MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 16: Due Tuesday, November 5

1. For the following function, show that $f_{xy} = f_{yx}$. That is, show that the order in which you take the partial derivatives doesn't matter.

$$f(x, y) = \cos(xy + y)$$

2. Find f_{xx}, f_{yy} and f_{yxy} for the following equations.
   (a) $f(x, y) = x^3 + x^2y + xy^2 + y^3$
   (b) $f(x, y) = e^{xy+1}$
   (c) $f(x, y) = x\cos(y) + y\sin(x) + (xy)^2$

3. Consider the function below and answer the following questions

$$f(x, y) = x^2 + y^2$$

   (a) What is this surface?
   (b) Find the first partial derivatives of f.
   (c) Graph the level curves. At the point $(1, 1)$, draw the vector $\langle f_x(1, 1), f_y(1, 1)\rangle$.
   (d) In what direction does the vector $\langle f_x(1, 1), f_y(1, 1)\rangle$ point (is z increasing or decreasing)?
   (e) Let $F(x, y, z) = x^2 + y^2 - z$. Find the first partial derivatives of F (i.e. F_x, F_y, and F_z). How do these partials relate to the partials of f?
   (f) Graph the surface. At the point $(1, 1, 2)$, draw the vector $\langle F_x(1, 1, 2), F_y(1, 1, 2), F_z(1, 1, 2)\rangle$.
   (g) Is the vector $\langle F_x(1, 1, 2), F_y(1, 1, 2), F_z(1, 1, 2)\rangle$ perpendicular to the surface?

4. If the limit exists, find it. If it does not, prove it.

$$\lim_{(x,y)\to(1,0)} \frac{4y^4\cos(x-1)}{(x-1)^4 + y^4}$$

5. If the limit exists, find it. If it does not, prove it.

$$\lim_{(x,y)\to(1,0)} \frac{5xy}{\sqrt{x^2 + y^2}}$$

6. Let $F(x, yz) = x^2 + y^2 - z^2 - 1$. Answer the following questions.

   (a) When $F(x, y, z) = 0$, what surface do you have? Is the point $(-1, 1, 1)$ on this surface?

   (b) Find $\langle F_x(-1, 1, 1), F_y(-1, 1, 1), F_z(-1, 1, 1)\rangle$.

   (c) Find the equation for the plane who is normal to the vector found in part (6b) and contains the point $(-1, 1, 1)$.

   (d) Justify the claim that the plane in part (6c) is tangent to the surface in (6a) at the point (-1, 1, 1). (Graphing is the best way to do this. You may graph the surface in two dimensions by setting one of the variables fixed, like $y = 1$.)