Public Disclosure and Dissimulation of Insider Trades*

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Abstract

Regulation requiring insiders to publicly disclose their stock trades after the fact complicates the trading decisions of informed, rent-seeking insiders. Given this requirement, we present an insider’s equilibrium trading strategy in a multiperiod rational expectations framework. Relative to Kyle (1985), price discovery is accelerated and insider profits are lower. The strategy balances immediate profits from informed trades against the reduction in future profits following trade disclosure and, hence, revelation of some of the insider’s information. Our results offer a novel rationale for contrarian trading: dissimulation, a phenomenon distinct from manipulation, may underlie insiders’ trading decisions.

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I. Introduction

Under US securities laws, insiders (including officers, directors, and beneficial owners of five percent or more of equity securities) associated with a firm must report to the Securities and Exchange Commission (SEC) trades they make in the stock of that firm. These reports are filed after the trade is completed and are publicly available immediately upon receipt by the SEC. The regulatory objectives of public disclosure of insider trades include curbing unfair enrichment by those with access to private information and preserving market integrity.

We study the effect of trade disclosure on the dynamic trading strategy of informed insiders. Specifically, we provide a solution to a discrete time analog of Kyle’s (1985) rational expectations trading model in which an insider, endowed with long-lived private information, must disclose the quantity he trades at the close of each round of trading. The ex-post public reporting of the insider’s trades changes the equilibrium strategy of the insider given that the market maker can infer information from the insider’s previous trade before the next round of trading. By playing a mixed strategy in every

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1Section 16(a) of the Securities Exchange Act of 1934 requires insiders to report their trades to the Commission within ten days following the end of the month in which the trade occurs. Section 13(d) requires any individual who acquires five percent or more of a firm’s stock to report it within ten days. Subsequent changes to the position must also be reported within ten days.

2From an empirical standpoint, trading by corporate insiders appears over time to be increasingly profitable. Seyhun (1986) finds that insiders tend to buy before an abnormal rise in stock prices and to sell before an abnormal decline. Earlier studies by Lorie and Niederhoffer (1968), Pratt and DeVere (1970), Jaffe (1974), and Finnerty (1976) draw similar conclusions. More recently, Seyhun (1992a) finds compelling evidence that insider trading volume, frequency, and profitability all increased significantly during the 1980s. Over the decade, he documents that insiders earned over 5% abnormal returns on average. Seyhun (1992b) determines that insider trades predict up to 60% of the variation in year-ahead returns. Accordingly, insider trading continues to be an economically important phenomenon.

3At an anecdotal level, there is support for this kind of reasoning by non-insider market participants: EDS officers and directors sold 479,207 shares over approximately six weeks leading up to a stock price drop of 30%. Several individuals made multiple trades including Joseph M. Grant, the chief financial officer, who sold stock on five dates between the February 9, 1988 and March 11, 1998 at prices from $43.50 to $44.63. The price dropped steadily from a mid-March high of just over $50 to under $35 by early June. Moreover, this was not the first time EDS insiders displayed good timing. ‘Two previous rounds of insider sales were followed by stock-price reductions in the 30% range in 1996 and 1997,’ said Bob Gabele, president of CDA/Investnet. ‘The current selling led us to suspect that there could very
round except the last one, the insider garbles the information conveyed by his trade. Nevertheless, public disclosure of insider trades accelerates the price discovery process and lowers trading costs by comparison to Kyle (1985). Compared to the benchmark case of no public disclosure, expected insider profits (equivalently, expected losses to liquidity traders) fall substantially. In the limit, as the number of trading rounds and incidents of disclosure per unit time becomes very large, the insider’s expected profits are halved. Price discovery corresponds to Kyle’s result in the limit as the number of trading rounds per unit time becomes large. Thus, besides reducing trading costs, disclosure may also lead to gains in market efficiency depending on the frequency with which the insider trades.

It might be supposed that an informed insider who is required to disclose his trades after the fact would surrender his entire informational advantage the first time he is compelled to disclose the quantity he has traded. In fact, we show that an insider earns the same expected profit, and dissipates a constant amount of his private information, in every trading round. To accomplish this, the insider’s trades include a random noise component. The random component may be a buy or a sell. The insider trades the sum of this quantity and an information-based component. While the insider in our model sometimes buys (sells) when his information is that the stock is overvalued (undervalued), he also may sell (buy) more aggressively than he would if his trades were not subject to disclosure after the fact. Except in the last round, the insider’s strategy places strictly positive probability on all trade quantities, both buy and sell, irrespective of the insider’s information. The strategy balances immediate profits from informed trades against the reduction in future profits following trade disclosure and, hence, revelation of some of the insider’s information. Our results show the optimality of adding a random noise

well be another drop in price this year,’ he added.” (Laura Saunders Egodigwe Wall Street Journal (May 6, 1998) p. C1.)
component to informed trades, thereby diminishing the market maker’s ability to draw
inferences from the public record. We call this dissimulation, a phenomenon distinct
from manipulation. Dissimulation provides a novel rationale for contrarian trading.

There are several elegant aspects to the insider’s trading strategy. First, the variance
of noise added by the insider to determine his trade satisfies two relationships: (i) the
insider’s disguise at the time he trades is optimized by setting the variance of his trade,
in the eyes of the market maker, equal to the variance of the liquidity demand; and,
(ii) the insider’s disguise at the time he reveals his trade is optimized by setting the
variance of the noise component of his trade equal to the variance of the information-
based component, as seen from the perspective of the market maker. Remarkably, the
insider’s strategy satisfies these two relationships in every period.

Second, the strategy applies for all values of the exogenous parameters, including
the number of trading periods. Dissimulation is thus a robust effect. Third, the strat-
egy yields expected profits to the insider that depend only on (i) the insider’s ex ante
information advantage, and (ii) the total liquidity available. For a given information
advantage and total liquidity, ex post public disclosure of the insider’s trade after every
trading round ensures that the insider’s total expected profits are constant irrespective of
the number of trading rounds. Fourth, the price adjustment per share traded is constant
over time. This implies that liquidity traders have no reason to reallocate their demands
over time. Fifth, the insider’s strategy space is richer than in other models rooted in
the framework of Admati and Pfleiderer (1989), where the insider is constrained to buy
or sell a single quantity. In our setting, the insider can trade any number of shares
he chooses. This relaxation in structure creates scope for dissimulation, but eliminate
incentives for manipulation in the sense of trading as if better informed. In our model,
the insider mixes before the last round no matter what news is received, and trades in
the opposite direction with positive probability.
Dissimulation, is distinct from the manipulation strategies considered by Fishman and Hagerty (1995), hereafter FH, and John and Narayanan (1997), hereafter JN.\textsuperscript{4} Dissimulation (i) occurs irrespective of the magnitude of the insider’s information advantage or market liquidity; (ii) occurs in equilibrium whether the insider’s news is good or bad; and, (iii) is not motivated by an uninformed insider’s attempt to pool with an informed insider. In contrast, the manipulation considered in FH is driven by (iii). Contrary to (ii), the insider considered by JN will never manipulate with good news and bad news simultaneously in equilibrium. The existence of manipulation in the equilibria considered by FH and JN depend importantly on the information advantage of the insider and the amount of liquidity in the market. Thus, different from (i), manipulation does not exist for all parameter values.

The remainder of our paper is organized as follows: Section II presents our analysis, and section III contains a discussion of our principal results.

\section{Analysis}

\subsection{Two-period Kyle Model}

Consider a standard Kyle (1985) two-period model in which there is one risky asset with a liquidation value, \( v \), that is normally distributed with mean \( p_0 \) and variance \( \Sigma_0 \).

\textsuperscript{4}In FH, an uninformed insider might imitate an informed insider with good news, buy shares in order to induce a positive price change, and then sell shares after the trade is publicly disclosed. The basic idea is for an uninformed insider to move the price by buying or selling stock in their company and to profit in expectation by undoing their position in the next round of trading. Thus, uninformed insiders the inability of market makers to distinguish uninformed trades from those of privately informed insiders. JN consider the case where insiders are informed for sure and there is an asymmetry in the likelihood that the insider receives good and bad news. A higher probability of good (bad) news may lead to a contrarian strategy under which an insider with bad (good) news trades as if he had good (bad) news and then profits by trading in the opposite direction in the following period. The insider is indifferent between trading in the correct direction, thereby reducing the insider’s information advantage for the second round, and trading in the opposite direction at an expected loss in order to sustain the advantage into the second round. In both studies, the results suggest that insiders may benefit from public disclosure of their trades.
Index the periods by \( n \in \{1, 2\} \). A single insider learns \( v \) at the start of the first period and places an order to buy or sell \( x_n \) shares of the risky asset at the start of period \( n \). A market maker receives these orders along with those of liquidity traders whose exogenously generated demands, \( u_n \), are normally distributed with mean 0 and variance \( \sigma_u^2 \). Assume \( v, u_1, \) and \( u_2 \) are mutually independent. The market maker observes only the total order flow at each date, \( y_n = x_n + u_n \), and sets the price, \( p_n \), equal to the posterior expectation of \( v \).

Let the insider’s trading strategy and market maker’s pricing rule be sets of real-valued functions \( X = \{X_1, X_2\} \) and \( P = \{P_1, P_2\} \) such that, given an initial price \( p_0 \),

\[
\begin{align*}
    x_n &= X_n(p_{n-1}, v), \quad n \in \{1, 2\} \\
    p_1 &= P_1(y_1), \\
    p_2 &= P_2(y_1, y_2).
\end{align*}
\]

Using \( \pi_n \) to denote the portion of the insider’s total profits directly attributable to his period \( n \) trade, it is apparent that

\[
\pi_n(x_n, p_n) = x_n(v - p_n), \quad n \in \{1, 2\}.
\]

A subgame perfect equilibrium is defined by \( X \) and \( P \) such that:

\[
E \left[ \sum_{k=n}^{2} \pi_k(x_k, p_k \mid p_{k-1}, v) \right] \geq E \left[ \sum_{k=n}^{2} \pi_k(\hat{x}_k, p_k \mid p_{k-1}, v) \right], \quad \text{for } n \in \{1, 2\}, \quad \text{and,}
\]

for any strategy \( \{\hat{X}_1, \hat{X}_2\} \),

\[
\begin{align*}
    p_1(x_1; p_1) &= E(v \mid y_1) \quad \text{and,} \\
    p_2(x_1, x_2; p_1, p_2) &= E(v \mid y_1, y_2).
\end{align*}
\]

The proposition below is based on a special case of a well-known result due to Kyle (1985).
Proposition 1: Given no public disclosure of insider trades, a subgame perfect linear equilibrium exists in which

\[ x_n = \beta_n(v - p_{n-1}), \quad n \in \{1, 2\}, \]

\[ p_n = p_{n-1} + \lambda_n y_n, \quad n \in \{1, 2\}, \quad (1) \]

\[ \lambda_1 = \frac{\sqrt{2K(2K-1)}}{4K-1} \sqrt{\Sigma_0} / \sigma_u, \quad \lambda_2 = \frac{1}{2\sigma_u} \sqrt{\Sigma_1} = \frac{1}{2\sigma_u} \sqrt{2K \Sigma_0 / 4K - 1}, \]

\[ \beta_1 = \frac{2K - 1}{4K - 1} \lambda_1, \quad \beta_2 = \frac{1}{2\lambda_2}, \]

\[ E(\pi_1) = \beta_1 \Sigma_1 = \frac{2K(2K-1)}{(4K-1)^2} \sigma_u \sqrt{\Sigma_0}, \quad E(\pi_2) = \beta_2 \Sigma_2 = \sqrt{\frac{2K}{4K - 1}} \sigma_u \frac{\sqrt{\Sigma_0}}{2}, \quad \text{and} \]

\[ \Sigma_1 = \frac{2K}{4K - 1} \Sigma_0, \quad \Sigma_2 = \frac{\Sigma_1}{2} \]

where

\[ \frac{\lambda_2}{\lambda_1} \equiv K = \frac{1}{6} \left( 1 + 2\sqrt{7} \cos \left( \frac{1}{3} \left( \pi - \arctan(3\sqrt{3}) \right) \right) \right) \approx 0.901. \]

Proof: See Appendix

The expressions in Proposition 1 provide a benchmark against which to compare an equilibrium for the case where the insider’s trade in the first period is publicly disclosed on completion of trading in that period.

B. Two-period Model with Public Disclosure of Insider Trades

Assume the insider’s trade in period one is publicly disclosed after trading in period one and before trading in period two. The pricing and trading strategies depicted in Proposition 1 are not an equilibrium in the new setting. To see this, suppose the market maker conjectures that the insider follows the first period strategy \( x_1 = \beta_1(v - p_0) \). Then, upon observing \( x_1 \), the market maker would infer \( v = x_1/\beta_1 + p_0 \). Accordingly he would
choose \( p_2 = x_1/\beta_1 + p_0 \), and \( \lambda_2 = 0 \). Understanding this, the insider would have incentive to choose \( \hat{x}_1 \neq x_1 \). Any such defection induces mispricing in the second period. Since the market marker chooses \( \lambda_2 = 0 \), the market depth is infinite and the second period profits of the insider are unbounded.

Clearly, no invertible trading strategy can be part of an equilibrium in this case. We show an equilibrium exists in which the insider’s first-period trade consists of an information-based linear component, \( \beta_1(v-p_0) \), and a noise component, \( z_1 \), where \( z_1 \) is normally and independently (of \( v \) and \( u_1 \)) distributed with mean 0 and variance \( \sigma_z^2 \).

Public disclosure of \( x_1 \) allows the market maker to update his beliefs from those formed on a basis of the first period order flow. In particular, let

\[
p^*_1 = p_0 + \gamma_1 x_1
\]

be the expected value of \( v \) given \( x_1 \) and \( y_1 \). The new price does not depend on \( p_1 \) or \( y_1 \) because \( x_1 \) is a sufficient statistic for \( \{x_1, p_1, y_1\} \) with respect to \( v \); that is, \( E(v \mid x_1, y_1, p_1) = E(v \mid x_1) \). In turn, \( p^*_1 \) replaces \( p_1 \) in the second period price

\[
p_2 = p^*_1 + \lambda_2 y_2.
\]

Applying the principal of backward induction, we can write the insider’s second period optimization problem for given \( x_1 \) and \( p^*_1 \) as

\[
x_2 \in \arg \max_x E[x(v - p_2)].
\]

Taking the first-order-condition (FOC) results in the familiar solution to the one period problem given a prior price of \( p^*_1 \):

\[
x_2 = \beta_2(v - p^*_1) = \frac{1}{2\lambda_2}(v - p^*_1),
\]

\[
E \left[ \pi_2(p^*_1, v) \right] = \frac{1}{4\lambda_2}(v - p^*_1)^2.
\]
Stepping back to the insider’s first period optimization problem, we have

\[ x_1 \in \arg\max_x E[x(v - p_1) + \pi_2(p_1^*, v)]. \]

Substituting for \( p_1 \) and \( p_1^* \) from (1) and (2), respectively, differentiating, and setting the result equal to zero leads to the following FOC:

\[
\left( \frac{\gamma_1^2}{2\lambda_2} - 2\lambda_1 \right) x_1 + \left( 1 - \frac{\gamma_1}{2\lambda_2} \right) (v - p_0) = 0. \tag{4}
\]

The second-order-condition is

\[
\frac{\gamma_1^2}{2\lambda_2} - 2\lambda_1 \leq 0.
\]

If our proposed mixed trading strategy,

\[
x_1 = \beta_1(v - p_0) + z_1,
\]

\[
z_1 \sim N(0, \sigma_{z_1}^2),
\]

is to hold in equilibrium, then the insider must be indifferent across all values of \( x_1 \). Thus, from (4) we seek values of \( \lambda_1, \lambda_2 \), and \( \gamma_1 \) such that, \( \lambda_1 > 0, \lambda_2 > 0 \),

\[
\frac{\gamma_1^2}{2\lambda_2} - 2\lambda_1 = 0, \quad \text{and} \quad 1 - \frac{\gamma_1}{2\lambda_2} = 0. \tag{5} \tag{6}
\]

Equations (5) and (6) imply

\[
\lambda_1 = \lambda_2 = \frac{\gamma_1}{2}. \tag{7}
\]

The breakeven conditions of the market maker now become

\[
p_1 = E(v \mid y_1) = p_0 + \lambda_1 y_1,
\]

where

\[
y_1 = \beta_1(v - p_0) + z_1 + u_1,
\]

\[
p_1^* = E(v \mid x_1) = p_0 + \gamma_1 x_1,
\]

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and
\[ p_2 = E(v | y_2, p_1^*) = p_1^* + \lambda_2 y_2. \]

Furthermore,
\[ \lambda_1 = \frac{\text{Cov}(v, y_1)}{\text{Var}(y_1)} = \frac{\beta_1 \Sigma_0}{\beta_1^2 \Sigma_0 + \sigma_{z_1}^2 + \sigma_u^2}, \quad (8) \]
\[ \gamma_1 = \frac{\text{Cov}(v, x_1)}{\text{Var}(x_1)} = \frac{\beta_1 \Sigma_0}{\beta_1^2 \Sigma_0 + \sigma_{z_1}^2}, \quad (9) \]
\[ \lambda_2 = \frac{\text{Cov}(v, y_2 | x_1)}{\text{Var}(y_2 | x_1)} = \frac{\beta_2 \Sigma_1}{\beta_2^2 \Sigma_1 + \sigma_u^2}, \quad (10) \]

where
\[ \Sigma_1 = \text{Var}(v | x_1) = \Sigma_0 - \gamma_1^2 (\beta_1^2 \Sigma_0 + \sigma_{z_1}^2). \quad (11) \]

Combining (7), (8), and (9) implies
\[ \beta_1^2 \Sigma_0 + \sigma_{z_1}^2 = \sigma_u^2. \quad (12) \]

In turn, (11) reduces to
\[ \Sigma_1 = \Sigma_0 - 4 \lambda_2^2 \sigma_u^2 \quad (13) \]

using (12) and (7). Conditional on $\Sigma_1$, the insider’s trading strategy from (3) and the market maker’s inference based on aggregate order flow are exactly as in Proposition 1, i.e.,
\[ \beta_2 = \frac{1}{2 \lambda_2} \quad \text{and} \quad \lambda_2 = \frac{1}{2 \sigma_u} \sqrt{\Sigma_1}. \quad (14) \]

Substituting this value for $\lambda_2$ into (13) yields
\[ \Sigma_1 = \frac{\Sigma_0}{2}. \quad (15) \]
From equations (7)—(15), we solve for the values of \( \lambda_1, \lambda_2, \beta_1, \beta_2, \gamma_1, \) and \( \sigma_{z_1}^2 \) given by the following proposition:

**Proposition 2:** In the two-period setting with public disclosure of insider trades, a subgame perfect equilibrium exists in which

\[
\begin{align*}
  x_1 &= \beta_1(v - p_0) + z_1, \\
  x_2 &= \beta_2(v - p_1^*), \\
  p_1 &= p_0 + \lambda_1 y_1, \\
  p_2 &= p_1^* + \lambda_2 x_2, \\
  p_1^* &= p_0 + \gamma_1 x_1, \\
  \lambda_1 &= \lambda_2 = \frac{1}{2\sigma_u} \sqrt{\frac{\Sigma_0}{2}}, \\
  \beta_1 &= \frac{1}{4\lambda_1}, \\
  \beta_2 &= \frac{1}{2\lambda_2}, \\
  \gamma_1 &= 2\lambda_1, \\
  \sigma_{z_1}^2 &= \frac{\sigma_u^2}{2}, \\
  E(\pi_1) &= E(\pi_2) = \frac{\sigma_u}{2} \sqrt{\frac{\Sigma_0}{2}}, \text{ and} \\
  \Sigma_1 &= \frac{1}{2} \Sigma_0.
\end{align*}
\]

Marginal trading costs must be the same across the two periods with disclosure for the insider to achieve the indifference in the first period demands necessary to sustain a mixed strategy, \( \lambda_1 = \lambda_2, \) in the case with disclosure.\(^5\) A disparity in such costs would create an incentive to deviate from a mixed strategy in order to exploit the lower cost. The additional price adjustment for insider trades when publicly disclosed, \( \gamma_1 - \lambda_1, \) equals the price adjustment (marginal cost) based on the first-period order flow, \( \lambda_1, \) reflecting a comparable resolution of uncertainty at each stage.

The lower first than second period trading intensity, \( \beta_1 < \beta_2, \) is consistent with the absence of a concern for the effect of trading in the last period on future expected

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\(^5\)The intuition is similar to the case of the Kyle (1985) continuous time version. As shown in Back, Cao, and Willard (1995), this result does not hold if there are more than two informed traders.
profits. Setting the $\text{Var}(x_1) = \beta_1^2 \Sigma_0 + \sigma_{z_1}^2 = \sigma_u^2$ serves to disguise the insider’s trades vis-à-vis those of liquidity traders, and setting $\sigma_{z_1}^2 = \beta_1^2 \Sigma_0 = \sigma_u^2/2$ sustains disguise for the information-based component of those trades once they are publicly disclosed.

The rate of price discovery, $\Sigma_1 = \frac{1}{2} \Sigma_0$, is comparable to Kyle (1985) in the limiting case as the number of periods in a given time horizon becomes very large. However, as we will show next, the rate is greater with public disclosure than without for any finite number of periods.

The same exogenous parameters imply different values for the endogenous parameters depending on whether the insider must disclose his trades after the fact. To distinguish the values, we add an upper bar to the endogenous parameters in the case of no disclosure (i.e., the values corresponding to Proposition 1). The next proposition compares the endogenous parameters across the two cases.

**Proposition 3:** In the two-period setting, the following orderings apply:

\[
\begin{align*}
\lambda_1 &< \bar{\lambda}_1, \\
\lambda_2 &< \bar{\lambda}_2, \\
\beta_1 &> \bar{\beta}_1, \\
\beta_2 &> \bar{\beta}_2, \\
E(\pi_1) &< E(\bar{\pi}_1), \\
E(\pi_2) &< E(\bar{\pi}_2), \quad \text{and} \\
\Sigma_1 &< \Sigma_1.
\end{align*}
\]

Proof: See Appendix.

Intuitively, the market maker sets the marginal cost of first period trades lower with public disclosure than without, $\lambda_1 < \bar{\lambda}_1$, under the rational conjecture that some of the insider’s trades are randomly generated in the former case. The insider trades more intensely with public disclosure than without in the first period, $\beta_1 > \bar{\beta}_1$; however, the effect in reducing the prior variance is mitigated by the random component of his trades.

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The prior variance is further reduced by the disclosure per se, resulting in a greater total reduction, $\Sigma_1 < \Sigma_1$.

Not surprisingly, second (last) period strategies vary with disclosure only to the extent of a difference in the prior variance upon entering that period. Since, as is evident from (1) and (16), second period marginal costs are increasing in prior variance, then second period marginal costs are lower with disclosure than without, $\lambda_2 < \bar{\lambda}_2$, and this fact combined with $\Sigma_1 < \Sigma_1$ implies greater second period trading intensities, $\beta_2 > \bar{\beta}_2$.

Finally, we observe that, in each period, expected insider profits are lower with disclosure than without, $E(\pi_n) < E(\bar{\pi}_n), n \in \{1, 2\}$, implying lower total expected trading costs to liquidity traders. Both this result and the greater rate of price discovery evident in prior variance reductions generalize to the $N$-period case which we turn to next.

C. $N$-period Model with Public Disclosure of Insider Trades

The principal new insights from this extension relate to how the magnitudes of trading costs and rates of price discovery vary with the frequency of insider trading with and without public disclosure. The next proposition characterizes our equilibrium in this case.

**Proposition 4:** In the $N$-period setting with public disclosure of insider trades, a subgame perfect equilibrium exists in which, given $p_0^* \equiv p_0$,

\begin{align*}
x_n &= \beta_n(v - p_{n-1}^*) + z_n, \\
p_n &= p_{n-1}^* + \lambda_n y_n, \\
p_n^* &= p_{n-1}^* + \gamma_n x_n, \\
\lambda_n &= \frac{1}{2\sigma_u} \sqrt{\Sigma_{n-1}} = \frac{1}{2\sigma_u} \sqrt{\Sigma_0} \sqrt{\frac{N}{N-n+1}}, \\
\beta_n &= \frac{1}{2(N-n+1)\lambda_n},
\end{align*}
\[ \gamma_n = 2\lambda_n, \]
\[ \sigma_{\gamma_n}^2 = \left( \frac{N - n}{N - n + 1} \right) \sigma_u^2, \]
\[ E(\pi_n) = \frac{\sigma_u}{2} \sqrt{\frac{\Sigma_{n-1}}{N - n + 1}} = \frac{\sigma_u}{2} \sqrt{\frac{\Sigma_0}{N}}, \]
\[ \Sigma_n = \frac{N - n}{N - n + 1} \Sigma_{n-1} = \frac{N - n}{N} \Sigma_0 \]

for \( n \in \{1, 2, \ldots, N\} \).

Proof: See Appendix.

In the result above, it is striking that the change in prior variance from one period to the next, \( \Sigma_{n-1} - \Sigma_n \), is constant. This can be compared to Kyle’s (1985) result for discrete trading which implies a slower rate of prior variance reduction. Since in Kyle (1985) the prior variance in each period depends on endogenous parameters which can only be characterized by difference equations, we resort to his numerical method to make this comparison.

[Figure 1]

Figure 1 plots the error variance of price, \( \Sigma_n \), with and without disclosure of insider trades for varying numbers of trading rounds. As the figure makes plain, only in the limit as the number of trading rounds per unit time becomes very large are the price discovery rates comparable with and without public disclosure of insider trades. The reduction in prior variance is always greater when disclosure is required. As the number of trading rounds grows smaller over a fixed time horizon, difference between the prior variances without and with disclosure grows wider.

[Figure 2]

Figure 2 plots the values of the market maker’s price adjustment, \( \lambda_n \), with and without disclosure of insider trades for varying numbers of trading rounds. Consistent with the intuition from the two-period case, the market maker’s price adjustments, \( \lambda_n \),
are constant over time, and account for half of the total price adjustment; i.e., $\lambda_n = \frac{1}{2}\gamma_n$. With no disclosure, price adjustments decrease over time in the discrete case. Also as the number of trading rounds per unit of time increases, the price adjustments at each point in time increase. As trading frequency approaches the continuous case, the price adjustments in the no disclosure case approach the total price adjustment in our analysis (i.e., for all $n$, $\hat{\lambda}_n = \gamma_n$ as $N \to \infty$). Hence, trading costs in the limiting case, measured by $\lambda_n$, are cut in half with public disclosure of insider trades.

Figure 3 plots expected insider profits when the insider must disclose his trades and when no such disclosure is required for the cases $N = 4$ and $N = 20$. Compared round by round to the no disclosure case, insider profits are lower with disclosure. In the limit as the number of periods grows large, insider profits with disclosure are half insider profits with no disclosure. As is well known, with no disclosure, the insider’s aggregate profits increase as the number of trading rounds increases, and expected insider profits in a trading round are lower in later rounds. When the insider must disclose, (i) the insider’s aggregate expected profits are $\sigma_u\sqrt{\sum_0^t}/2$ irrespective of the number of trading rounds; and, (ii) given $N$, expected profits are constant over trading rounds. Intuitively, constant expected profit per trading round follows from the indifference over trade quantities that, in turn, is necessary to a mixed strategy.

III. Conclusion

Our analysis extends Kyle’s (1985) model to provide for ex post disclosure of insider trades. Such disclosure raises the prospect that market makers may deduce the insider’s private information. The insider thwarts this prospect by adding noise to his demands. We call this dissimulation. Dissimulation is costly since it causes the insider at times to trade in a manner inconsistent with his private information. Liquidity traders un-
ambiguously benefit from lower expected trading costs by comparison to Kyle’s results without public disclosure.

While public disclosure of insider trades may cut insider profits by as much as a half, insider trading remains profitable. Our results suggest (i) insiders continue to trade on private information after public disclosure of trades from earlier rounds and before public release of that information; (ii) price adjustments to insider trades are comparable to price adjustments to the public disclosure of those trades; and, (iii) price adjustments to insider trades before and after public disclosure of those trades remain the same. To our knowledge, no empirical study addresses these predictions.\(^6\)

Our results are robust with respect to discretion by liquidity traders to allocate their trades over successive rounds. The equivalence of price adjustments that characterize equilibria in our model implies no incentive for liquidity traders to reallocate their demands over time if provided with such discretion. Accordingly, the clustering phenomena depicted by Admati and Pfleiderer (1988) in a context of successive short-lived information and Bushman et al. (1997) in a context of long-lived information without public disclosure should not arise in our setting with long-lived information and public disclosure.

An important issue for securities market regulators is the role of insider trading in price discovery. In the words of Carlton and Fischel (1983, page 868), “The greater the ability of market participants to identify insider trading, the more information such trading will convey.” We show formally the extent to which an insider is able to profit from long-lived private information even though he must disclose his trades after the

\(^6\)Damodaran and Liu (1993) report significant price adjustments, ultimately related to public announcements of appraisals by real estate investment trusts, at the time of insider trading and public reports of insider trades, but not at the time appraisals are announced. An open question germane to our predictions is whether appraisals in their study are “fully” reflected in prices before the appraisals are publicly released due to repeated insider trading, or because public disclosure of insiders’ trades allows market participants to deduce their appraisal information.
fact. To maximize his expected profits, the insider dissembles his information by adding a random component to his trades. Despite this, the analysis shows that information is reflected more rapidly in price with disclosure of insider trades than without (i.e., the error variance of price is always smaller with disclosure than without).
REFERENCES


Appendix

Proof of Proposition 1: Tailoring Theorem 2 of Kyle (1985) to the two-period case, we have the following relationships:

\begin{align*}
\alpha_2 & = 0, \\
\alpha_1 & = \frac{1}{4\lambda_2(1 - \alpha_2\lambda_2)} = \frac{1}{4\lambda_2}, \\
\beta_1 & = \frac{1 - 2\alpha_1\lambda_1}{2\lambda_1(1 - \alpha_1\lambda_1)}, \\
\beta_2 & = \frac{1 - 2\alpha_2\lambda_2}{2\lambda_2(1 - \alpha_2\lambda_2)}, \\
\lambda_1 & = \frac{\beta_1\Sigma_1}{\sigma_u^2}, \quad \text{(A1)} \\
\lambda_2 & = \frac{\beta_2\Sigma_2}{\sigma_u^2}, \quad \text{(A2)} \\
\Sigma_1 & = (1 - \beta_1\lambda_1)\Sigma_0, \quad \text{and} \quad \text{(A3)} \\
\Sigma_2 & = (1 - \beta_2\lambda_2)\Sigma_1. \quad \text{(A4)}
\end{align*}

Substituting for \(\alpha_1\) and \(\delta_1\) in the expressions for \(\beta_1\) and \(\beta_2\) yields

\begin{align*}
\beta_1 & = \frac{2\lambda_2 - \lambda_1}{\lambda_1(4\lambda_2 - \lambda_1)}, \quad \text{and} \quad \text{(A5)} \\
\beta_2 & = \frac{1}{2\lambda_2}. \quad \text{(A6)}
\end{align*}

Combining (A3) and (A5),

\begin{align*}
\Sigma_1 & = \left(1 - \frac{2\lambda_2 - \lambda_1}{4\lambda_2 - \lambda_1}\right)\Sigma_0. \quad \text{(A7)}
\end{align*}

Combining (A6) and (A8),

\begin{align*}
\Sigma_2 & = \frac{\Sigma_1}{2}. \quad \text{(A8)}
\end{align*}

This fact, (A1), and (A2) give

\[ \frac{\beta_1\Sigma_1}{\lambda_1} = \frac{\beta_2\Sigma_2}{\lambda_2}, \]
which, after substituting from (A6) and (A8) and rearranging implies

$$\beta_1 = \frac{\lambda_1}{4\lambda_2^2}. \quad \text{(A9)}$$

Equating the right hand sides of (A5) and (A9) results in a polynomial in $\lambda_2/\lambda_1$,

$$0 = 8\left(\frac{\lambda_2}{\lambda_1}\right)^3 - 4\left(\frac{\lambda_2}{\lambda_1}\right)^2 - 4\left(\frac{\lambda_2}{\lambda_1}\right) + 1,$$

which has three real roots at approximately $-0.623$, $0.223$, and $0.901$. From the second order conditions of Theorem 2 of Kyle (1985), $\lambda_1 > 0$ and $\lambda_2 > 0$, hence the first root is extraneous. The second root implies the error variance of price at the beginning of the second round is negative by (A7), which is also impossible. Hence, $\lambda_2/\lambda_1 = K$, where $K \approx 0.901$. Tedium calculations show that $K$ has precisely the form given in the statement of Proposition 1. Substituting (A5) into (A1) yields an expression for $\lambda_1$ in $K$ and the exogenous parameters, $\sigma_u^2$ and $\Sigma_0$. It is straightforward to derive $E(\pi_1)$ and $E(\pi_2)$.

$$E[\pi_1] = E[x_1(v - p_1)]$$
$$= E[x_1(v - p_0 - \lambda_1(x_1 + u_1))]$$
$$= E[\beta_1(v - p_0)^2 - \lambda_1(\beta_1(v - p_0)^2)]$$
$$= E[(1 - \beta_1\lambda_1)\beta_1\Sigma_0]$$
$$= \beta_1\Sigma_1$$
$$= \frac{2K(2K - 1)}{(4K - 1)^2}\sigma_u\sqrt{\Sigma_0}.$$ 

Similarly,

$$E[\pi_2] = E[x_2(v - p_2)]$$
$$= E[x_2(v - p_1 - \lambda_1(x_1 + u_1))]$$
$$= E[\beta_2(v - p_1)^2 - \lambda_2(\beta_2(v - p_1)^2)]$$

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\[ E[(1 - \beta_2\lambda_2)\beta_2\Sigma_1] = \beta_2\Sigma_2 = \sqrt{\frac{2K}{4K-1}} \frac{\sigma_u}{2} \sqrt{\Sigma_0}. \]

QED.

**Proof of Proposition 3:**

\[ \bar{\Sigma}_1 = (1 - \bar{\beta}_1\bar{\lambda}_1)\Sigma_0 = \left(1 - \frac{2\lambda_2 - \bar{\lambda}_1}{4\lambda_2 - \lambda_1}\right)\Sigma_0 = \frac{2\bar{\lambda}_2}{4\lambda_2 - \lambda_1}\Sigma_0 > \frac{\Sigma_0}{2} = \Sigma_1. \quad (A10) \]

\[ \bar{\lambda}_2 = \frac{\sqrt{\bar{\Sigma}_1}}{2\sigma_u} > \frac{\sqrt{\Sigma_1}}{2\sigma_u} = \lambda_2 \quad \text{by (A10)}. \quad (A11) \]

\[ \bar{\beta}_2 = \frac{1}{2\lambda_2} < \frac{1}{2\lambda_2} = \beta_2 \quad \text{by (A11)}. \]

\[ \bar{\lambda}_1 = \frac{\sqrt{2K(2K-1)}}{4K-1} \frac{\sqrt{\Sigma_0}}{\sigma_u} > \frac{1}{2\sqrt{2}} \frac{\sqrt{\Sigma_0}}{\sigma_u} = \lambda_1. \]

\[ \bar{\beta}_1 = \sigma_u \frac{\sqrt{2K-1}}{2K\Sigma_0} < \frac{1}{\sqrt{2}} \frac{\sqrt{\Sigma_0}}{\sigma_u} = \beta_1. \quad (A12) \]

\[ E(\bar{\pi}_2) = \frac{\bar{\beta}_2}{2} \frac{\Sigma_1}{2} = \frac{1}{4\lambda_2} \frac{\bar{\Sigma}_1}{2} = \frac{\sigma_u \sqrt{\bar{\Sigma}_1}}{2} > \frac{\sigma_u \sqrt{\Sigma_1}}{2\sqrt{2}} = \frac{\beta_1}{2} \frac{\Sigma_1}{2} = E(\pi_2), \quad \text{by (A11)}. \]

\[ E(\bar{\pi}_1) = \frac{\beta_1\Sigma_1}{\beta_1^2\Sigma_0 + \sigma_u^2} > \frac{\beta_1\sigma_u^2\Sigma_0}{\beta_1^2\Sigma_0 + \sigma_u^2}, \quad \text{by (A12)}, \]

\[ > \frac{\beta_1\sigma_u^2\Sigma_0}{\beta_1^2\Sigma_0 + \sigma_u^2\sigma_1^2 + \sigma_u^2} \]

\[ = \frac{\beta_1\Sigma_0}{2} = \beta_1\Sigma_1 = E(\pi_1). \]

QED

**Proof of Proposition 4:** In the proof, we reverse the indexing of periods so that the final period is indexed by 1 and the first period by M. To avoid confusion with notation in the stated proposition, let \( q_m^* \) be the price prior to entering trading round \( m - 1 \). For example, in a two period world, \( E(v) = q_3^* \). That is, the price prior to the first trading round is the unconditional expectation of \( v \). After the first trading round and the first disclosure, price is \( q_2^* \).
Rewriting endogenous parameters characterized in Proposition 3 using this relabelling, we hypothesize for all $m \in \{2, 3, \ldots, M\}$:

\[
\begin{align*}
\lambda_m &= \frac{1}{2\sigma_u} \sqrt{\frac{\Sigma_{m+1}}{m}}, \\
\beta_m &= \frac{1}{2m\lambda_m}, \\
\gamma_m &= 2\lambda_m, \\
\sigma^2_{x_m} &= \left(\frac{m-1}{m}\right) \sigma_u^2, \\
\Sigma_m &= \frac{m-1}{m} \Sigma_{m+1},
\end{align*}
\]

and, the insider’s objective function can be written as

\[
E \left[ (v - q^*_M - \lambda_m x_m)x_m + \frac{1}{4\lambda_{m-1}} (v - q^*_m - \gamma_m x_m)^2 \right].
\]

The proof is by induction. From Proposition 2, (A13)-(A17) for $M = 2$. Now show that if (A13)-(A17) hold for $m \in \{2, 3, \ldots, M - 1, M\}$, then (A13)-(A17) also hold for $m = M + 1$. Use (A18) and expected profits for period $M + 1$ of $E [(v - q_{M+1})x_{M+1}]$ to write the $M + 1$ period problem as:

\[
E \left[ (v - q^*_M - \lambda_{M+1} x_{M+1})x_{M+1} + \frac{1}{4\lambda_M} (v - q^*_M - \gamma_{M+1} x_{M+1})^2 \right].
\]

Recall $q^*_{M+1} = E(v \mid q^*_{M+2}, x_{M+1}) = q^*_{M+2} + \gamma_{M+1}x_{M+1}$. Using this relationship and equation (A15), rewrite the insider’s objective function as

\[
E \left[ (v - q^*_M - \lambda_{M+1} x_{M+1})x_{M+1} + \frac{1}{4\lambda_M} (v - q^*_M - \gamma_{M+1} x_{M+1} - \frac{\gamma_M}{2} x_M)^2 \right]
\]

\[
= E \left[ (v - q^*_M - \lambda_{M+1} x_{M+1})x_{M+1} + \frac{1}{4\lambda_M} (v - q^*_M - \gamma_{M+1} x_{M+1} - \frac{\gamma_M}{2} x_M)^2 \right]
\]

\[
= E \left[ (v - q^*_M - \lambda_{M+1} x_{M+1})x_{M+1} + \frac{1}{4\lambda_M} (v - q^*_M - \gamma_{M+1} x_{M+1} - \frac{\gamma_M}{2} x_M)^2 \right].
\]
The first equality follows from \( x_M = \beta_m(v - q_{M+1}^*) + z_M \) and the last equality follows on substitution of \( q_{M+1} = q_{M+2}^* + \lambda_{M+1}(x_{M+1} + u_{M+1}) \), after taking the expectation with respect to \( z_M \) and \( u_{M+1} \). Expression (A19) is equivalent to (A18) for \( M + 1 \) periods.

For the insider to choose \( x_{M+1} \) in equilibrium, the first order condition on (A19),

\[
\left( \frac{\gamma_{M+1}^2}{2\lambda_M} - 2\lambda_{M+1} \right) x_{M+1} + \left( 1 - \frac{\gamma_{M+1}}{2\lambda_M} \right) (v - q_{M+2}^*) = 0,
\]

must hold. Since the insider must be willing to play a mixed strategy in which \( x_{M+1} \) may be any real number, the insider must be indifferent across all choices of \( x_{M+1} \). Hence, it must be that

\[
\frac{\gamma_{M+1}^2}{2\lambda_M} - 2\lambda_{M+1} = 0, \quad \text{and} \quad 1 - \frac{\gamma_{M+1}}{2\lambda_M} = 0. \quad \text{(A20)} \]

Equations (A20) and (A21) are equivalent to

\[
\lambda_{M+1} = \lambda_M = \frac{\gamma_{M+1}}{2}. \quad \text{(A22)}
\]

Since \( q_{M+1} \) is the conditional expectation of \( v \), given the aggregate order flow, and \( q_{M+1}^* \) is the conditional expectation of \( v \) given the order of the insider alone, we have

\[
\lambda_{M+1} = \frac{\beta_{M+1}\Sigma_{M+2}}{\beta_{M+1}^2\Sigma_{M+2} + \sigma_{z_{M+1}}^2 + \sigma_u^2}, \quad \text{and} \quad \frac{\gamma_{M+1}}{2\lambda_M} = \frac{\beta_{M+1}\Sigma_{M+2}}{\beta_{M+1}^2\Sigma_{M+2} + \sigma_{z_{M+1}}^2}, \quad \text{(A23)} \]

\[
\Sigma_{M+1} = \Sigma_{M+2} - \gamma_{M+1}^2(\beta^2\Sigma_{M+2} + \sigma_{z_{m}}^2) \quad \text{(A24)}
\]

Combining (A22) – (A24) implies

\[
\beta_{M+1}^2\Sigma_{M+2} + \sigma_{z_{M+1}}^2 = \sigma_u^2. \quad \text{(A25)}
\]

In turn, (A25) reduces to

\[
\Sigma_{M+1} = \Sigma_{M+2} - 4\lambda_M^2\sigma_u^2. \quad \text{(A26)}
\]
using (A26) and (A22). Substituting for $\lambda_M$ from (A13) yields

\[
\Sigma_{M+1} = \Sigma_{M+2} - \frac{\Sigma_{M+1}}{M}, \quad \text{or} \\
\Sigma_{M+2} = \frac{M}{M+1} \Sigma_{M+1}.
\] (A27)

Now consider (A23),

\[
\lambda_{M+1} = \frac{\beta_{M+1} \Sigma_{M+2}}{\beta_{M+1} \Sigma_{M+2} + \sigma_{m+1}^2 + \sigma_u^2} \\
= \frac{\beta_{M+1} \Sigma_{M+2}}{2 \sigma_u^2}, \quad \text{[by (A26)],} \\
= \frac{\beta_{M+1} \Sigma_{M+1}}{M+1}, \quad \text{[by (A27)].}
\]

Since $\lambda_{M+1} = \lambda_M$ by (A22), (A23) implies

\[
\frac{\beta_{M+1} \Sigma_{M+2}}{\beta_{M+1} \Sigma_{M+2} + \sigma_{m+1}^2 + \sigma_u^2} = \frac{\beta_M \Sigma_{M+1}}{\beta_M \Sigma_{M+1} + \sigma_{M+1}^2 + \sigma_u^2} \\
\beta_{M+1} \Sigma_{M+2} = \beta_M \Sigma_{M+1}, \quad \text{[by (A26)],} \\
\beta_{M+1} = \frac{M}{M+1} \beta_M, \quad \text{[by (A27)],} \\
= \frac{1}{2(M+1)\lambda_M}, \quad \text{[by (A14)],} \\
= \frac{\sigma_u}{M+1} \sqrt{\frac{M}{\Sigma_{M+1}}}, \quad \text{[by (A13)].}
\]

Substituting this last expression and (A27) into (A23) gives

\[
\lambda_{M+1} = \frac{1}{2\sigma_u} \sqrt{\frac{\Sigma_{M+2}}{M+1}}.
\]

Substituting for $\beta_{M+1}$ in (A26), we have

\[
\sigma_{m+1}^2 = \left( \frac{M}{M+1} \right) \sigma_u^2.
\]

Finally,

\[
E[\pi_m] = E[x_m(v - q_m)] \\
= E[x_m(v - q_{m+1}^* - \lambda_m(x_m + u_m))]
\]

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\[ \begin{align*}
&= E[\beta_m (v - q^*_m)^2 - \lambda_m (\beta_m (v - q^*_m) + z_m)^2] \\
&= E[(1 - \beta_m \lambda_m) \beta_m \Sigma_{m+1} - \lambda_m \sigma^2_{z_m}] \\
&= \frac{\sigma_u}{2} \sqrt{\frac{\Sigma_{m+1}}{m}}.
\end{align*} \]

Q.E.D.
Fig. 1. Error variance of price given number of trading rounds.

This figure contrasts the error variance of price, $\Sigma_n$, (i) when the insider must disclose each trade ex post, and (ii) when no such disclosures are made. Exogenous parameters are normalized by setting $\Sigma_0 = 1$, $\sigma_u^2 = 1/N$, and assigning trading round $n$ to time $n/N$. When the insider must disclose, the value of $\Sigma_n$ following each disclosure declines linearly over time independent of the number of periods (solid line). Dots correspond to the Kyle solution without disclosure of the insider’s trades. Large dots correspond to the case $N = 4$. Small dots correspond to the case $N = 20$. 

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Fig. 2. Liquidity parameter given number of trading rounds.

This figure contrasts the value of the liquidity parameter, \( \lambda_n \), when (i) the insider must disclose his trade after each trading round, and (ii) when no such disclosures are made. Exogenous parameters are normalized as in figure 1. Also as in figure 1, dots correspond to the Kyle solution without disclosure of quantities traded by the insider; large dots correspond to the case \( N = 4 \); and, small dots correspond to the case \( N = 20 \). Remarkably, when the insider must disclose, the value of \( \lambda_n \) is \( \frac{1}{2} \) in every period, irrespective of the number of periods (solid line).
Fig. 3. Insider profits given number of trading rounds.

This figure contrasts expected insider profits, $E(\pi_n)$, when (i) the insider must disclose his trade after each trading round (triangles), and (ii) when no such disclosures are made (dots). Exogenous parameters are normalized as in figure 1. Large dots and triangles correspond to the case $N = 4$. Small dots and triangles correspond to the case $N = 20$. 