Employee Stock Options

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Abstract

This paper examines the valuation of employee stock options (ESOs). Because ESOs are inalienable, the employee's optimal exercise policy differs from the policy a naive reading of the finance literature would suggest. The employee prefers to exercise options before maturity under certain conditions on risk aversion, investment opportunities, and wealth. Since the ESOs’ cost to the employer depends on the employee's exercise policy, this finding has implications for changes to the accounting treatment of ESOs under consideration by the Financial Accounting Standards Board. Numerical examples suggest the employer's cost is much less than the options' Black-Scholes value.

Key words: Management compensation, Stock options, Exercise Policy, Valuation

JEL classification: G12, M41, C61

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1. Introduction

This paper presents an analytical framework for exploring the valuation consequences of the features that distinguish employee stock options (ESOs) from traded stock options (TSOs). The framework includes the Black-Scholes formula as a special case. Simple numerical examples convey vividly the importance of these features to the valuation of ESOs.

If the strike price of an employee stock option is no less than the price of the stock at the date of grant, current U.S. GAAP provides that no expense be recorded by the employer for financial statement purposes at the date of grant or at any date in the future. Despite this, the exercise of an employee stock option triggers a dilution of the claims of the firm’s existing shareholders. In response to perceived inadequacies in disclosure, the Securities Exchange Commission requires proxies issued after January 1, 1993 to disclose either the potential realizable value of each option grant or the present value of each option grant “under any option pricing model” for each of a corporation’s most highly compensated executive officers.\(^1\) The Financial Accounting Standards Board (FASB) is also exploring revisions to the accounting for ESOs. The FASB issued an Exposure Draft, “Accounting for Stock-based Compensation,” on June 30, 1993. The proposed accounting standard adopts grant date accounting.\(^2\) Minutes of Board and Task Force on Accounting for Stock Compensation meetings indicate concern that a naive application of the Black-Scholes option pricing formula will misstate the expense to the firm associated with granting an option to an employee.

An understanding of the exercise policy of the employee is crucial to determining the cost of the option to the employer. This paper highlights the effect of risk aversion on the decision by the employee to exercise an ESO and illustrates how risk aversion reduces the cost incurred by the employer. The paper also demonstrates how knowledge of an employee’s exercise policy should be used to value an ESO.

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1 See Regulation S-K §229.402.
2 Under grant date accounting, the cost of the option to the employer is its fair value at the date of grant. Some adjustment may be made for employee turnover during the vesting period. The expense may be reported ratably over the time between the date of grant and the date the options vest. The fair value is not adjusted for subsequent changes in the price of the underlying stock, although it will be adjusted at the time of exercise if the expected time of exercise differs from the actual time of exercise.
Previous research on ESOs falls into three categories: (i) estimates of the cost of options for financial reporting purposes, (ii) analysis of the incentive effects of options, and (iii) description and documentation of the relationships among tax, financial reporting and economic effects of stock options. This paper considers items (i) and (iii) allowing for the possibilities that options may be exercised earlier than at maturity and employees may be risk averse. In recent independent work, Kulatilaka and Marcus (1994) offer some of the intuitions presented here. Both papers contain numerical simulations that illustrate the effect of early exercise on the cost of options to the employer corporation. Kulatilaka and Marcus (1994) and Lang and Huddart (1994) describe the effect of volatility on employer cost. The present paper proves early exercise is optimal for sufficiently risk averse employees under plausible conditions. This paper also documents the set of states in which the employee prefers to exercise an option before maturity. The intuition provided for the early exercise decision suggests some testable predictions about employee exercise behavior.

Section 2 distinguishes employee stock options from the more familiar traded stock options. The section also establishes conditions under which an employee will exercise her option only at maturity. If any of these conditions is relaxed, the employee may exercise her option before maturity. Section 3 develops the binomial model of stock returns that is the workbench for the intuitions about exercise policy. The section illustrates how risk aversion can precipitate early exercise. Section 4 characterizes the optimal policy, provides some intuition for the policy, and relates the exercise policy to the employer’s cost. Section 5 concludes the paper.

3 In the first category, Smith and Zimmerman (1976) use option pricing theory to bound the cost of an ESO to be expensed in the employer’s financial statements. Noreen and Wolfson (1981) examine how closely option pricing models approximate observed prices for long term warrants that resemble ESOs. Foster et al. (1991) show that pro forma adjustments to reflect revisions to the accounting for ESOs proposed by the FASB are material to the financial statements of the corporations in their sample. In the second category, Noreen (1976) and Hemmer (1993) consider the role of options in mitigating moral hazard. Lambert et al. (1991) consider the incremental incentive effects of options on an employee in the presence of other types of compensation. In the third category, Walter (1987) and Scholes and Wolfson (1992) describe the differences in tax, financial reporting, and economic effects of various kinds of options. Changes in the form of the option contract are associated with changes in the tax regime [Hite and Long (1982)] and with the financial reporting characteristics of the firm [Matsunaga et al. (1992) and Matsunaga (1993)]. Lambert et al. (1989) document reductions in dividends following the introduction of ESO plans that are not dividend protected. Huddart and Lang (1994) provide some preliminary evidence on the relationships among stock price levels, volatility, and employee exercise decisions.
2. Characteristics of Employee Stock Options

It is important to distinguish ESOs from TSOs. While the exercise of a TSO has no effect on the welfare of holders of the underlying stock, the exercise of an ESO is dilutive since the corporation issues new stock to the optionee. Thus, ESOs are a type of warrant. While TSOs usually mature within one year of the date of issue, ESOs may be exercised in a window of time that extends over many years (see figure 1). ESOs, in common with TSOs, are usually “American” not “European” options (i.e., they can be exercised at any time during the exercise window, not just at maturity).\(^4\)

![Figure 1](image)

An important restriction on ESOs is that they are inalienable: they cannot be sold by the employee to whom they are issued. Besides, it is impractical for a corporate officer to implement a trading strategy that would have the same effect as selling the option. Section 16(c) of the Securities Exchange Act of 1934 prohibits an officer from taking a net short position in any equity security of her employer. All transactions in equity securities or derivatives must be reported by the employee to the SEC and the employer. Employers are thus in a position to monitor officer compliance with company policies that prohibit employees from undoing the incentive effects of the options they receive. Many companies do impose short sale restrictions on their employees, suggesting that incentive effects of options are important and monitoring of compliance with short sale prohibitions is practicable.

The pricing and optimal exercise policies for TSOs rely on the construction of a portfolio containing a riskless asset and the stock underlying the option that duplicates the return of the option [Black and Scholes (1973)]. Merton (1973) proves that a traded call option on a stock paying no dividends (or an option protected from dividends by suitable adjustments of the exercise price) should never be exercised before maturity. This logic

\(^4\) Additionally, corporate compensation committees may impose conditions on the exercise of ESOs, like attainment of specific accounting or performance targets, that do not apply to TSOs. After options vest, there are typically no restrictions on when they may be exercised. Since May 1991, the insider trading rules in Section 16 of the Securities Exchange Act of 1934 permit employees to sell stock immediately after options are exercised. Most options granted since 1986 are non-qualified stock options for tax purposes. There are no restrictions on the immediate sale of stock acquired on exercise. Conversations with corporate controllers and stock option program administrators indicate over 95% of the stock acquired on exercise is sold immediately.
does not apply to ESOs. Since the employee cannot trade freely in either the option or the underlying stock, the value of the option to the employee, the optimal exercise policy, and the cost to the employer do not follow the classical arguments in the option pricing literature. Nevertheless, it is possible to show that the employee will not exercise the option early under certain conditions. Throughout the paper, I assume the stock pays no dividends, there are no transactions costs, and tax rates are constant over time.

The next proposition shows a risk neutral employee will exercise only at maturity an option on a stock that pays no dividends. The risk neutral case is a useful benchmark for comparison with the subsequent analysis of the behavior of risk averse employees. We can expect the exercise policy of a risk averse employee to resemble that of a risk neutral employee as the wealth of the employee (excluding the ESOs she owns) grows large or as the fraction of the total wealth of the employee represented by ESOs becomes small.

**Proposition 1:** Assume the employee can invest the proceeds from exercising the option in either the stock or a riskless asset. It is optimal for a risk neutral employee to exercise the option only at maturity when the employee believes the expected return on the stock is at least as great as the after-tax return on the riskless investment which in turn is non-negative.

All proofs are in the appendix. The intuition for this result is straightforward. The expected return on the stock is at least as great as the expected return on the riskless asset. Thus, if a risk neutral employee were to exercise the option, she would invest the proceeds in the stock. However, the expected payoff from holding the option is always greater than the expected payoff from the stock that could be purchased by exercising the option. Also, exercising the option early accelerates the payment of the exercise price. The next section develops the exercise policy of a risk averse employee who holds an inalienable ESO.
3. Model

A common rationale for granting an option to an employee is to motivate the employee to take actions likely to increase the stock price. Presumably, shareholders would like to know the cost of the option to decide whether the benefits, in terms of retaining and motivating the employee to whom the option is granted, exceed the cost. This problem is exceedingly complex if the productive actions or trading strategy of the employee have an effect on the evolution of the stock price, and strategy or actions differ depending on the employee’s stake in the corporation. As a first step in attacking this challenging problem, this paper assumes the parameters of the stock price process are stationary irrespective of employee actions, the strike-to-market ratio, or whether the employee holds stock or unexercised options on the stock of her employer. This strong assumption requires some explanation.

Where the employer grants stock options to low- and mid-level employees, it seems likely that the effect of any individual employee’s action or exercise strategy on the stock price is negligible. Accordingly, the incentive effect of options for these employees should be small. Hence, the effect of granting options to low- and mid-level employees on the evolution of the stock price also should be small. For such broad-based plans, the stationarity assumption seems reasonable.

The cost of options issued under broad-based plans is a major component of compensation at some companies. For instance, Pepsi has in place a stock option plan under which one third of its employees receive yearly options on stock with a market value equal to ten percent of salary [Burchenal and Weslock (1991a) and (1991b)]. Granting options in lieu of salary may be attractive to firms simply because it increases accounting earnings. This observation is consistent with claims by executives at many companies that adoption of the FASB’s proposal will cause them to issue fewer options to low-level employees than previously. The analysis in this paper speaks directly to the problem of costing such plans.

Where the employer grants stock options to executives, the stationarity assumption is less plausible. Suppose that granting an employee a stock option results in a distribution of future stock prices that is preferred by shareholders to the distribution that obtains in the absence of the option. That is, granting the option creates incentives that “improve”
the stock price process. Exercising the option dilutes the ownership of existing equity holders. When the number of shares under option is small relative to the number of shares outstanding, the dilution cost is the difference between the market price of the stock on the date of exercise and the strike price. If granting options increases the expected value of the stock, the dilution cost is larger. Thus, imposing the stationarity assumption when options increase the value of the stock biases the cost of the option downward. However, any increase in the cost of the option due to incentive effects is less than the concomitant expected increase in the value of the stock outstanding at the time the option is granted. Ignoring the incentive effect in valuing the option provides a benchmark cost of the option that can be compared to alternative forms of compensation.5

Minutes of deliberations by the FASB and the Task Force, as well as the Exposure Draft, indicate an objective of a new financial accounting standard is to “level the playing field” between companies that choose to compensate their employees in different ways by expensing employment services acquired in exchange for stock options at the fair value of the stock option. It would be problematic for the FASB to adopt a rule that allowed or required the expense for financial reporting purposes associated with granting an option to be adjusted for anticipated incentive effects. First, the determination of the incremental cost due to the feedback among option grant, stock price evolution, and option value would be subjective. Second, GAAP generally requires expenses to be recorded at the fair value of the asset sacrificed. For instance, if an employee receives a salary of $100,000 per year, the expense to the employer is $100,000 each year irrespective of the return realized on the employee’s services. Likewise, if an employee receives an option with an expected cost (ignoring the incentive effect) of $100,000, the expense to the employer should be $100,000, irrespective of the return realized.δ For these reasons, stationarity is an appealing simplification for purposes of accounting standard setting.

Next, this section develops the binomial model of stock returns that is the workbench for the intuitions presented in this paper. The examples and propositions below employ the

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5 Sale of stock acquired on exercise of the option reduces incentives to improve the stock price process. Firms that use options as incentive devices typically issue options at regular time intervals. Thus, key employees always hold unexercised options. This mitigates the loss of incentives that occurs when an employee exercises options and sells stock.

δ Adopting grant date accounting for ESOs will not redress inconsistencies between the expense reported for a stock option and the expense reported for a stock appreciation right (SAR).
binomial model of stock returns explained in Cox et al. (1979). Figure 2 depicts changes in stock price over time as an ingrown tree. At every node of the tree, the stock price moves up with probability $p$ or down with probability $1 - p$. On an uptick, the stock price increases by a factor of $s > 1$. On a downtick, the stock price falls by a factor of $1/s$. Thus, if the initial stock price is 1 and after $i$ periods there have been $k$ upticks (and $i - k$ downticks), the stock price in period $i$ is $s^j$ where $j = 2k - i$. Denote by $(i, j)$ the node at time $i$ in which the stock price is $s^j$.

[Figure 2]

Suppose the employee seeks to maximize her expected utility of wealth at or after the maturity of the option. Her preferences exhibit constant relative risk aversion, captured by the von Neumann-Morgenstern utility function for wealth $W$ of $U(W) = W^\gamma$ for $\gamma \in (0, 1]$. Reductions in $\gamma$ correspond to increases in relative risk aversion. Initially, the employee’s wealth consists solely of the option granted to her by her employer. Employees neither make additions to nor withdrawals from that wealth. To pay the exercise price, the employee must sell some of the stock she acquires on exercise. To keep the analysis simple, the numerical examples assume no taxes are imposed on the optionee. Given the power utility function assumed in this paper, $U(W) = W^\gamma$, the optimal exercise policy is unchanged if tax is imposed at the date of exercise on the difference between the market value of the stock and the exercise price of the option, provided the rate of tax is constant over time. Besides the ESO, the investment universe consists of a riskless asset with a per period return of $d$, and stock in the employer corporation. Interpret the return on the riskless asset as the return on the market portfolio in an economy without systematic risk. It follows that the expected return on the stock equals the return on the riskless asset (i.e., $ps + (1 - p)/s = d$). Without loss of generality, assume $s \geq 1$ and $p \geq \frac{1}{2}$. Then the return on the riskless asset is non-negative (i.e., $d \geq 1$). All tax rates are assumed to be zero.

To simplify the analysis, assume that the employee’s portfolio can consist of only one type of security (riskless asset, stock, or option) at any time. Allowing the employee

\footnote{If there is systematic risk in the economy, the expected return on the stock will exceed the expected return on the riskless asset. The general pattern of the employee’s optimal exercise strategy does not change from the exercise strategy documented here. Of course, the employee’s exercise decision depends on the returns available on alternative investments. If the return on the riskless asset is reduced, the set of circumstances under which the employee chooses to exercise her option is smaller.}
more flexibility in the investment of the proceeds from exercise increases the attractiveness of early exercise. Thus, early exercise by risk averse employees would occur more often than the analysis below suggests if the restriction on the composition of the portfolio after exercise were relaxed. With the restriction, the employee’s investment strategy following the exercise of her option does not depend on her investment horizon: she always invests the proceeds in the riskless asset.

The example below illustrates how the optimal option exercise policy depends on the risk aversion of the employee.

Example. A risk averse employee may exercise an in-the-money option before maturity to hold a riskless asset that pays a return equal to the expected return of the stock. Consider the tree in figure 2. The employee seeks to maximize her expected utility for wealth at time 3. At time zero, her employer grants her an option to buy stock at a strike price of \( X = 1 \) at either time 1, time 2, or time 3. Suppose the probability of an uptick is \( p = \frac{2}{3} \) at every node. Following an uptick the stock price doubles (i.e., \( s = 2 \)). After a downtick, the stock price falls by half. The expected return from holding the stock is equal to the expected return from holding the riskless asset, \( ps + (1 - p)/s = d \), so \( d = 1.5 \).

If she exercises the option at node \((2, 2)\), the employee receives \( s^2 - 1 \) to be invested for one period at rate \( d \) for an expected utility of \( U(d(s^2 - 1)) \). If she waits, the employee receives \( s^3 - 1 \) with a probability of \( p \) and \( s - 1 \) with a probability of \( 1 - p \) for an expected utility of \( pU(s^3 - 1) + (1 - p)U(s - 1) \). The employee will wait to exercise if and only if the expected utility from waiting exceeds the expected utility from exercising now. This relationship holds more generally. The employee will exercise the option at \((1, 1)\) if and only if \( U(d^2(s - 1)) > pEU[B(2, 2)] + (1 - p)EU[B(2, 0)] \) where \( EU(\cdot) \) denotes expected utility and \( B(i, j) \) is the stochastic payment realized by the employee from carrying out the optimal strategy at \((i, j)\).

A risk neutral employee \( (\gamma = 1) \) only exercises the option at nodes \((3, 3)\) and \((3, 1)\). When the employee has square root utility \( (\gamma = \frac{1}{2}) \), she strictly prefers to exercise the option at \((2, 2)\) to waiting until either \((3, 3)\) or \((3, 1)\) obtains. When the employee is more risk averse \( (\gamma = \frac{1}{4}) \), she finds it optimal to exercise the option at \((1, 1)\) as well. Early exercise means paying the exercise price early. Acceleration of this payment reduces the
risk of the final payoff by allowing the employee to switch from an investment in the risky option to the riskless asset, at the cost of lowering the expected return. This alternative may be attractive to a risk averse employee.

Smith and Zimmerman (1976) argue that the cost of an option to an employer is at least the difference (if positive) between the current stock price and the exercise price discounted back from the date of expiry to the date of grant. Because of the difficulty inherent in estimating the volatility of a stock that is not publicly traded, the recent Exposure Draft encourages non-public entities to determine the cost of the option using this so-called “minimum value method.” This terminology is misleading. It is possible that the cost of the option is less than the minimum value when employees are risk averse and the option can be exercised before maturity.

To illustrate, suppose the employer grants a risk averse \( \gamma = \frac{1}{2} \) employee an option to buy stock at a price of 1 when the current stock price is 4. Suppose the option matures in one period. The employee can exercise the option immediately. Also, \( p = \frac{2}{3}, s = 2, \) and \( d = 1.5 \). Notice that the employee faces exactly the same payoffs as she does at node (2, 2) of the example sketched above and illustrated in figure 2. The lower bound on the option’s cost suggested by Smith and Zimmerman is \( 4 - \frac{1}{1.5} = 3.3 \). But, a risk averse employee exercises the option at node (2, 2). In that case, the cost to the employer is 3, which is less. If the employee were either risk neutral or able to sell the option, then the option would not be exercised early (i.e., Smith and Zimmerman’s bound would apply). The bound and the theory behind it do not apply when employees are risk averse and the option is inalienable. Contrary to assertions in the Exposure Draft [FASB (1993), paragraph 115], minimum value is not necessarily a reasonable surrogate for fair value even when the strike price is near (or below) the stock price at the date of grant.

In very simple examples, I have shown that it may be optimal to exercise an ESO before maturity. How prevalent is exercise before maturity in more realistic settings? Anecdotal evidence suggests employees often exercise options before maturity. Proposition 2 implies exercise before maturity is both optimal and commonplace.

\[8\] This is the value of a short position in a forward contract on the stock.
Proposition 2: Suppose (i) the return on the riskless asset is positive; (ii) the expected rate of return on the stock is at least as great as the after-tax return on the riskless asset; (iii) the stock price process has no drift ($p = \frac{1}{2}$); and, (iv) at both nodes that can be reached next period, the employee prefers to exercise (rather than hold) the option. There exists a threshold level of risk aversion such that an employee who is more risk averse than the threshold will exercise her option this period. An employee who is less risk averse than the threshold will exercise her option next period.

An option that is not exercised on or before maturity is worthless, so an in-the-money option will certainly be exercised at maturity. If there is one period until maturity, and the option is sure to be in-the-money next period, then there is a threshold level of risk aversion such that an employee who is more risk averse than the threshold will exercise her option one period before maturity. If there are two periods until maturity, and the employee prefers (in the event of either an uptick or a downtick this period) exercising the option in one period to holding the option until maturity, then there is a threshold level of risk aversion such that an employee who is more risk averse than the threshold will exercise her option two periods early. Applying this logic iteratively, a sufficiently risk averse employee will exercise the option in the third and earlier periods before maturity. The numerical results in the next section confirm that the employee will exercise her option at a significant subset of the nodes occurring before maturity.

4. Optimal Exercise Policy

The optimal exercise policy for an ESO can be determined from a recursive algorithm. The algorithm is a straightforward extension of the machinery used in the example. The first subsection presents the optimal exercise policy in a specific case (using plausible parameter values) and gives the intuition for the regions of (time, stock price) space where exercise is optimal. The second subsection illustrates the impact of early exercise on the cost of the option to the employer. In addition, implications for accounting policy are considered.

4.1 Intuition for the Exercise Policy

Figure 3 plots the optimal option exercise policy for a risk averse manager as a region of (time, stock price) space. The only parameter that changes across the first two panels
of the figure is the employee’s coefficient of risk aversion. These panels assume the option
vests on the date of grant. The third panel assumes the option vests two years after the
date of grant. The early exercise region can be interpreted as comprising an upper and
lower arm. The lower arm is negligible in figure 3a. The two arms are distinct in figures 3b
and 3c. The early exercise region grows as the employee becomes more risk averse.

[Figure 3]

To develop an intuition for the early exercise region, consider first an employee con-
strained to hold all her wealth in the stock if she exercises the option. Suppose the stock
price is low (i.e., close to the exercise price), but that the option is in-the-money. If the
employee exercises the option early, she sells just enough of the stock she gets to fund the
exercise price of the option. Her final payoff is the value of the remaining stock on the
last day of her investment horizon. At the time she exercises the option, this payoff is
uncertain, but its distribution resembles a standard lognormal distribution. As the stock
price approaches the strike from above, the ratio of the payoffs from waiting to exercise
and from exercising now becomes infinite. Thus, the stock price must be strictly greater
than the strike price for the employee to prefer immediate exercise.

From Proposition 1, a risk neutral employee never exercises the option early. The
reason is that the expected payoff from holding the option to maturity always exceeds the
expected payoff from early exercise. Besides the expected payoff, a risk averse employee
also cares about the shape of the payoff distribution. The payoffs from exercising the
option now or at maturity differ in this regard. The possible payoffs from holding the
option to maturity resemble a standard lognormal distribution shifted left (by payment
of the exercise price) and truncated at zero. The probability mass from the truncated
section of the distribution is assigned to the zero payoff. Since there is significant chance
the option will expire out-of-the-money, holding the option entails a large probability of a
zero payoff. Exercising the option early to hold the stock offers a lower expected return,
but there is no chance of a zero payoff. If the employee evaluates the payoff distribution
from exercising the option now as sufficiently less risky than holding the option until the
next period, she will exercise now. The reduction in risk compensates her for the lower
expected payoff.
Now consider the case when the current stock price is high. Conditional on the current stock price being high, there is little chance the option will mature out-of-the-money. Thus, the truncated portion of the payoff distribution is tiny. Again, to build intuition, suppose the employee is constrained to hold all of her wealth in stock if she exercises her option. The principal difference between exercising the option early and at maturity is that when the employee exercises the option early, she pays the exercise price sooner. Early payment of the exercise price is costly because of the time value of money. Thus, exercising at maturity dominates exercising early.

As the stock price increases, the payoff distribution from holding the option shifts from a truncated distribution that places large probability on the zero payoff to one that closely resembles the return from holding stock. As this transformation takes place, the employee crosses a threshold at which she prefers holding the option to exercising the option and holding the stock. This argument roughly establishes the bounds on the lower early exercise region as a function of the stock price. The employee does not benefit by exercising an at-the-money option. She may exercise a just-in-the-money option. She will not exercise a deep-in-the-money option to hold stock. The upper exercise region depicted in figure 3 exists only when the proceeds from exercise can be invested in something other than the employer’s stock.

Now suppose the employee can invest the proceeds from exercising the option in either the riskless asset or the stock. If the employee prefers stock to the option, then she prefers the riskless asset to the option. Thus, the lower exercise region continues to exist when the riskless asset is added to the set of possible investments.

Next, consider the upper exercise region. The employee prefers the riskless asset to the stock. It does not follow that the riskless asset is preferred to the option everywhere. The risk-adjusted return on the option may well exceed the return on the riskless asset when there remains a long time to maturity or when the stock is close to the option’s strike price. In either case, the “upside potential” (more technically, the delta) of the option is large compared to the difference between the current stock price and the exercise price.

\[9\] In the binomial model, when the current stock price is high enough, there is no chance the option will mature out-of-the-money. Consider node (2,2) of the Example.
When the option is deep-in-the-money and the time to maturity is short, the payoff distribution of the option closely resembles that of the stock. Then, the employee finds it worthwhile to exercise the option early to invest the proceeds in the riskless asset, since she prefers the riskless asset to the stock. The resemblance between the payoff distributions from holding the option to maturity and from exercising the option to hold the stock is strongest when the stock price is very high. As the stock price decreases, the expected return from holding the option increases. Holding the option may dominate holding the riskless asset below some threshold, as in figure 3.

4.2 Numerical Example

This section shows the significance of early exercise on the value of the option to the employee and the cost of the compensation to the employer using the recursive algorithm developed above. For financial accounting purposes, the employer (i.e., the corporation issuing the option) is a reporting entity. The appropriate cost to record in the entity’s books is the subject of debate among accounting rule makers, audit firms, compensation consultants, and their clients. From an economic standpoint, the corporation is a nexus of contracts. The cost of the option is not incurred by the corporation, but by the shareholders of the corporation. The cost to the shareholders is the dilution they suffer when the option is exercised. I use the term ‘cost to the employer’ to mean the aggregate of the costs incurred by the shareholders. This notion of cost is appropriate for grant date accounting.

Suppose the employer grants the employee an option to buy 10,000 shares of the firm’s stock at any time over the next five years. Suppose the initial stock price and the strike price are both $20. Also suppose the stock price evolves according to a lognormal process with a mean of $\mu = 10\%$ and a volatility of $\sigma = .3$ per year. The employee’s objective is to maximize her expected utility after five years. Assume the employee exhibits preferences described by the utility function $U[W] = W^{\gamma}$ where $\gamma = \frac{1}{4}$. The employee has no wealth other than the option. These parameters correspond to figure 3b.

With these parameters, the employee is indifferent, at the date of grant, between the option and a sure payment today that gives the same utility. The sure payment today that

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10 For a process with $n$ steps per year, the parameters $p$ and $s$ corresponding to mean $\mu$ and volatility $\sigma$ are $p = \frac{1}{2} + \frac{1}{2} \frac{\mu}{\sigma} n^{-\frac{1}{2}}$ and $s = \exp(\sigma n^{-\frac{1}{2}})$. In this numerical example, a forty-step binomial model with $p = .558926$ and $s = 1.1119$ represents the stock price process. See Cox et al. (1979) for details.
makes the employee indifferent is $36,961. If the employee receives a European option, she foregoes the opportunity to exercise early. Imposing this restriction, the corresponding sure payment today is $30,881. Thus, the value the employee places on the American option is about 20% more than the value she places on the European option.

A risk averse employee chooses an exercise policy that maximizes her expected utility. The policy that maximizes expected utility does not, in general, coincide with the policy that maximizes the expected value of the option. Therefore, the expected cost to the employer of granting the ESO, given an exercise policy that maximizes expected utility, is less than the expected cost of an option, given an exercise policy that maximizes expected value. Option pricing models for tradable options effectively assume the exercise policy maximizes expected value. Thus, the expected cost to an employer who issues an ESO is less than the naive use of options pricing models would suggest.

To unambiguously determine the cost incurred by shareholders at the date of grant of an employee stock option, one ideally would like to have a competitive market for such claims in which third parties quote a price at which they will assume the obligation to honor the call option. This market, if it existed, would clear at a price that satisfies the FASB’s concept of “fair value.” The price the employer would pay to defease the obligation is the cost of the option to the employer.

For every employee exercise policy, there exists a portfolio of the firm’s stock and a riskless asset, called the hedging portfolio, that exactly mimics the payoff from the employee stock option. Provided the third party implements the trading strategy that replicates the option’s payoff, the third party bears no risk. This is because closing out the riskless asset and stock positions yields an amount that, added to the exercise price, equals the funds the employee requires to purchase the underlying stock.\footnote{If the option ends up out-of-the-money, the value of the hedging portfolio is zero.} Competition among third parties will drive the cost of defeasing the option down to the cost of the hedging portfolio.

Since $p$ and $s$ can be inferred from the past evolution of the stock price, the cost of the option to shareholders at the date of grant can be calculated in the absence of a competitive market for ESO obligations (but with knowledge of the employee’s exercise
policy) by calculating the cost at the date of grant of acquiring the hedging portfolio. The cost of the unexercised option at node \((i, j)\) valued at time \(i\) is the expected cost of the option at time \(i + 1\) discounted back one period.\(^{12}\) Denote the cost of the hedging portfolio at time \(i\) and node \(j\) by \(C(i, j)\). On exercise of a non-qualified option, the employee triggers a tax deduction in the hands of the employer that yields a tax savings in that period of \(T(s^j - X)\). Also, at exercise the employee pays the employer \(X\) for an asset worth \(s^j\). The after-tax, after-dilution cost of the option to the employer is therefore \((s^j - X)(T - 1)\). Then

\[
C(i, j) = \begin{cases} 
0 & \text{if the option expires out-of-the-money at this node;} \\
\frac{pC(i+1,j+1)+(1-p)C(i+1,j-1)}{d} & \text{if time remains to maturity and the employee does not exercise;} \\
(s^j - X)(T - 1) & \text{if the employee exercises the option at this node.}
\end{cases}
\]

The employer’s cost at the time of grant is \(C(0, 0)\).

Continuing the example above, the cost at the date of grant to defease a European option is $108,879. This is also the Black-Scholes value of the option. The cost to defease an American option, assuming the employee adopts the exercise strategy that maximizes her utility, is $94,229, about 13 percent less than a European option. Thus American options are less costly to the employer and more valuable to the employee than European options.

Implementation of grant date accounting requires knowledge of the fair value of the option at the date of grant. The results in this section suggest the fair value of an ESO to the employer may be much less than the Black-Scholes value of the option. Estimation of the fair value of the ESO requires knowledge not only of the dynamics of the stock price, but also of the employee’s exercise policy. In turn, the exercise policy hinges on the employee’s wealth, risk aversion, and the set of alternative investments. These factors considerably complicate the determination of an option’s value for purposes of implementing grant date accounting. In general, exercise of the option depends on the price path of the stock; it depends only partly and only indirectly on the time since the date of grant.

\(^{12}\) The mechanics behind this reasoning are well known. They are carefully explained in Cox et al. (1979).
Under plausible assumptions, the naive application of a traded stock option valuation formula yields an upwardly biased measure of cost. The FASB has suggested reducing the cost of an ESO by reducing the time parameter in the Black-Scholes option pricing formula from the time to maturity to the expected time to exercise [FASB (1993), paragraphs 129 and 197]. This proposal assumes the employee exercises at all nodes \((i, j)\) where \(i\) is greater than some \(I\). This adjustment will counteract the bias. There is no reason to believe this adjustment will result in a more precise estimate of the option’s cost since the boundary of the exercise region described in this section and figure 3 does not conform to this proposal. However, adjustments that do conform to the exercise policy presented in figure 3 can be incorporated into the binomial model.

A striking feature of figure 3 is the existence of a ceiling at which the employee chooses to exercise the option. Once the stock price reaches the ceiling, the employee exercises the option. The examples suggest a close approximation of the employee exercise policy may be obtained by assuming options are exercised as soon as the price of the stock reaches some critical multiple of the strike price.\(^{13}\) The descriptive validity of this simple heuristic for an employee’s exercise decision can be tested by examining whether the exercise decisions of employees with respect to a particular stock option are clustered at the point where the stock price first crosses the ceiling. Assuming these predictions are validated, these observations suggest the cost to the employer of granting stock options would be closely approximated by a binomial model in which exercise is assumed to take place the first time the stock reaches a specified multiple of the strike price. Pricing such options using the binomial model is straightforward and involves only one more parameter than the Black-Scholes model, namely the ceiling.

There are, of course, restrictions on the exercise of ESOs that must be considered in calculating their cost. The most significant restriction is the vesting period for the option. Lengthening the vesting period can only reduce the set of circumstance under which the employee exercises the option before maturity. The effect of imposing a vesting period is

\(^{13}\) The ceiling is not perfectly horizontal. The ceiling is lower as the time remaining until the option matures grows short. Since the ceiling is only approximately horizontal, a more refined heuristic would make the multiple a function of the time remaining until the option expires. Also, a more refined heuristic would take account of the possibility exercise will be triggered when the stock price crosses into the lower exercise region.
shown in figure 3c. Postponing the vesting date reduces the value of the option to the employee and increases the cost of the option to the employer.\footnote{Incentive considerations are likely to influence the choice of vesting period. Postponing exercise by lengthening the vesting period may yield desirable incentive effects. The increased cost to the employer may be offset by these effects.} If the stock price is above the ceiling when the vesting conditions lapse, the employee will exercise her option immediately. Since some corporations grant ten-year options that do not vest for as long as five years, there are many instances where options are exercised as soon as they vest. Thus vesting provisions (i) truncate the exercise ceiling near the grant date and (ii) add a vertical lip to the exercise region at the time they lapse. Incorporating this adjustment into the binominal model is also straightforward.

Table 1 sheds light on the magnitude of errors induced in the valuation of ESOs under methodology proposed in the FASB's exposure draft when employees exercise options according to the optimal exercise policy developed in this paper. For two different terms, two vesting schedules, and three assumptions about risk aversion, the table presents the Black-Scholes value of the option, the modified Black-Scholes value proposed by the FASB, and value determined by the binomial model given the optimal exercise policy under the assumptions used in this paper.

In the modified Black-Scholes formula proposed by the FASB, the time parameter is set equal to the expected time until the option is exercised instead of the time until the option expires. In table 1, the expected time to exercise is computed under the assumption the employee adopts the exercise policy that maximizes her expected utility. The (unmodified) Black-Scholes value is the maximum expected cost of the option to the employer under any exercise strategy. The table indicates that the modified Black-Scholes value overstates the cost of the option to the employer relative to the cost of the option determined by the binomial model assuming the optimal exercise strategies depicted in figure 3. The overstatement is greatest when the vesting period is short and the employee is very risk averse. The overstatement is 16\% of the modified Black-Scholes value for a five year option that vests immediately and is granted to an employee with a coefficient of risk aversion of 0.125.

The table demonstrates that, under plausible conditions, the employee’s exercise policy has a significant effect on the determination of the cost of the option at the date of
grant. Moreover, the modified Black-Scholes value proposed by the FASB does not closely approximate the theoretically correct value derived from the binomial model given exact knowledge of the employee’s exercise strategy. The disparity between the modified Black-Scholes value and binomial value is greatest when the employee is very risk averse and the vesting period is short.

This disparity is exacerbated by the \textit{ex post} adjustment to the modified Black-Scholes valuation also proposed by the FASB. The adjustment would require firms to charge to income the difference between the value of the option computed at the date of grant, using the expected time until exercise, and value obtained using the actual elapsed time to exercise. This “correction” increases the cost of options exercised later than expected (or that expire out of the money) and decreases the cost of options exercised earlier than expected. In fact, under the assumption of the binomial model used here, options exercised early are the ones which, \textit{ex post}, proved most costly to the employer. Options exercised late are the ones which, \textit{ex post}, proved least costly to the employer.

5. Conclusion

This paper considers the optimal dynamic exercise strategy of an employee who receives a stock option. Knowledge of the exercise policy is necessary to estimate the cost of the option to the employer for financial statement or other purposes. As the set of circumstances under which the employee exercises the option (i.e., the exercise region) grows larger, the cost of the option to the employer decreases. The exercise region is jointly determined by the employer, who sets the vesting schedule, and the employee, who decides whether to exercise when exercise is permitted.\textsuperscript{15} The cost to an employer who issues an ESO is less than options pricing models based on certain arbitrage arguments would suggest. It may even be less than a putative lower bound (the “minimum value”) that has been suggested for financial accounting purposes.

Risk aversion induces early exercise when either (i) the option is near-the-money and the time to maturity is very short or (ii) alternative less risky investments are available.

\textsuperscript{15} In this context, it is interesting to note emergence of “quickie options” that vest after 1 year [Byrne (1994)].
and the option is deep-in-the-money.\textsuperscript{16} These findings have implications for the FASB members as they re-evaluate an accounting standard for stock-based compensation plans. Likely, the Board will not prescribe the Black-Scholes formula because the assumptions underlying that pricing methodology do not apply to ESOs. Moreover, this paper suggests that for plausible parameter values, the Black-Scholes value is too high. Issuing detailed rules on the mechanics of a more general procedure (like the one used in this paper) is problematic because it is difficult to determine such inputs to the model as employee risk preferences. Reducing the time parameter in an option pricing model from the time to maturity to some shorter interval (e.g., the expected time to exercise) counteracts the bias induced in pricing models by early exercise. However, this paper shows that such adjustments are imprecise because the employee’s exercise decision depends on the stock price path. These complexities are not captured by the adjustments proposed by the FASB in the Exposure Draft. The examples presented in this paper suggest the FASB’s proposal to adjust the time parameter in the Black-Scholes model will overstate the cost of the option to the employer corporation. Additionally, the adjustment proposed by the FASB to correct for differences between expected and actual times to exercise will change employer corporations’ income statements more (less) for options that, \textit{ex post}, cost less (more) than originally estimated.

Many extensions of this paper are possible. The option pricing machinery used in this paper can be applied to value more complicated kinds of options for which the Black-Scholes approach is invalid. For instance, it is possible to value reload options and options with performance vesting requirements.

There are important questions concerning employee stock options that remain unanswered. An important assumption underlying this analysis is that the stock price process is stationary. While this is a reasonable approximation for employees covered by broad-based stock option plans, it seems less plausible for top executives. Because options provide incentives to employees, granting options may ‘improve’ the stock price process. Ignoring the incentive effect on the stock price process provides a conservative cost of the option.

\textsuperscript{16} Additionally, changes in tax rates may induce a preference either for early exercise or exercise at maturity. Adverse private information held by the employee favors early exercise. Other reasons for early exercise not considered in the paper include consumption or liquidity needs of the employee, and exercise induced by termination of the employee or payment of dividends.
Consideration of the links among compensation, employee actions, and stock price is an interesting topic for future research. Another open question is the empirical prevalence of early exercise of employee stock options. This paper suggests several testable relationships among exercise decisions, risk aversion, and stock price mechanics.
Appendix I.

Proof of Proposition 1: Suppose the employee contemplates exercising at time 0 an option that matures at time 1 with an exercise price of $X$. Let the stock price at time $i$ be $S_i$. Let $E(S_1)$ be the expected price of the stock at time 1. Suppose the riskless asset pays $d$ after tax at time 1 per dollar invested at time 0.

$$1 \leq d \leq \frac{E(S_1)}{S_0} \quad (I.1)$$

Obviously, when the strike price is above the market price, it is not worthwhile to exercise the option. Suppose $S_0 > X$. If the option is a non-qualified option (NQO) for tax purposes, tax on the difference between the market price and the strike price is payable at the ordinary rate on the date of exercise. Tax at the capital gains rate is payable on subsequent appreciation when the stock is sold. Thus, it is necessary to show

$$\max\left\{ d, E\left(\frac{S_1 - (S_1 - S_0)g}{S_0}\right) \right\} (S_0 - X)(1 - t) \leq E\left(\max\{0, (S_1 - X)(1 - t)\}\right)$$

where $g$ is the capital gains tax rate and $t$ is the ordinary tax rate. Since the ordinary tax rate, $t$, enters symmetrically, it is sufficient to show

$$(S_0 - X) \max\{d, \frac{E(S_1)}{S_0}(1 - g) + g\} \leq E(\max\{0, S_1 - X\}).$$

Now,

$$(S_0 - X) \max\{d, \frac{E(S_1)}{S_0}(1 - g) + g\}$$

$$\leq \max\{d(S_0 - X), \frac{E(S_1)}{S_0}(S_0 - X)\}$$

$$\leq \max\{d(S_0 - X), E(S_1) - X\}, \quad \text{because } E(S_1) \geq S_0,$$

$$= E(S_1) - X, \quad \text{by (I.1),}$$

$$\leq E\left(\max\{0, S_1 - X\}\right).$$

If the option is an incentive stock option (ISO) for tax purposes, tax on the difference between the market price of the stock on the day it is sold and the strike price of the option at the capital gains rate is payable at the capital gains rate. No tax is payable
on the exercise date. Holding the ISO to maturity dominates exercising the ISO early to hold the stock along every stock price path. The argument for the NQO when \( g = t \) shows that holding the ISO to maturity dominates exercising an ISO early to hold the alternative investment.

**Proof of Proposition 2:**

The employee’s utility up to the time the option is exercised is multiplicatively separable from the utility of holding either the riskless asset or the stock over the remainder of the employee’s investment horizon. Therefore, when the employee prefers to exercise (rather than hold the option) next period, the employee prefers to exercise (rather than hold the option) this period if and only if

\[
pU[Ss-X] + (1-p)U[S/s-X] < \max(U[d(S-X)], pU[(S-X)s] + (1-p)U[(S-X)/s]).
\]

The result follows from the two lemmas below. The inequalities in the lemmas are the inequality above, where each term has been divided by \( X^\gamma \), \( V \) replaces \( S/X \), and \( p \equiv \frac{1}{2} \). From (iv), the employee prefers to exercise rather than hold the option next period, so the option must be in-the-money next period. Hence \( V > s \). Lemma 1 supposes the employee must invest the proceeds from exercising the option in the stock. Lemma 2 supposes the employee must invest the proceeds from exercising the option in the riskless asset. Lemma 1 shows that any sufficiently risk averse employee will exercise her option early to hold the stock. Lemma 2 shows that any sufficiently risk averse employee will exercise her option early to hold the riskless asset. An employee who is more (less) risk averse than either (both) the threshold given by Lemma 1 or (and) the threshold given by Lemma 2 will exercise her option early (at maturity), establishing the Proposition.

**Lemma 1:** For every \( s \) and \( V \), there exists a \( \hat{\gamma} \) satisfying \( 0 < \hat{\gamma} < 1 \) such that

\[
\frac{1}{2}(sV - 1)^\gamma + \frac{1}{2}(V/s - 1)^\gamma \lesssim \frac{1}{2}[s(V - 1)]^\gamma + \frac{1}{2}\left[\frac{1}{s}(V - 1)\right]^\gamma \quad \text{if} \quad \gamma \lesssim \hat{\gamma}.
\]

**Lemma 2:** For every \( s \), \( V \), and \( d \), there exists a \( \hat{\gamma} \) satisfying \( 0 < \hat{\gamma} < 1 \) such that

\[
\frac{1}{2}(sV - 1)^\gamma + \frac{1}{2}(V/s - 1)^\gamma \lesssim [d(V - 1)]^\gamma \quad \text{if} \quad \gamma \lesssim \hat{\gamma}.
\]
**Proof of Lemma 1:** Let $a_1 = \ln(V/s - 1)$, $a_2 = \ln(V - 1)$, $a_3 = \ln(sV - 1)$, and $b = \ln s$. The lemma is equivalent to showing $f(\gamma) = e^{\gamma a_3} + e^{\gamma a_1} - e^{\gamma a_2}(e^{\gamma b} + e^{-\gamma b})$ is non-positive for $0 < \gamma < \hat{\gamma}$, positive for $\hat{\gamma} \leq \gamma$, and $f(1) > 0$. This function can be rewritten $f(\gamma) = e^{\gamma a_1}g(\gamma)$, where $g(\gamma) \equiv 1 + e^{\gamma c_2} - e^{\gamma c_1}(e^{\gamma b} + e^{-\gamma b})$, $c_1 \equiv a_2 - a_1$, and $c_2 \equiv a_3 - a_1$.

Because $e^{\gamma a_1}$ is non-negative, the sign of $g$ is the sign of $f$. Thus it must be shown that $g(\gamma)$ is non-positive for $0 < \gamma < \hat{\gamma}$, positive for $\hat{\gamma} \leq \gamma$, and $g(1) > 0$. Since $g(0) = 0$, it is sufficient to show that: (i) $g(\gamma)$ is monotonically decreasing on some interval $(0, \hat{\gamma})$; (ii) $g(\gamma)$ is monotonically increasing on $(\hat{\gamma}, \infty)$; and (iii) $g(1) > 0$. To establish this, consider $g'(\gamma)$ and the following claims.

$$g'(\gamma) = c_2 e^{\gamma c_2} - (c_1 + b)e^{\gamma (c_1 + b)} - (c_1 - b)e^{\gamma (c_1 - b)}$$
$$= e^{\gamma (c_1 - b)}h(\gamma), \text{ where } h(\gamma) = [c_2 e^{\gamma (c_2 - c_1 + b)} - (c_1 + b)e^{2\gamma b} - (c_1 - b)]$$

**Claim 1:** $c_2 > c_1 + b$ and $c_2 - c_1 + b > 2b$.

Note that these two inequalities are equivalent. The first inequality reduces to showing $a_3 > a_2 + b$ or $sV - 1 > (V - 1)s$. This last inequality holds because $s > 1$. Q.E.D.

**Claim 2:** $h(\gamma)$ increases monotonically without bound on $(0, \infty)$.

$$h'(\gamma) = c_2 (c_2 - c_1 + b)e^{\gamma (c_2 - c_1 + b)} - 2b(c_1 + b)e^{2\gamma b}$$
$$= 2b(c_1 + b)e^{2\gamma b}[\frac{(c_2 - c_1 + b)c_2}{2b(c_1 + b)}e^{\gamma (c_2 - c_1 - b)} - 1]$$
$$> 0 \text{ for } \gamma \geq 0 \text{ by Claim 1}.$$

Also, $\lim_{\gamma \to \infty} h'(\gamma) = \infty$ so $\lim_{\gamma \to \infty} h(\gamma) = \infty$. Q.E.D.

**Claim 3:** $g'(0) < 0$.

$$g'(0) = c_2 - 2c_1 = a_3 - 2a_2 + a_1 = \ln \left( \frac{V^2(s+1/s)V + 1}{V^2 + 2V + 1} \right).$$

The argument of the logarithm is less than 1 because $s + 1/s > 2$. Q.E.D.

**Claim 4:** There exists a $\bar{\gamma} > 0$ such that $g'(\gamma) < 0$ for $\gamma < \bar{\gamma}$ and $g'(\gamma) > 0$ for $\gamma > \bar{\gamma}$. 23
The sign of $g'(\gamma)$ is given by $h(\gamma)$ because $e^{\gamma(c_1 - b)} > 0$ for all $\gamma$.

Claim 2 shows that $h(\gamma)$ increases monotonically without bound. Hence, $g'$ changes sign at most once. Claim 3 shows $g'(0) < 0$, so $\bar{\gamma} > 0$. Q.E.D.

Since $g'(0) < 0$ and $g'(0)$ changes sign once only, $g$ is decreasing and then increasing on $\gamma \in (0, \infty)$. Also,

$$g(1) = \frac{s + 1/s - 2}{V/s - 1} > 0$$

since $V > s$ and $s + 1/s > 2$. Because $g(1) > 0$ and $g(0) = 0$, $g$ changes sign exactly once in $(0, 1)$. □

Proof of Lemma 2: Let $a_1 = \ln(V/s - 1)$, $a_2 = \ln(V - 1)$, $a_3 = \ln(sV - 1)$, and $b = \ln d$. The proposition is equivalent to showing that $f(\gamma) = e^{\gamma a_3} + e^{\gamma a_1} - 2e^{\gamma(b + a_2)}$ is non-positive for $0 < \gamma < \bar{\gamma}$, positive for $\bar{\gamma} \leq \gamma$, and $f(1) > 0$. This function can be rewritten $f(\gamma) = e^{\gamma a_1} g(\gamma)$ where $g(\gamma) \equiv 1 + e^{\gamma c_2} - 2e^{\gamma c_1}$, $c_1 \equiv b + a_2 - a_1$, and $c_2 \equiv a_3 - a_1$. Because $e^{\gamma a_1}$ is non-negative, the sign of $g$ is the sign of $f$.

Claim: $c_1 < c_2 < 2c_1$.

To show that $c_1 < c_2$, it is enough to show $a_3 > b + a_2$. This reduces to $d(V - 1) < sV - 1$ or $1 - d < (s - d)V$. The last expression holds when $1 < d < s$ or $1 < d < (s - d)V$. The last expression holds when $1 < d < s$. To show that $c_2 < 2c_1$, it is enough to show $a_1 + a_3 < 2(b + a_2)$. This reduces to $(V/s - 1)(sV - 1) < d^2(V - 1)^2$ or $[2d^2 - (s + 1/s)]V < (d^2 - 1)(V^2 + 1)$. Now $s + 1/s > 2$ for $s > 1$, so it suffices to show $2V \leq V^2 + 1$. This last expression holds for any $V > 0$. Q.E.D.

Now $g'(\gamma) = c_2 e^{\gamma c_2} - 2c_1 e^{\gamma c_1}$. This quantity is less than zero if and only if

$$\gamma < \bar{\gamma} \equiv \frac{\ln \left( \frac{2c_1}{c_2} \right)}{c_2 - c_1}.$$ 

The claim establishes that the numerator and denominator are both positive. Thus, $g(\gamma)$ is decreasing in the range $(0, \bar{\gamma})$ and increasing in the range $(\bar{\gamma}, \infty)$. Also,

$$g(1) = \frac{V(s + 1/s - 2d)}{V/s - 1} > 0$$

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since $V > s$ and $\frac{1}{2} s + \frac{1}{2} \frac{1}{s} > d$ by assumption. Since $g(0) = 0$ and $g(1) > 0$, there is a unique $\hat{\gamma}$ satisfying $\bar{\gamma} < \gamma < 1$ at which $f(\gamma) = 0$. Moreover, $f(\gamma) < 0$ for $\gamma < \hat{\gamma}$ and $f(\gamma) > 0$ for $\gamma > \hat{\gamma}$. ■
References


Huddart, Steven and Mark Lang, 1994, Roundtable on Stock Option Valuation: Presentation to the Financial Accounting Standards Board, April 18, 1994, mimeo.

Kulatilaka, Nalin and Alan J. Marcus, 1994, Early Exercise and the Valuation of Employee Stock Options, Working paper, Boston University.


Figure 1. Institutional features of employee stock options. Employee stock options often are not exercisable for some years after issuance. The option may be exercised before expiry. The employee may sell the stock at any time after she exercises the option.
Figure 2. The binomial model of stock price evolution. An ingrown tree depicts stock price movements. At every node, the probability of an uptick is $p$. The probability of a downtick is $1 - p$. On an uptick (downtick), the stock price increases (decreases) by a factor of $s$ ($1/s$).
Figure 3. Graphical representations of optimal exercise policies under various assumptions. This figure plots three instances of the optimal exercise policy for an ESO that matures after 5 years and is owned by a risk averse employee. The parameters that vary across the three cases are the risk aversion of the employee and the vesting schedule. In (a), the employee is assumed to have a terminal utility for wealth given by $U(W) = W^{1/2}$. In (b) and (c), the employee has utility $U(W) = W^{1/4}$. In (a) and (b), the option vests immediately. In (c), the option does not vest until two years after the grant date. In all three cases, the employee prefers holding the riskless asset to holding the stock. The optimal policy in (time, stock price) space is shown by

- $b$: Exercise, but only if the proceeds from exercise can be invested in the riskless asset;
- $s$: Exercise, even if the proceeds from exercise must be invested in the stock. A fortiori, exercise if the proceeds can be invested in the riskless asset;
- $u$: The employee cannot exercise the option because it has not vested;
- $+$: Hold the option until next period.

A forty-step binomial model with parameters $p = .558926$ and $s = 1.1119$ approximates the evolution of the price of a stock with annual mean return $\mu = .1$ and volatility $\sigma = .3$. 
<table>
<thead>
<tr>
<th>Term</th>
<th>10 years</th>
<th>5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vesting date (years after grant)</td>
<td>5 5 5 0  0 0</td>
<td>2 2 2 0  0 0</td>
</tr>
<tr>
<td>Coefficient of risk aversion (γ)</td>
<td>0.5 0.25 0.125 0.5 0.25 0.125</td>
<td>0.5 0.25 0.125 0.5 0.25 0.125</td>
</tr>
<tr>
<td>Expected time to exercise (years)</td>
<td>9.41 7.95 6.08 9.39 7.69 4.80</td>
<td>4.96 4.12 2.81 4.96 4.11 1.99</td>
</tr>
<tr>
<td>Black-Scholes value ($000)</td>
<td>155 155 155 155 155 155</td>
<td>109 109 109 109 109 109</td>
</tr>
<tr>
<td>Modified Black-Scholes value ($000)</td>
<td>151 140 121 151 137 106</td>
<td>109 97 76 109 97 61</td>
</tr>
<tr>
<td>Binomial value ($000)</td>
<td>149 136 116 149 132 96</td>
<td>108 94 70 108 94 51</td>
</tr>
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</table>

Table 1. Option value under different valuation models. This table presents three measures of the cost of employee stock options. The first measure is the Black-Scholes value of the option in which the time parameter equals the time from grant until maturity. The second is the modified Black-Scholes value of the option proposed by the FASB. In the modified Black-Scholes formula, the time parameter equals the expected time from grant until the option is exercised. The expected time to exercise is computed for an employee who adopts the exercise policy that maximizes her expected utility under the assumptions used in this paper. The third measure is the value determined by the binomial model given the optimal exercise policy. Valuations for 5- and 10-year options with and without a vesting requirement are presented. The values are based on an option to buy 10,000 shares of the firm’s stock at $20 per share. The strike price equals the value of the stock at the date of grant. A forty-step binomial model approximates the evolution of the price of a stock with annual mean return of \( \mu = 0.1 \) and volatility of \( \sigma = 0.3 \). The employee who holds the option has a terminal utility for wealth given by \( U(W) = W^{\gamma} \) where \( \gamma \) is either 0.5, 0.25, or 0.125.