MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 22: Due Tuesday, December 3

Note: This homework will be worth 5 points. Think of this as a mini-review packet. Please check back to examples in the notes and previous homework assignments if you get stuck. As always, you can email me if you have questions.

Functions of Several Variables

1. Find and sketch (i) the domain and (ii) the range of the following functions.
   (a) $f(x,y) = \sqrt{2x - y}$
   (b) $f(x,y) = \dfrac{\sqrt{y - x^2}}{1 - x^2}$
2. Sketch the level curves of the following function
$$f(x,y) = x^3 - y$$

Limits and Continuity

3. Find the limit or justify why it does not exist.
   (a)
$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$$
   (b)
$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{xy+1} - 1}$$

Partial Derivatives

4. Find the indicated partial derivative for the function.
   (a) $f(x,y) = \sin(2x + 3y)$; f_{yyx}
   (b) $f(x,y) = \sqrt{1 + xy^3}$; f_{yxx}

Directional Derivatives & Gradients

5. Find the directional derivative of $f(x,y) = \sqrt{xy+1}$ at $(2,4)$ in the direction of $\vec{v} = \langle 3,5 \rangle$.

6. Find the maximum rate of change of $f(x,y) = yx^2$ at the given point and the direction in which it occurs at $(1,1)$.

Tangent Planes and Linear Approximations

7. Find the tangent plane of the following function at the point $(3,1,0)$

$$f(x,y) = \ln(x-2y)$$

8. Given that a function f is differentiable at the point $(1,1)$, estimate the value of $f(0.8,1.3)$ for the given information

   - $f(1,1) = 12$
   - $f_x(1,1) = 100$
   - $f_y(1,1) = -40$

Chain Rule

9. Find $\partial z/\partial t$ and $\partial z/\partial s$ for the following functions:

   (a) $z = xy^2 + x$, $x = 10st$, $y = 2t + s$

   (b) $z = \sqrt{x^2+y^2}$, $x = se^t$, $y = te^s$

Maximum and Minimum Values

10. Find the critical points for the following functions. Classify each point as a local max, local min, or saddle point (when possible).

    (a) $f(x,y) = (x-y)(1-xy)$

    (b) $f(x,y) = xy - x^2y - xy^2$

11. Find the absolute maximum(s) and minimum(s) for the following functions over the given domain D.

    (a) $f(x,y) = 4x + 6y - x^2 - y^2$, $D = \{(x,y) \mid 0 \le x \le 4, 0 \le y \le 5\}$

    (b) $f(x,y) = x^3 - 3x - y^3 + 12y$, $D = \{(x,y) \mid -2 \le x \le 2, -2 \le y \le 3\}$