A Queued-Code Based on LDPC Block Codes

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***Abstract*— A 'queued-code' is a novel code which allows for instantaneous rate adaptation, to match the current channel state information (CSI), assumed known to the transmitter. Previously, information theoretic codes were analyzed in the regime of large delay bounds. Numerical results had demonstrated the performance gain of these codes over fixed rate codes, in a range of situations. This paper presents queued-codes which are designed using low density parity check (LDPC) block codes. Simulation results demonstrate performance gain of these codes over fixed rate LDPC block codes. The simulation results are consistent with the numerical results obtained for the information theoretic codes, which is a satisfying result. Further, this paper argues for the use of rate adaptation within a codeword, as opposed to rate adaptation across codewords, when CSI is available at the transmitter.**

Keywords: Coding theory, rate adaptive transmission scheme, LDPC, queuing, mobile wireless.

I. Introduction

Traditionally, Quality of Service (QoS) guarantees for delay-sensitive applications, in terms of data rate, error rate and delay bound, have been provided by performing a queuing analysis of networks. [1] considered a pure queuing model which assumed ideal channel codes, which could achieve instantaneous Shannon capacity. Analysis showed that the effect of channel variation on link performance could be captured by a single function called 'effective capacity'. However, a real system will have to use channel codes to mitigate the effect of noise, operating at rates below channel capacity. Therefore, we introduced the concept of a 'queued-code' in [2] which considered the queuing model (for QoS analysis, as in networking) along with an accurate physical layer model for channel coding (for combating noise, as in information theory).

The queued-code combines canonical models from information theory (channel coding) and queuing theory (server+queue). It encompasses the pure coding framework, and thus, is expected to have a better performance than the pure coding approach. Numerical results in the large delay regime [2], [3] showed that the queued-code can achieve a significant performance gain, compared to a pure coding approach, over a range of scenarios. However, those papers considered information theoretic codes which allow rate adaption on a symbol-by-symbol basis, with arbitrary precision in the choice of rates. In this paper, we present a practical queued-code, based on the intuition provided by the information theoretic codes. For simplicity of presentation, we concentrate on block codes, instead of streaming codes, which yield a larger exponent compared to the block codes [2]. In particular, the queued-code is based on low density parity check (LDPC) block codes which provide a large memory, similar to the information theoretic codes, and yet at the same time have efficient decoding algorithms [4], [5], [6]. Simulations using the queued-codes are used to explore an intuition provided by [2], [3], that rate adaptation *within* a codeword, allows one to obtain a larger error exponent, under a delay constraint. Note the difference of this approach with the conventional approach, in which case, rate over each codeword is constant.

Usually, rate adaptive codes are designed by constructing a family of codes, each code for a particular channel state. One way of generating a family of channel codes from a fixed base code, is by puncturing a low rate mother (base) code [7]. With this design, the rate remains constant over the length of a codeword, irrespective of the fading rate. The queued-code in this paper, however, adapts the rate of the code instantaneously, *within* a codeword. Hence, the code can match the channel state within the codeword, by adapting the rate instantaneously, while it can also utilize the large memory provided by a large block length of the code.

There have been other attempts at providing rate adaptation, using channel state information (CSI), to achieve a higher system reliability. One such class of schemes is the hybrid automatic repeat-request (H-ARQ) schemes [8], [9]. In *type-I* H-ARQ schemes, an encoded packet is transmitted; encoded using a code which can perform both error detection and error correction. If the received packet is detected in error, the receiver attempts to correct the errors. If the packet could not be decoded at the receiver, because of large channel noise, the packet is rejected, and a request for packet retransmission is made. A packet is transmitted repeatedly until it is received correctly at the receiver. A disadvantage of this scheme is that the code rate is fixed, once the code is chosen. For a fading channel, a rate adaptive scheme would be preferred, which can adapt the code rate, based on CSI. A *type-II* H-ARQ scheme, remedies the situation, by allowing transmission of incremental redundancy, each time a retransmission is requested [9], [10], [11], [12]. In subsequent transmissions for the same packet, the transmitter only transmits additional parity bits. The parity bits are transmitted until the packet can be correctly decoded, or some maximum number of retransmissions occur. Thus, rate adaptive transmission is achieved, using feedback of success or failure of decoding. There are some key differences between these schemes and the code presented in this paper. The queued-code can achieve rate adaptation within a code block, while the H-ARQ schemes perform rate adaptation across blocks. Further, the H-ARQ

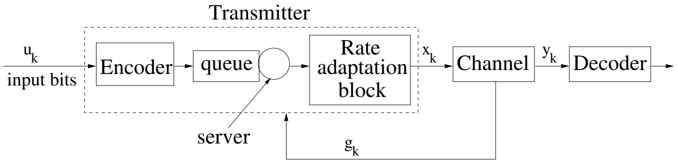

Fig. 1. A queued-code

schemes utilize the information about CSI only in terms of success or failure of decoding. The queued-code, on the other hand, uses the channel gains for the entire packet. Thus, it uses CSI at a much finer scale and hence, is expected to perform better than H-ARQ schemes.

The pure coding schemes approach the problem by designing various codes for different CSI values. It is desirable to have a single encoder and decoder for the various codes. This is achieved by using Rate Compatible Punctured Codes (RCPC) [7]. Rate-compatibility restriction ensures that all code bits of high rate codes are used by low rate codes. In particular, rate compatible LDPC codes have been shown to have good performance over non-uniform channels [13], [14]. However, each of these approaches allow rate adaptation on a block by block basis, i.e., the rate within a block remains constant. When CSI is not available, LDPC codes can be designed for fading channels, based on channel statistics, by designing the node degree distributions [15]. i.e., given the channel statistics, optimum distributions for the number of edges connected to bit nodes and check nodes is determined. However, this assumes that only the channel statistics are known, not the instantaneous CSI. We, in contrast, assume that the channel fading is slow enough, so that the channel gain can be accurately estimated and fed back to the transmitter. Thus, knowing the instantaneous channel gain, we can precisely modify the instantaneous rate of the channel code, to match the CSI. Note that in this paper, we do not attempt to show the best design of such codes, for a given channel. We only aim to demonstrate the utility of rate adaptation, within a block code, as suggested by theory [2], [3].

Section II presents the system model, and the problem setup. Section III provides an overview of the information theoretic queued-code, and the intuition provided by analysis of those codes. The construction of a queued-code, based on LDPC block codes, is then presented. Simulation results for block fading channels are presented in Section IV. Section V concludes the paper.

II. Problem Formulation

We begin by formally describing the system model, shown in Fig. 1. The discrete-time memoryless channel (DMC) model [16] is specified by the conditional probability distribution $p(y_k|g_k, x_k),\ k = 1, 2, \ldots$, where x_k, y_k are the channel input symbol and output sample respectively, at time k, while g_k is the channel state information CSI (e.g., gain) at time k. The CSI is assumed to be known both to the transmitter and the receiver. We assume a block fading channel [17], which has blocks of equal gain g_k, and gains in different blocks are independent and identically distributed (i.i.d.). The fading block length, i.e. the time duration in terms of number of coded symbols for which the CSI remains constant, is denoted by T_c. In simulations, we will assume that the noise is Gaussian.

Fig. 1 shows the system model. The transmitter consists of an encoder followed by a queue+server and a rate adaptation block. Thus, it combines the canonical models of channel coding and queuing. The input to the encoder are information bits, denoted by $\{u_k\}_{k=1}^{K}$. The encoder produces coded bits $\{v_k\}_{k=1}^{N}$ which are modulated using BPSK modulation, to produce coded symbols $\{x_k\}_{k=1}^{N}$ at the output, where $x_k = 2v_k - 1$. The coded symbols are fed into queue+server. The encoder has a fixed rate $\mu = K/N$, called the *base rate*. The queue+server and the rate adaptation block then adapt the rate of the channel code. The server chooses an instantaneous rate $R(g_k) = \alpha_k\mu$ in each fading block, based on the current CSI g_k. The rate $R(g_k)$ is achieved through the rate adaptation block, which can either increase or decrease the base rate, by using puncturing or repetition. Its purpose is to reconcile the variable symbol rate out of the server, with the constant symbol rate transmitted over the channel. The instantaneous rate can be increased ($\alpha_k > 1$) by puncturing $T_p(\alpha_k - 1)/\alpha_k$ symbols in a given block of T_p symbols. e.g., $T_p = 60$, $\alpha = 1.5$ increases the base rate by a factor of 1.5, by puncturing 20 symbols out of 60 symbols. Punctured symbols are not transmitted and hence, the effective rate increases in that fading block. The instantaneous rate can be decreased ($\alpha_k < 1$) by repeating some (or all) of the symbols in a given block. Repetition of symbols produces more coded symbols for the same number of information bits. Hence, repetition decreases the effective rate in a given fading block. At the receiver, the punctured symbols are re-inserted as erasures, into the received stream. For BPSK modulation, this means that the punctured symbols are replaced by 0. For repeated symbols, the receiver averages the symbols, before feeding them into the decoder, thus obtaining an SNR gain. The location of symbols that are either punctured, or repeated, is assumed known to the receiver (e.g., by using a known pseudo-random sequence for puncturing/repeating symbols), and is considered part of the system design. Note that a constant rate of information bits at the input of the transmitter, leads to a variable number of symbols, at the output of queue+server. The rate adaptation block matches the variable symbol rate produced by queue+server with the constant symbol rate transmitted over the channel. A given packet is transmitted, until all the information bits have been encoded and transmitted into the channel. Thus, the time to transmit a packet D, measured in terms of number of transmitted symbols, becomes a random variable, which depends on the particular sequence of CSI realized. At the receiver, a packet is decoded, when it has been received completely.

Let P_{err} be the probability of bit decoding error. Ideally, given the system design described above, base rate μ and the channel statistics, we would like to choose an $R(g)$ function, to minimize P_{err}. This was the problem formulation in [2], [3]. However, the aim of this paper is to present a practical construction of channel codes based on [2], [3], and to demonstrate the utility of rate adaptation within a codeword, in contrast to rate adaptation across codewords.

Hence, we do not attempt to design the 'optimal' channel codes, given the system design explained above, or to find the optimal rate function $R(g)$. Instead, we present specific code designs, based on the intuition obtained from [2], [3], that show a significant gain over the fixed rate codes. Note that our formulation encompasses a pure coding framework (by choosing $R(g_k) \equiv \mu$) which does not allow queuing, and thus, is expected to perform better than the latter.

III. A Queued-Code Based on LDPC Block Codes

A. Background

The key intuition in this paper comes from [2], [3]. Those papers presented a queued-code, which was obtained by combining ideas from queuing and coding theory. The analysis of those channel codes was carried out in the large delay regime, using ideas from large deviation theory (effective capacity), and information theory (random coding bounds). The analysis led to finding asymptotic limits of delay-constrained communications, in terms of error exponents. Numerical results in a variety of scenarios showed that the queued-code achieves a larger error exponent, compared to a pure coding approach, leading to a better Quality of Service (QoS). The QoS was defined as the probability of bit decoding error, given a delay constraint. The key insight in those papers was that a queued-code can help achieve a better QoS, by instantaneously matching the transmission rate (within a codeword) with the current CSI, while simultaneously utilizing the long memory provided by the channel code. The papers considered streaming codes, since, block codes require additional delay to buffer the data before a block of data can be encoded. Thus, the idea of streaming codes was important, in a delay-constrained scenario, because it allows for stream decoding of the channel code. However, the idea of using rate adaptation within a codeword, to match the instantaneous rate with the current channel state, and thus achieving a better QoS, is also important by itself. In this paper, we explore this idea of *instantaneous* rate adaptation. To simplify the presentation, we use block codes to explore this idea. In particular, we use low density parity check (LDPC) codes, which have been shown to achieve excellent performance in AWGN and fading channels [6], [18], [19]. The key motivation for using LDPC codes was that these codes provide a large memory, similar to information theoretic codes, while allowing a simple decoder.

Numerical results in [2], [3] showed that the gain provided by the queued-code depends on the fading rate. If the channel is fast fading, i.e. there are a large number of fades per codeword so that ergodic capacity can be achieved, choosing a fixed rate code is sufficient. However, if the fading is slow, there may be an outage if the code rate is larger than the average capacity for the particular sequence of channel gains. By adapting the code rate, within a codeword, the channel code can better match the channel realization, and hence, reduce the outage probability. Thus, the simulations results in this paper, will consider scenarios where fading is slow. For a slow fading channel with $T_c \geq N$, each coded block (packet) experiences the same channel gain, and hence, the adaptive code reduces to a constant rate code, within each fade. Thus, it reduces to a traditional rate adaptive system, where each packet has a fixed rate, and rate adaptation is performed on a packet by packet basis. In order to show the benefit of rate adaptation within a codeword, we will explore scenarios where $T_c < N$.

B. An Instantaneously Rate Adaptive LDPC code

[2], [3] presented an information theoretic code that allows rate adaptation on a symbol by symbol basis. The transmitter consisted of a queue+server followed by a variable rate encoder. The instantaneous rate of the code was determined by the server, which chose an instantaneous server capacity $R(g_k)$, based on the current CSI g_k. The variable rate encoder receives bits at a rate $R(g_k)$, encodes them into a sequence of symbols, and transmits them into the channel. Variable rate encoding is achieved by encoding different number of information bits into a fixed number of symbols. Thus, the rate of coded symbols produced by the encoder is independent of the $R(g)$ function. In this paper, we present a queued-code, which we call an instantaneously rate adaptive code, based on the information theoretic code, but with some modifications, in order to ease the code design. The instantaneously rate adaptive LDPC code is constructed by modifying a constant rate LDPC code with rate μ, called the *base code* (encoder in Fig. 1). The construction of the instantaneously rate adaptive LDPC code is described in Section II.

The instantaneously rate adaptive LDPC code is constructed by modifying a constant rate LDPC code with rate μ. The instantaneous rate of the code is modified, based on the current CSI g_k, by using puncturing or repetition. As noted in Section II, the time to transmit a packet D, measured in terms of number of transmitted symbols, is a random variable, whose value depends on the particular sequence of CSI realized. $R(g)$ function can be designed such that $\mathbf{E}[R(g)]$ is sufficiently greater than μ, so that $Pr\{D > (1+\epsilon)N\}$ can be made sufficiently small, where N is the block length of the LDPC code and ϵ is a small fraction. For simulation results, we will present some statistics of D. The channel code presented in this paper will be appropriate for applications which have an average latency requirement.

We note that the variability of D is the result of considering a block code instead of a streaming code. Block codes were considered to simplify the presentation, and to demonstrate the utility of rate adaptation within a codeword, independent of other effects. We also note that a simple rate adaptation method was applied to the fixed rate codes, by puncturing and repetition of the coded symbols. Again this was done for simplicity in presentation, and to clearly demonstrate the utility of instantaneous rate adaptation.

[2], [3] assumed a fluid model for bits, and hence all possible non-negative rates were allowed for the queued-code. In a practical code, in general, it may not be possible to control the rates with arbitrary precision. However, the use of LDPC codes of large memory allows us to finely control the rates, by puncturing or repeating only some of the bits in a large block of symbols, if the fading is slow. In the next section, we present simulation results, to demonstrate the benefit of rate adaptation within a codeword.

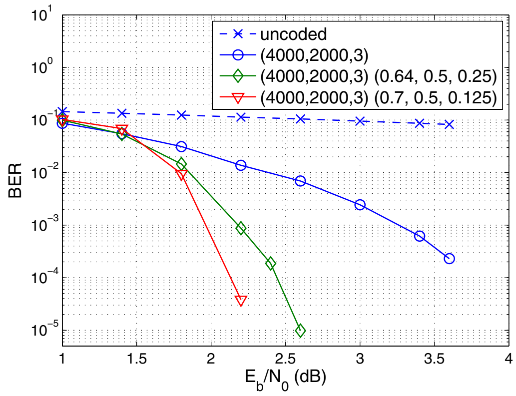

Fig. 2. BER performance for rate $1/2$ codes over a 3-state channel

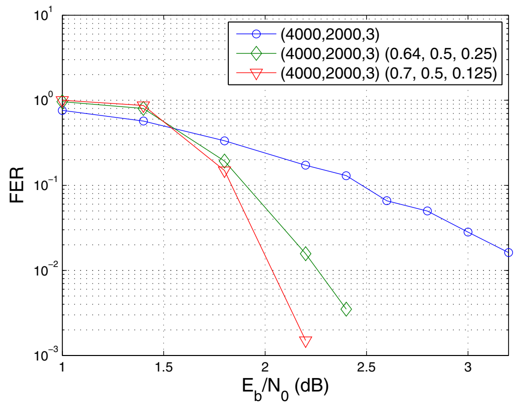

Fig. 3. FER performance for rate $1/2$ codes over a 3-state channel

IV. Simulation Results

We compare a queued-code with a fixed rate code. The variable rate code admits the realization of a fixed rate code, by setting $R(g_k) \equiv \mu$, and hence, with a correct choice of the rate function, cannot perform worse than a fixed rate code.

Assume a simple Gaussian channel model, $y_k = \sqrt{g_k}x_k + n_k$, where the noise n_k is Gaussian with variance σ_n^2, and x_k are BPSK modulated symbols transmitted at a fixed power 1 per symbol. The channel gains can take one of the finite number of values, $g_k \in \{g_1, g_2, \ldots, g_n\}$. We assume a block fading model, i.e., channel gains remain constant over T_c symbols, and are i.i.d. across blocks. The channel gain statistics are specified by the channel probability mass function $p(g) = (p_1, p_2, \ldots, p_n)$. A range of average signal-to-noise-ratio (SNR) $E_b/N_0 = \mathbf{E}[g]/(2\mathbf{E}[R(g)]\sigma_n^2)$ and fading block-length T_c are simulated. Bit error probability (BER) and Frame error probability (FER) for both the queued-code and the fixed rate code are estimated, using Monte-Carlo simulations. For the queued-code, the rate function $R(g)$ is chosen, such that $\mathbf{E}[R(g)] > \mu$. We did not attempt to exhaustively optimize the $R(g)$ function for the queued-code. Instead, various $R(g)$ functions were evaluated, guided by the numerical results in [2], [3].

Figures 2, 3, 4 and 5 show that the queued-code provides a performance gain over the pure coding system for a wide range of E_b/N_0 and T_c. Fig. 2 shows the variation of bit error rate (BER) with E_b/N_0 for $\mu = 1/2$ and $T_c = 400$. A 3-state channel was simulated with $p(g) = (0.2, 0.7, 0.1)$. The possible channel gains were 9, 6, 0 dB. A $(4000, 2000, 3)$ LDPC code was chosen of block length $N = 4000$, rate $1/2$ and column weight 3, available at [20]. With this T_c, each coded block of the fixed rate code experiences 10 fades. Thus, the constant rate code has a certain outage probability. The number of fades experienced by the variable rate code depends on the particular sequence of CSI. Two variable rate codes were simulated with the following rate functions, $R_1(g) = (0.64, 0.5, 0.25)$, $R_2(g) = (0.7, 0.5, 0.125)$. $R_2(g)$ represents a more aggressive rate adaptation compared to $R_1(g)$. The variable rate code with rate function $R_1(g)$ achieves more than 1 dB of gain over the fixed rate code, at a BER of 10^{-4}. The variable rate code with rate function $R_2(g)$ achieves a larger gain, showing that a more aggressive rate adaptation can lead to a larger gain. For $R_1(g)$, the average delay was 3987, standard deviation was 250.4, and the 95-percentile was 4401 (i.e., a 10% increase). For $R_2(g)$, the average delay was 3990, standard deviation was 373.5, and the 95-percentile was 4629 (i.e., around 15% increase). Note that the queued-code performs well in terms of the average delay metric, and hence, is appropriate for applications with average latency requirements. Further, a more aggressive rate adaptation results in a larger variability in delay, as expected. Thus, we can trade-off the gain resulting from rate adaptation with the delay statistics. The performance difference between the fixed rate code and the variable rate code increases with SNR and thus, a larger gain is expected at lower BER. This trend is consistent with the numerical results in [2], [3]. Fig. 3 shows the variation of FER with E_b/N_0 for the same setup, as described above. The variable rate codes achieve substantial gains over the fixed rate code, in terms of FER as well. Again, a more aggressive rate adaptation provides a larger gain.

Fig. 4 shows a similar variation of BER with E_b/N_0, for channel codes of different rates. The following channel codes were chosen, a $(1920, 1280, 3)$ LDPC code, of rate $1/3$ and block length 1920, and a $(4200, 1400, 3)$ LDPC code, of rate $2/3$ and block length 4200. The first code was obtained from [20], while the second code was generated using R. Neal's software for LDPC codes [21], using random code construction. The same 3-state channel was used for simulations, as for the previous figure. The $R(g)$ functions for the queued-codes corresponding to rate $1/3$ and rate $2/3$ LDPC codes were chosen as $(0.43, 0.33, 0.17)$ and $(0.86, 0.67, 0.33)$ respectively. The figure shows that the variable rate code performs better than the fixed rate code, for LDPC codes of various rates. For rate $1/3$ codes, the variable rate code achieves a gain of 1 dB, at BER of 10^{-4}. For rate $2/3$ codes, the variable rate code achieves a larger gain, of almost 2 dB, at a BER of 10^{-4}.

Fig. 5 shows the variation of BER with fading block length T_c at $E_b/N_0 = 2$ dB. The simulations were performed for the $(4000, 2000, 3)$ LDPC code. Again, the same 3-state channel was used. The $R(g)$ function was chosen as $(0.64, 0.5, 0.25)$, for the variable rate code. The figure shows that the performance difference between the fixed rate code and the variable

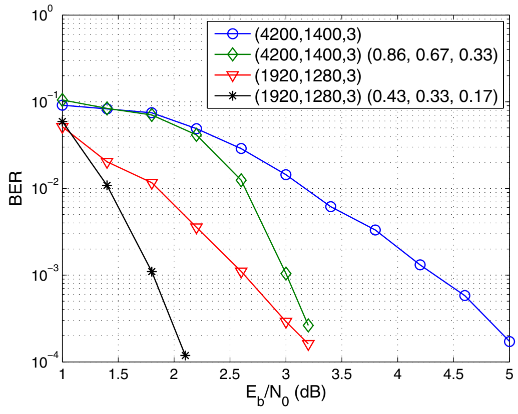

Fig. 4. BER performance for rate $1/3$ and rate $2/3$ codes

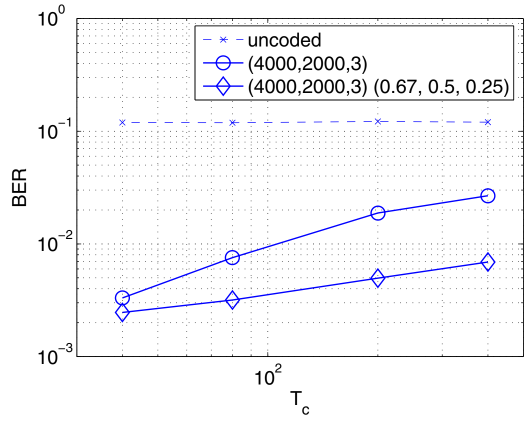

Fig. 5. BER performance with fading block length

rate code increases with the fading block length T_c. Again, this result is consistent with the numerical results in [2], [3].

Similar results were obtained for other channel models. e.g., for a 5-state channel with possible channel gains 12, 9, 6, 3, 0 dB and $p(g) = (0.15, 0.25, 0.45, 0.1, 0.05)$, simulations were performed for the $(4000, 2000, 3)$ LDPC code. The fading block length T_c was chosen as 400. A variable rate code with $R(g) = (0.65, 0.59, 0.5, 0.25, 0.25)$ achieved a gain of 1 dB, at a BER of 10^{-4}.

V. Conclusion

This paper presented a practical channel code corresponding to the information theoretic queued-code, presented in our previous papers. The queued-code was implemented by adding rate adaptation to LDPC block codes. Simulation results for some simple fading channels showed that queued-codes perform better than fixed rate LDPC codes, over a range of scenarios. The simulation results were consistent with the numerical results obtained for the information theoretic queued-code, which is a satisfying result. Thus, it was shown that it is possible to construct practical queued-codes, by using simple rate adaptation techniques with standard block codes. In future, we will explore the construction of streaming queued-codes. Further, the simulation results showed the benefit of rate adaptation within a codeword, as opposed to rate adaptation across codewords, in a fading scenario. Thus, it was shown that channel codes can achieve better performance, under fading scenarios, by adapting the instantaneous rate to match the channel state, if CSI is available at the transmitter.

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