Indoor Navigation without the use of GPS utilizing Intelligent Data Algorithms

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Abstract

Navigation in today’s world is dominated by the Global Positioning System (GPS). GPS is becoming commercially and socially accepted as the best position reporting system. While this system offers users unprecedented position accuracies, GPS marginally works indoors. The GPS system relies on satellites to solve for a user’s position. These satellites have a very low signal power and most receivers have trouble acquiring/tracking indoors due to the significant amount of obstructions. To solve this problem, a methodology is discussed to perform indoor navigation with commercially available parts. Three separate data sources are combined in this indoor navigation system: an inertial navigation system, distance/wall sensors, and a digital map of the building. Also provided in the paper, are the results from a software simulation of the indoor navigation system. By intelligently combining the three separate data sources, better accuracies than a standalone GPS receiver can be achieved (0.5 meters vs. 5 meters).

Keywords

Indoor Navigation, Inertial Navigation System, INS, Distance Sensors, Wall Sensors

1 Introduction

The Global Positioning System (GPS) is an intoxicating system, offering a user real-time 5-meter horizontal absolute RMS (root-mean-square) position accuracy. GPS works by measuring the distance from a user’s current location to multiple satellites in orbit. These distances are called pseudoranges. From the pseudoranges, a position solution is formulated. But the satellite signals have low power, so GPS does not always work indoors. Just because GPS does not work well indoors, does not mean that indoor navigation is not wanted. Indoor navigation will have an affect on a wide variety of applications: watches, laptops, PDAs, cell phones, etc. Maybe soon, when a person instant-messages a friend, their current location will also pop up.

Indoor navigation is not a new idea; robot hobbyists have been doing small-scale indoor navigation for many years. The systems used have mostly relied on simple dead reckoning. Simple dead reckoning is easy and the devices are usually small with little power consumption. Normally with this method, position accuracies quickly degrade over time. Alternative to cheap dead reckoning is to use an Inertial Navigation System (INS). An INS is a more complex dead reckoning system that senses motion and rotations directly. Accurate INSs of the past have been limited to military vehicles or relatively high value commercial applications such as commercial airlines. These devices were expensive and large and not suited for robotics, human wearing, or other small applications.

Only until now has technology been ready to take on the indoor navigation problem. Smaller and faster computers allow for more powerful algorithms. Computers can store huge amounts of data, such as a digital map of a building. Sensors have also gotten more accurate and less expensive. An INS adequate for this task can be bought relatively
cheaply. With the coming age of Micro-Electro-Mechanical Systems (MEMS), an average INS will also be very small and unnoticeable. Distance sensors (to detect a distance to an object) are commercially available with sub-meter accuracies.

2 Indoor Navigation Filter Theory

Fusing of an INS, distance sensors, and a digital map of a building with an indoor navigation filter will provide the solution to the indoor navigation problem. The components will not function accurately alone. The INS position accuracies will grow unbounded over a short amount of time. The distance sensors will not know what direction they are sensing without INS attitude outputs. The digital map provides the estimated ranges for the distance sensor range measurements. Within the overall indoor navigation filter is the distance sensor filter. This filter specifically computes the user’s horizontal position from the difference between measured and estimated ranges. This indoor navigation system is concerned with a user’s horizontal position and not vertical. The user is assumed to always be on the same floor of the building. The indoor navigation filter in Figure 1 intelligently combines the data sources and keeps the position uncertainties relatively low.

2.1 Inertial Navigation System

The main purpose of an inertial navigation system for the indoor navigation filter is to determine the orientation of the distance sensors. The INS also has the benefit of being able to determining a user’s velocity and position. To do all of this, an INS usually incorporates six sensors: three accelerometers and three gyroscopes. Accelerometers measure the users acceleration while the gyroscopes measure rotation rate.

The main types of INSs are gimbaled and strapdown. In a gimbaled system, the gyroscopes mechanically rotate the accelerometers, keeping the accelerometer axis aligned with the Navigation coordinate frame. The Navigation coordinate frame is the plane tangent to the Earth’s surface at a given location (also referred to as the local tangent frame). In a strapdown system, there are no moving parts. It is up to the computer to resolve the acceleration into the Navigation coordinate frame. Today, almost all systems are strapdown because the computation power is available and there are no moving parts; thus resulting in a smaller, lighter, low cost system.

To get position, the INS has to integrate the user’s accelerations as measured by the accelerometers. This is done in the Navigation coordinate frame. This way the system can keep track of how much the user has moved East, North,
and Up. In a strapdown system, the accelerometers measure acceleration in the Body coordinate frame. The Body coordinate frame is directly related to the Navigation coordinate frame though roll, pitch, and heading. As the INS rotates, so does the coordinate frame. The INS accelerometers measure acceleration in the Body coordinate frame, but they need to be in the Navigation coordinate frame since that is where a user’s position is defined.

The solution to this is to use a direction-cosine-matrix to rotate the accelerations from the Body coordinate frame into the Navigation coordinate frame. Equation 1 shows the calculation of rotating the acceleration vector measured in the Body coordinate frame to an acceleration vector in the Navigation coordinate frame.

$$\ddot{A}_N = C_{N/B} \ddot{A}_B$$

(1)

$\ddot{A}_N$ is the acceleration vector in Navigation coordinate frame. $\ddot{A}_B$ is the acceleration vector in Body coordinate frame. $C_{N/B}$ is the Body-to-Navigation direction-cosine-matrix (defined later). Equation 1 is not used because it is better to transform incremental velocity ($\Delta \dot{V}$, deltavees) from Body to Navigation than to go directly from acceleration. Fried & Kayton (1997) explain it as, “The use of incremental velocities instead of instantaneous acceleration is important to preserve the correct velocity in the presence of changing acceleration during a sampling interval.”

$$\Delta \dot{V}_B(t) = \frac{1}{2} \ddot{A}_B(t-k)dt + \frac{1}{2} \ddot{A}_B(t)dt$$

(2)

$$\Delta \dot{V}_N(t) = C_{N/B}(t)\Delta \dot{V}_B(t)$$

(3)

Equation 2 integrates $\ddot{A}_B$ into $\Delta \dot{V}_B$ using trapezoidal integration. Equation 2 is not very useful for the reason provided by May (2002), “In general, the accelerometers provide a pulse train whose frequency is proportional to acceleration. This pulse train is accumulated, and therefore, only deltavees are available anyway.” Equation 3 then rotates the $\Delta \dot{V}_B$ from the Body coordinate frame into the Navigation coordinate frame. Equation 2 and Equation 3 do not take into account sculling errors. Sculling errors are the errors due to the fact that the direction-cosine-matrix is also changing with time. Neglecting the effect of sculling will have a negligible effect due to the low accuracies of the sensors used in the INS for this indoor navigation filter.

$$\ddot{V}_N(t) = \ddot{V}_N(t-k) + \Delta \dot{V}_N(t)$$

(4)

$$\dddot{P}_N(t-k) = \dddot{P}_N(t-k) + \frac{1}{2} \dddot{V}_N(t-k)dt + \frac{1}{2} \dddot{V}_N(t)dt$$

(5)

Equation 4 uses $\Delta \dot{V}_N$ to update the user’s velocity in the Navigation coordinate frame ($\dddot{V}_N$). Equation 5 updates the user’s position ($\dddot{P}_N$) from velocity ($\dddot{V}_N$) using trapezoidal integration.

If roll ($\phi$), pitch ($\theta$), and heading ($\psi$) of the user are known, the direction-cosine-matrix from Body to Navigation is defined in Equation 6 (Farrel & Barth, 1998).

$$C_{N/B} = \begin{bmatrix}
\cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\theta) + \cos(\psi)\sin(\theta)\sin(\phi) & \sin(\psi)\sin(\theta)\sin(\phi) + \cos(\psi)\sin(\theta)\cos(\phi) \\
\sin(\psi)\cos(\theta) & \cos(\psi)\cos(\theta) + \sin(\psi)\sin(\theta)\sin(\phi) & -\cos(\psi)\sin(\theta)\sin(\phi) + \sin(\psi)\sin(\theta)\cos(\phi) \\
-\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\theta)\cos(\phi)
\end{bmatrix}$$

(6)

Because the INS uses gyroscopes, roll, pitch, and heading cannot be computed directly. The gyroscopes measure the rate of change of roll, pitch, and heading. Integrating these rates independently to get the desired answer will not
work because the coordinate frame is also rotating. So a pure roll rate measurement in one time frame may be coupled with pitch in the next.

To calculate $B_N^C$ from roll, pitch, and heading rates, quaternions are used. A Quaternion is a compact mathematical way to depict rotations. Initialization of a quaternion ($b$) from a direction-cosine-matrix is given in Equation 7 (Barth & Farrel, 1998).

$$b = \begin{bmatrix}
\frac{C_{g_{11}} - C_{g_{22}}}{4b_1} \\
\frac{C_{g_{21}} - C_{g_{33}}}{4b_2} \\
\frac{C_{g_{32}} - C_{g_{11}}}{4b_3} \\
\frac{1}{2}\sqrt{4C_{g_{11}} + C_{g_{22}} + C_{g_{33}}}
\end{bmatrix}$$

An initial direction-cosine-matrix is obtained through special initialization procedures. When an INS starts out on a stationary platform, the only accelerations are due to the earth’s gravity. Also, the only rotation during this alignment is due to the earth’s rotation. To mathematically level the INS, a direction-cosine-matrix is computed such that no accelerations are sensed in the north and east direction, and no angular rotation is sensed around the east gyroscope.

Extraction of roll ($\phi$), pitch ($\theta$), and heading ($\psi$) from a quaternion ($b$) is given in equations 8, 9, 10 (Farrel & Barth, 1998). Equation 9 is written wrong in Barth & Farrel (1998). Using a similar equation from Fried & Kayton (1997), Equation 9 has been corrected.

$$\phi = \arctan \left( \frac{2(b_2b_3 - b_1b_4)}{1 - 2(b_1^2 + b_2^2)} \right)$$

$$\sin(\theta) = -2(b_1b_3 - b_2b_4)$$

$$\psi = \arctan \left( \frac{2(b_1b_2 - b_3b_4)}{1 - 2(b_2^2 + b_3^2)} \right)$$

The INS gyroscopes measure roll rate ($p$), pitch rate ($q$), and heading rate ($r$). To use these measurements, integrate the instantaneous rate measurements over the measurement-sampling interval to get $P$, $Q$, and $R$. Equation 11 shows this mathematically. Equation 12 shows how to update the quaternion from the integrated rates (Barth & Farrel, 1998).

$$P = \int_{t_i}^{t} p(t) dt \quad Q = \int_{t_i}^{t} q(t) dt \quad R = \int_{t_i}^{t} r(t) dt \quad \dot{\theta} = (P, Q, R)$$

$$b(t) = \begin{bmatrix}
\cos \left( \frac{P}{2} \right) & \frac{R}{2} & -Q & -O \\
-\frac{R}{2} & \cos \left( \frac{P}{2} \right) & P & Q \\
-\frac{Q}{2} & P & \cos \left( \frac{P}{2} \right) & O \\
-\frac{O}{2} & Q & -P & \cos \left( \frac{P}{2} \right)
\end{bmatrix} b(t_i)$$

Equations 1 to 12 contain all the information needed to provide INS data to the indoor navigation filter.
2.2 Distance Sensors

Distance Sensors are the way that the indoor navigation filter senses the environment (walls of a building). Distance sensors are relatively cheap for good accuracy. Most distance sensors are an active sensor, meaning that they send out a pulse (light or ultrasonic) to detect an object. Two important factors of distance sensors are their effective range and accuracy. Light type sensors are more common for distances less than a meter, while ultrasonics have a range up to tens of meters. The main disadvantage of ultrasonic sensors is that they have a lower accuracy than light type sensors. Ultrasonic sensors normally have an accuracy of tenth of a meter, while light type sensors have an accuracy of hundredth of a meter.

The distribution of the distance sensors on the physical structure of the indoor navigation system is important. Figure 2 shows a 2-D example of position estimation with range measurements. The uncertainty of the position depends upon both the quality of the range measurement and the geometry of range sensors. The gray areas in Figure 2 show the uncertainty in a position solution from ranging at different angles. The setup in the middle provides the best geometry. Figure 3 shows examples of good and bad distance sensor arrangements. The total number of distance sensors need to be distributed evenly.

![Figure 2](image1.png)

*Figure 2 The shaded region represents the uncertainty in the position estimate based on different geometric configurations (Enge & Misra, 2001)*

![Figure 3](image2.png)

*Figure 3 Examples of good and bad geometry arrangements for the distance sensors*

To take all the distance sensor measurements and get a position, a distance sensor filter is needed. The distance sensor filter, which is part of the indoor navigation filter, will take an arbitrary amount of sensor measurements and figure out the user’s position (fully explained later). To calculate an estimated distance/range from the user to the wall requires knowledge of where the walls are. The digital building map is used for this. Using the difference between the measured range and the estimated range, the filter uses a least-squares solution to solve for position. Table 1 defines some parameters to be used in the distance sensor filter. Figure 4 shows pictorially the definition of the sensor mount angle.

![Figure 4](image3.png)

*Figure 4 Sensor Mount Angle*
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^n$</td>
<td>$n^{th}$ sensor mount angle with respect to the INS in the Body coordinate frame (Figure 4)</td>
</tr>
<tr>
<td>$\rho^n_B$</td>
<td>Measured range for the $n^{th}$ distance sensor in the Body coordinates frame</td>
</tr>
<tr>
<td>$E\rho^n$</td>
<td>Estimated range for the $n^{th}$ distance sensor</td>
</tr>
<tr>
<td>$W(x, y, \psi)$</td>
<td>Function to calculate the distance/range to a wall from the digital building map</td>
</tr>
<tr>
<td>$\text{bias}^n$</td>
<td>Sensor bias for the $n^{th}$ distance sensor (initially zero)</td>
</tr>
</tbody>
</table>

Equations 13 to 19 are the equations to solve for the user’s position from the range measurements.

$$
\tilde{\rho}_N^n = C_{N/B} \begin{bmatrix} \cos(\alpha^n) \\ \sin(\alpha^n) \end{bmatrix} \left( \rho^n_B - \text{bias}^n \right)
$$ (13)

$$
M_N^n = \sqrt{\left(\rho_{N,x}^n\right)^2 + \left(\rho_{N,y}^n\right)^2}
$$ (14)

$$
E\rho^n = W(x, y, \psi + \alpha^n)
$$ (15)

$$
Z^n = M_N^n - E\rho^n
$$ (16)

$$
H = \begin{bmatrix} -E\rho_1^n & -E\rho_2^n \\ E\rho_3^n & E\rho_4^n \\ -E\rho_5^n & -E\rho_6^n \\ E\rho_7^n & E\rho_8^n \end{bmatrix}
$$ (17)

$$
\delta\hat{x} = (H^T H)^{-1} H^T Z
$$ (18)

$$
\hat{\rho} = \hat{\rho} + \delta\hat{x}
$$ (19)

When the distance sensors make a range measurement, the measurement is in the Body coordinate frame. Equation 13 rotates the measurement ($\rho_B^n$) into a vector in the Navigation coordinate frame ($\tilde{\rho}_N^n$). $C_{N/B}$ is created from the INS attitudes as described in Equations 6 to 21. Equation 14 calculates the measured horizontal range in the navigation coordinate frame ($M_N^n$). $E\rho^n$ is the estimated horizontal range to the closest wall calculated from the digital map based on current location and current direction/heading (Equation 15). Taking the measured range and subtracting what the filter thinks the range should be (the estimated range), an error is formed (Equation 16). $Z$ is an array of the error measurements from all the distance sensors at time $t_k$. Equation 17 is the way to fill the geometry matrix ($H$) (Enge & Misra, 2001). There is no bias term in the $H$ matrix because a bias is not common to each range measurement. Unlike GPS, each range measurement has its own bias term and cannot be solved for directly in this least-squares solution. The least-squares equation is defined in Equation 18 (Enge & Misra, 2001). Before equation 19 can be used, a reasonability check is performed. This will keep the filter from moving the user through a wall.
bias for each distance sensor gets calculated by building a range measurement difference table. Using past residuals from each distance sensor, a bias can be calculated for each distance sensor by averaging the measurement errors over the length of the table.

A problem that the distance sensor filter has to take into account is the fact that this is a non-linear problem. This is non-linear because as the filter updates the user’s position, the part of the wall that is ranged from also changes. This can be demonstrated in Figure 5. To deal with the non-linearity, the filter is run in a loop until the filter has stopped moving the position (homes in on an answer).

2.3 Digital Map of a Building

Maps of the building are very important to the indoor navigation filter. Since computers can store lots of data relatively cheaply, it is feasible to store a building’s internal structures. It is assumed in this paper that the user has a pre-made map.

3 Simulation

A simulation of the indoor navigation system has been developed to demonstrate the concepts described in the previous section. Figure 6 shows a high-level block diagram of the simulation.

The simulation starts by getting a user’s state from the trajectory generator. The state of the user is position, velocity, acceleration, jerk, attitude, attitude rates, and attitude accelerations. Attitude is roll, pitch, and heading. The trajectory generator follows a set of operator entered time sequenced commands. As time progresses, the commands are followed. The trajectory generator in the simulation is kinematically correct: the derivative of position is velocity, the derivative of velocity is acceleration, and the derivative of acceleration is jerk. Since the
The trajectory equations that make the user turn/move from waypoint to waypoint are done by applying jerk and attitude acceleration to the user states. This way, only jerk and attitude acceleration are non-continuous in time. By integrating jerk and attitude accelerations up to position and attitude, true motion is achieved. Table 2 describes the parameters used in the simulation. Figure 7 shows the true user trajectory and true wall positions.

Table 2 Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Rate</td>
<td>100 Hz</td>
</tr>
<tr>
<td>Number of Monte Carlo Runs</td>
<td>20</td>
</tr>
<tr>
<td>Digital Map</td>
<td>10 meter by 10 meter room</td>
</tr>
<tr>
<td>Waypoint and Heading Commands</td>
<td>Walk around the room, shown in Figure 7</td>
</tr>
<tr>
<td>Human Wobble</td>
<td>± 1.0° in pitch and roll</td>
</tr>
<tr>
<td></td>
<td>± 0.5° in heading</td>
</tr>
</tbody>
</table>

Figure 7 True user trajectory and true wall positions used in the simulation

The indoor navigation filter is intended for human use. Therefore, a parameter to make the motion of the user more human-like is added (human wobble). This wobble accounts for the wave-like motion humans exhibit during walking.

At every distance sensor update, true range measurements for each distance sensor are calculated using the true user state and the true digital building map. Each calculated range has to have the roll, pitch and heading of the user calculated in it. These range measurements are then corrupted to simulate a true distance sensor. The inertial navigation system is simulated in the same manner. The indicated map is made at the beginning of the simulation by corrupting the true map wall coordinates. This simulates a map survey error.

Measurements used in the simulation are dependent on the accuracies of the components. Table 3 shows the numbers used in the simulation.
Table 3 Sensor Errors in Simulation

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>5 mg ($\sigma$) noise</th>
<th>0.1 mg ($\sigma$) bias</th>
<th>ADXL202 MEM Accelerometer (Analog Devices Inc, 2002)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gyroscope</td>
<td>0.05 deg/sec ($\sigma$) noise</td>
<td>0.001 deg/sec ($\sigma$) bias</td>
<td>Gyrochip II (GlobalSpec Inc, 2002)</td>
</tr>
<tr>
<td>Map Error</td>
<td>0.2 meter ($\sigma$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance Sensor</td>
<td>0.1% of the range ($\sigma$) noise</td>
<td>0.01 meter ($\sigma$) bias</td>
<td>range measurements are within 0.05 to 12 meters each sensor gets its own unique bias Ultrasensor Configurable Material Sensor (Senix Corporation, 2002)</td>
</tr>
<tr>
<td>Distance Sensor update rate</td>
<td>1 second</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Distance Sensors</td>
<td>4 arranged 90° apart</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 Results

All figures in this section are based on Table 2 and Table 3 specifications unless otherwise noted. The figures were created by running the simulation 20 times in a Monte-Carlo fashion. The results shown are the RMS of the 20 runs’ radial position accuracy. On the same figure, a straight line of the form $y = mx + b$ is fit to the RMS data. The slope ($m$) is the error growth over time (meters per second). The bias $b$ (meters) is the initial accuracy at the beginning of the simulation.

As described in the Distance Sensor section, the indoor navigation filter is sensitive to the arrangement of the distance sensors. Figure 8 shows the difference between the four geometry arrangements of Figure 3.

![Figure 8 The difference between distance sensor geometries](image)
The repeated spikes in Figure 8 result from the user’s trajectory and its relationship to the walls in the room. The user’s trajectory is repeating about every minute. Between distance filter updates, the user’s position is slave to the INS. At one point in the trajectory, the position of the user is such the distance sensor filter cannot make an update. This is because the user’s assumed position is too near a wall. This also causes the estimated ranges to fall below the sensors minimum operating range. To contribute to the problem, the distance filter does not allow moving the user through a wall. Figure 9 shows the user’s indicated position and the indicated wall positions.

![Figure 9 Indicated user's trajectory and indicated wall positions used in the simulation](image)

The distance sensor update rate is also an important factor. Between updates, the INS controls the indicated position. The INS is nowhere near as accurate as the distance sensor measurements. So the longer interval between updates, the more the user’s position drifts. It is important to have an update rate compatible with the dynamics of the user and the amount of INS drift. Figure 10 shows three update rates: 1 second, 10 seconds, and 20 seconds. 20 seconds is the worst because the user has to rely on the INS too much. Also, the user has finished about a third of the trajectory without ever having taken one distance sensor position update.

![Figure 10 The difference between distance sensor update rates](image)
No matter how the digital map of the building is created, there will be residual map errors (survey errors). The map error will show up directly in the RMS radial position accuracy. Figure 11 shows the error from having no map error, a 0.2 meter ($\sigma$) error, and a 0.5 meter ($\sigma$) error. The map error is very important because the indoor navigation filter cannot do anything special to mitigate the effects. The better the map, the better the position fix.

![Figure 11 The difference between map survey errors](image)

Figure 12 presents the result of an hour run. The error growth is due to the INS heading error growing over time. After about 55 minutes, there appears to be no data. This is because the indoor navigation filter gets lost. The heading error is so large (20 degrees) that the filter cannot work. Using other sensors can clamp the heading error, but they are not discussed in this paper.

![Figure 12 Simulation of the indoor navigation filter over one hour](image)
5 Conclusion

Indoor navigation is starting to come of age. With GPS not available indoors, new techniques will need to be thought out. This paper offers one methodology to solve the indoor navigation problem. By using commercially available hardware and intelligent algorithms, indoor navigation without GPS is possible. The indoor navigation problem is solved specifically by combining multiple data sources. Combining different data sources reduces the individual sort-comings of each sensor alone. Using the intelligent algorithms discussed in this paper minimizes those errors.

The simulation proves the theory and gives an insight into some interesting aspects of the indoor navigation system. The arrangement of the distance sensors on the user/platform is important. By planning a good layout for the distance sensors, better position accuracies can be achieved. Because the INS has a large error growth rate, the frequency at which distance sensor measurements are made has to be taken into account. By letting the INS transition too long between measurements will cause the indoor navigation filter to get “lost.” A way to get better accuracies is to get more accurate maps. Map errors will “make or break” the indoor navigation system. The simulation results of the indoor navigation system are very good. Position accuracies of less than half a meter for about hour is the kind of accuracy that is required by a modern indoor navigation system.

Recommendations for further study are …

Add a 3-axis magnetic north sensor to clamp the attitudes errors from drifting.

Use a Kalman filter to estimate INS errors and distance sensor misalignment errors.

Combine ultrasonic sensors with optical sensors to achieve the optimum accuracies for variable distances.

Use two digital cameras in a stereographic setup to determine ranges.

6 References


Penn State ARL NRDC (2001), *Trajectory Output Generator Control Loops*. Penn State ARL, Warminster PA.
