MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 20: Due Tuesday, November 19

1. Find the tangent "plane" at (3, 2, 6) of the function

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

and use it to approximate $f(3.02, 1.97, 5.99)$.

Hint: Instead of $z = f(x, y)$, extrapolate the techniques of tangent planes and linear approximations in the case where $w = f(x, y, z)$.

2. (This is easier than it seems) Suppose that the price of a carburetor (a car part) is a function of how many cars are owned internationally and the price of steel. You've collected data on carburetor prices in 2011 and found that

- When there were \$1 billion cars on the planet and the price of steel was \$90 a ton, the price of a carburetor was \$200. That is, $f(1 \text{ billion}, 90) = 200$.
- When the number of cars increased by 1 million (=0.001 billion), the price of steel increased by \$2. To keep your units in billions of cars, you estimate that $f_x(1 \text{ billion}, 90) = 2000$.
- When the price of steel increased by \$10, the price of a carburetor increased by \$5. Therefore, you estimate that $f_y(1 \text{ billion}, 90) = 5$.

Estimate the cost of a carburetor if there are 2 billion cars on the road and the price of steel jumps to \$140 per ton. Is this a good estimate?

3. Suppose $f(3, 1) = 2$, $f_x(3, 1) = -1$, and $f_y(3, 1) = 10$. How much does z change if x increases by 0.5 and y increases by 0.7?

4. Use the Chain Rule to find dz/dt for the following.
   (a) $f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3$, where $x = 3t$ and $y = t^2$
   (b) $f(x, y) = \cos(xy)$, where $x = 1/t$ and $y = t^3 + t$
   (c) $f(x, y) = x^2 + y^2$, where $x = \sin(2t)$ and $y = \cos(2t)$

5. Use the Chain Rule to find $\partial z/\partial t$ and $\partial z/\partial s$ for the following.
   (a) $f(x, y) = \arctan(x - y)$, where $x = t^2 + s^2$ and $y = 2st$
   (b) $f(x, y) = \cos(x)\sin(y)$, where $x = s^2t^2$ and $y = st$