, 1)$ (pictured below). (Hint: Use the fact that every triangle can be described as half a parallelogram.)



5. A plane contains the points $A(1, 0, 1)$, $B(-2, 1, 3)$ and $C(4, 2, 5)$. Find a nonzero vector that is perpendicular to that plane. (Remember, any two vectors will make a plane.)

6. Consider these four points

$$P(1, 3, 2), Q(3, -1, 6), R(5, 2, 0), \text{ and } S(3, 6, -4)$$

   Do they lie on the same plane? (Hint: If they did, then what would be the volume of the parallelepiped they form? What's the *volume* of a piece of paper?)