

MATH 231: Calculus of Several Variables
Section 1, 107 Ag Sc & Ind Bldg,
TR 9:05 AM - 9:55 AM

Homework 12: Due Tuesday, Oct 15

1. Find the arc length of the following equations.

(a) $\vec{r}(t) = (3t + 1)\vec{i} + (t - 1)\vec{j} + (-2t + 10)\vec{k}$ for $0 \leq t \leq 1$

(b) $\vec{r}(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle$ for $0 \leq t \leq \pi/4$

(c) $\vec{r}(t) = \left\langle t, \frac{\sqrt{2}t^2}{2}, \frac{t^3}{3} \right\rangle$ for $0 \leq t \leq 1$

(d) $\vec{r}(t) = \langle 1, t^2, t^3 \rangle$ for $0 \leq t \leq 1$

2. Find the unit tangent vector and calculate the curvature of

$$\vec{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$$

3. Find the unit tangent vector and calculate the curvature of

$$\vec{r}(t) = \langle t^2, \sin t - t \cos t, \cos t + t \sin t \rangle$$

4. Which of the following could be a tangent vector for the equation below?

$$x = t^2, \quad y = \ln(t^2), \quad z = 12$$

(a) $\vec{v} = \langle 2, 0, 12 \rangle$

(b) $\vec{v} = \langle 2, 2, 0 \rangle$

(c) $\vec{v} = \langle 1, 1, 0 \rangle$

(d) $\vec{v} = \langle 1, 0, 12 \rangle$

(e) None of the above

5. Consider the displacement function

$$\vec{r}(t) = \langle 2t - 1, 4t^2 + 2t, 3 \cos t \rangle$$

Which of the following is true?

(a) \vec{r} passes through the point $(-1, 0, 0)$

(b) $\langle 2, 2, 0 \rangle$ is a tangent vector of $\vec{r}'(t)$

(c) At time $t = 0$, the object has a speed of $\sqrt{10}$

(d) Acceleration changes only in the z direction.

(e) None of the above

6. Which of the following functions has constant curvature? (There may be more than one answer)

(a) $y = x$

(b) $\vec{r}(t) = \langle 10t, -2t, t^2 \rangle$

(c) $\vec{r}(t) = \langle \sqrt{2}, t - 9, 10 - 1/t \rangle$

(d) $\vec{r}(t) = \langle t^3, 0, 0 \rangle$

(e) None of the above

7. Which of the following vectors could be the unit tangent vector of

$$\vec{r}(t) = \langle t^2, 3t, 7 \rangle$$

(a) $\langle 0, 0, 0 \rangle$

(b) $\frac{\langle 2, 3, 0 \rangle}{\sqrt{13}}$

(c) $\langle 1, 0, 0 \rangle$

(d) $\frac{\langle 1, 3, 7 \rangle}{\sqrt{59}}$

(e) None of the above

8. Can curvature be negative? Explain.

9. Can displacement be zero but arc length be nonzero? Explain.