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† Paul Fischer, Diego Garcia, Dan Givoly, Jenny Gaver, Florin Şabac Richard Sansing, Phil Stocken, Amy Sweeney, Yun Clement Zhang, and seminar participants at Dartmouth College, the 7th Carnegie-Mellon University Accounting Conference, and the 2006 American Accounting Association meetings provided many useful comments.

Send correspondence to:
Steven Huddart
Smeal College of Business
Pennsylvania State University
University Park, PA 16802-1912
telephone: 814 865–3271
facsimile: 814 863–8393
e-mail: huddart@psu.edu
web: www.personal.psu.edu/sjh11

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1. Introduction

We derive optimal contracts between a firm’s shareholder and its privately-informed manager when the manager has the discretion to manipulate a public report of firm value by deferring or accelerating recognition of value. For low realizations of firm value, the manager overstates value in his report and for high realizations of firm value, he understates value. The distribution of reported value in the model has the key characteristic of the distribution of firms’ earnings reports noted in the empirical literature, namely a low frequency of earnings reports below a threshold and a high frequency at the threshold, which we call a “divot”. Moreover, the revision in price at the disclosure of the manager’s report resembles the S-shaped price response noted in the empirical literature.

We explore the possibility that the slope of the price response to earnings news is related to the nature of the associated earnings management. Degeorge et al. (2001) find evidence of a divot in the distribution of earnings relative to analysts’ consensus EPS forecasts—exactly the spot about which the stock price response is steepest according to Freeman and Tse (1992) and subsequent studies. It is worth exploring, therefore, whether and how the non-linearity of returns and earnings may be related to the nature and intensity of earnings management at this point. Our analysis links the price response at the earnings announcement to the underlying earnings management. Our model implies that price responds most sharply to earnings news in the neighborhood of the divot.

Existing theory can be broken into three categories according to whether papers adopt a behavioral approach, a signaling approach, or a contracting approach. Behavioral theories of why managers strive to report earnings that meet or beat earnings expectations rely on some agents having preferences that violate the axioms of rational

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1 Another (partial) explanation is that earnings consists of transitory and permanent components. An increase in the permanent component of earnings implies earnings are higher in every future period, accordingly a multiplier of $1 + 1/r$, where $r$ is an appropriate discount rate, relates the increase in permanent earnings to the change in firm value. Transitory earnings changes imply no change in future earnings and so the multiplier is 1, which is smaller. To the extent that earnings further away from the forecast contain a larger transitory component, the multiplier on the surprise in earnings (i.e., operationalized as the difference between the consensus forecast and the announced earnings) is smaller, which could produce the S-shape. We suppress such consideration by normalizing the discount rate so that $r = 0$ and supposing that all innovations in value are transitory.
choice. For instance Burgstahler and Dichev (1997) suppose managers’ preferences are described by Kahneman and Tversky’s (1979) value function. Other arguments rely on agents using decision heuristics that imply sub-optimal use of available information. For instance, Degeorge et al. (1999) suppose that bankers and directors exhibit a “threshold mentality” such that their reaction to an earnings report is discontinuous about the threshold. In contrast to these behavioral approaches, we require the manager and shareholder in our model to be fully rational.

One class of models with rational actors are models of costly signaling such Guttman, Kadan, and Kandel (2006). In that paper, as in our paper, the manager incurs a personal cost when he distorts reported earnings. Investors are rational, so the stock price is the expected value of the firm conditional on the available information. Also as in our paper, it is possible that the manager’s disclosure strategy fully reveals his private information or that the manager makes the same report for some range of firm values, in which case reported earnings do not reveal all the manager’s private information. Our model and Guttman et al.’s are structured quite differently, however. In Guttman et al.’s analysis, the manager’s compensation is an exogenously-specified function of the stock price. Although it might be more efficient to make the manager’s pay also depend on his report, this is ruled out. In our model, in contrast, the compensation contract is endogenous, may take any form, and is conditioned on all publicly-available information. Thus, the manager’s pay is the unique function of his report that minimizes the shareholder’s costs of hiring the manager.

In both models, the manager makes the same report for a range of private values. In Guttman et al. this is because investors expect the manager to do exactly that. The manager does what the shareholders expect because the beliefs of the shareholders about the manager’s private information were the manager to announce earnings that are off the equilibrium path are assumed to be such that the manager prefers to do what the shareholders expect. That is, particular off-equilibrium beliefs are necessary to support the equilibrium. A consequence of this approach is that there are an infinite number of pooling equilibria (as well as a separating equilibrium). Our screening model yields a unique equilibrium and, because the game starts with the uninformed shareholder
offering a report-contingent contract to the manager, off-equilibrium beliefs need not be specified.

A parsimonious and rigorous economic theory of earnings management would (i) induce the manager to report value above fundamental value in some states and report value below fundamental value in others (i.e., to manage earnings), (ii) require that contracts between managers and shareholders be optimal, (iii) impose rational stock pricing, (iv) not be supported by *ad hoc* beliefs about off-equilibrium behaviors, and (v) have implications consistent with observed empirical regularities, including the divot in the distribution of earnings, the S-shaped response of price to reported earnings, and a compensation schedule that is an increasing concave function of the performance measure. Our goal is such a theory. This paper is an exploration of whether these strong empirical regularities can emerge endogenously from an optimal contracting framework in which a report of firm value that is manipulated upward in one period implies lower reported value in the future vice versa. In the next section, we develop the model, characterize optimal contracts under various assumptions, and then explore the implications of the optimal contracts on the distribution of reported value. In section 3, we discuss the stock price response to the manager’s report of value. In section 4, we outline some possible extensions. Section 5 concludes.

2. Model

In this section, we lay out the contracting problem, transform the contracting problem into an efficient allocation problem, characterize the efficient allocation in the general case (first without and then with a solvency constraint), and develop intuition for the optimal contracts, the manager’s reporting strategy, and the price response to the managers’ reports by solving three specific examples of the general problem.

We consider a contracting problem between a risk-neutral shareholder and a risk-neutral manager. We assume the manager is essential to the operation of the firm, so the contract must be such that the manager prefers working for the firm to receiving his reservation wage in his next best alternative. The manager, who is privately informed about the value of the firm, $x$, signs an employment contract, $R$, with the shareholder.
The manager then reports a value, $y$, to the shareholder. The contract $R$ must result in the manager choosing to work for the firm after learning $x$, so the participation constraint must be satisfied for every realization of $x$. Since the manager’s report is the only publicly-available information, stock price (which we derive later) is a function of $y$ only. Thus, restricting contracts to be functions of $y$ alone is without loss of generality. Hence, the contract $R$ depends on $y$ alone and is crafted so that the manager works for the firm at the lowest expected cost and no matter the realization $x$.

Since the manager is not subject to moral hazard with respect to his efforts as an employee of the firm, the solution to this contracting problem would be for the shareholder to pay the manager a fixed wage irrespective of his private information were it not for additional assumptions about the structure of the problem that connect the manager’s report to the manager’s compensation. We suppose that the manager can distort his report about the value of the firm away from $x$ by incurring a personal cost. The mechanics of the distortion are chosen so that overstatements (i.e., $y > x$) or understatements (i.e., $y < x$) in one period must reverse in subsequent periods. When the manager’s report differs from the underlying value of the firm, the difference $y - x$ is interpreted as the amount of manipulation (e.g., earnings management) undertaken by the manager.

The opportunity to manipulate accounting reports arises in part because accounting requires estimates that determine various components of earnings. For example, estimates of the value of consideration received in non-cash transactions affect reported revenue; estimates of uncollectible receivables affect bad debt expense, estimates of ending inventory values affect cost of goods sold; and estimates of future compensation rates, interest rates, life expectancy, turnover, and rates of return on pension plan assets affect defined benefit pension plan and health care expense. Reported earnings, which are revenues net of expenses, therefore depend on each of these estimates. Since the estimates are inherently unverifiable, managers have scope to manipulate the estimates and distort earnings to further their interests. In addition, accounting rules often afford managers discretion over how to account for a class of transactions. For example, managers have discretion to choose revenue recognition, inventory flow, depreciation and amortization policies; they may use that discretion to distort earnings.
Besides manipulating estimates and selecting accounting policies, managers may also adjust a reported performance measure by altering firm operating activities. Such alterations can result in economic costs that are not reflected in the measure. For example, managers may forego expenditures on research and development, advertising, and maintenance because financial accounting rules dictate that such expenditures are expensed immediately, while the benefits are recognized in future earnings. As another example, managers may engage in channel stuffing (i.e., actions that cause customers to accelerate purchases that would otherwise occur after the end of a fiscal period to boost revenues for the current period). Channel stuffing is costly because it is achieved by offering excessive discounts, cannibalizing future periods’ sales, and incurring the costs of future returns. As a final example, managers may set up off-balance sheet vehicles to create immediate revenue flows or defer expense flows.

In the case of manipulation accomplished through accounting estimate or accounting policy changes, such costs could include the manager’s effort directed at justifying to the firm’s auditors why certain reserve accounts are under- or over-stated. In the case of manipulation accomplished through operational changes, such costs could include the loss attributable to inefficient operations. All such costs are wasteful from a social perspective. For simplicity, we assume these costs are borne personally by the manager. We further suppose that the cost of manipulating the report is \( g(x - y) \) where \( g \geq 0, \ g(0) = g'(0) = 0, \ g'' > 0, \) and \( g(y - x) = g(x - y). \)

\(^2\) This assumption means that, at a personal cost, the manager can falsify the firm’s state. The “costly state falsification” framework has been applied to other reporting problems including Crocker and Morgan’s (1998) analysis of the design of insurance contracts and Crocker and Slemrod’s (2006) demonstration that earnings management is a necessary component of the efficient contract in a moral hazard problem when the manager may expend resources to inflate reported earnings.

The manager’s decision to manipulate is driven by the prospect that he will be employed by the firm in the future. Given that the manager’s pay this period depends

\(^2\) We might think about distinguishing estimate manipulation from operational manipulation by giving them different costs to the shareholder and manager. Bruns and Merchant (1990) present survey evidence that most accountants view earnings management accomplished via estimate manipulation as less acceptable than operational manipulation. Such considerations are outside the scope of our analysis.
on his report this period and his future pay depends on his future reports, a higher report today implies that future reports are lower by the amount of the overstatement in the current period and conversely. Whether it is worthwhile for the manager to raise or lower the value he reports in the current period depends on the concomitant anticipated cost or benefit of lowering or raising the value he reports in the future as well as the personal cost incurred by the manager in manipulating the report. If \( R(y) \) is an increasing function, then it follows that reserves are valuable to the manager because they may be drawn down to increase his pay in a future period.

Note that the manager’s private information is represented by the single variable \( x \). If some other variable relevant to contracting were private information that impacted \( R \), then the problem appears intractable. The solution to the contracting problem we consider does not depend, e.g., on whether the level of reserves is unknown, since the level of reserves does not affect the manager’s choice of earnings manipulation. The manager’s reporting strategy depends on cost of shifting value into and out of reserves. We assume this cost is independent of the level of reserves. In keeping with the notion that fundamental value is never directly observed by market participants, we assume that the shareholder never observes \( x \) directly. Instead, the shareholder and other market participants draw inferences about \( x \) from the disclosed value, \( y \).

The value to the manager of manipulating the report is that value not reported in the current period is added to a reserve that is hidden from the shareholder. To make the benefit of this hidden reserve concrete and to keep the problem tractable, we do not model the strategic interaction in the future periods explicitly. Instead, we suppose that the marginal benefit to the manager of adding to the hidden reserve is \( \beta \) per unit, where \( 0 < \beta < 1 \). Correspondingly, if the manager reports a value above the realized value, reserves are lower in the future, which implies an opportunity cost for the manager of \( \beta \) per unit. Choosing \( \beta > 0 \) implies there is some future benefit to higher reserves. Assuming that the contracts offered to the manager in future periods pay the manager more when the values he reports in the future are higher, then \( \beta \) can be interpreted as the expected increase in the manager’s future compensation from creating a reserve now that may be drawn down in the future to increase reported value.
Consistent with the foregoing discussion, we write the manager’s preferences as

\[ V(R(y), y \mid x) = R(y) - g(x - y) + \beta(x - y). \]  

(1)

In keeping with the nature of accounting manipulation accomplished via changes in accounting estimates, over- and under-reporting do not directly destroy value. For example, reserves taken against earnings in the current period (and so not reported to the shareholder in the current period) will be available to the shareholder in a future period. Manipulation nevertheless imposes indirect costs on the shareholder in two forms. First, the manipulation of earnings is personally costly to the manager. Since the shareholder must design a pay plan that meets the manager’s reservation wage constraint, the personal cost of manipulation undertaken by the manager ultimately is borne by the shareholder. This cost is an efficiency cost in that it reduces the total surplus that is split between the manager and shareholders. Second, the manager’s manipulation activities shift reports of value between periods with the goal of extracting more compensation from the shareholder. Thus, the benefit \( \beta \) per unit of earnings manipulation that the manager expects to derive in the future is also a cost imposed on the shareholder, although this cost is simply a transfer. We suppress all time-value-of-money considerations so that a unit of value reported in the future is worth the same as a unit of earnings reported today.

The shareholder’s preferences are

\[ \Pi(R, y \mid x) = y - R(y) - \beta(x - y) + b, \]  

(2)

where \( b \) is the change in hidden reserves. Because the change in the reserve is the amount of earning manipulation, \( x - y \), we rewrite the shareholder’s preferences as

\[ \Pi(R, y \mid x) = x - R(y) - \beta(x - y) \]  

(3)

and suppress further consideration of \( b \). The timeline summarizes the game we study.

[Figure 1]

7
Assume $x$ is distributed on $X \equiv [\underline{x}, \overline{x}]$. The density and cumulative distribution functions of $x$ are $f$ and $F$, respectively. The manager’s reporting choice, $y(x)$ should maximize $V(R(y(x)), y \mid x)$.

An efficient compensation contract is a function $R(y)$ that maximizes the expected value of the firm net of the cost of paying the manager,

$$
\max_{R(y)} \int_{\underline{x}}^{\overline{x}} \Pi(R, y \mid x)f(x)dx
$$

subject to

$$
y(x) \in \arg \max V(R(y), y \mid x) \quad \text{for all } x \in X
$$

and

$$
V(R(y), y \mid x) \geq \overline{U} \quad \text{for all } x \in X.
$$

We rule out contracting solutions in which the shareholder sells the firm to the manager by further supposing that the manager is subject to a liquidity constraint that prevents him from buying the firm. One might object to (4) on the grounds that (3) implies the shareholder chooses $R$ to maximize the firm’s fundamental value, $x$, when instead the shareholder ought to maximize the firm’s stock price conditioned on the release of $y$.

Under the assumption that the market price of the stock is $E(x \mid y) - R(y) - \beta(x - y)$, it is readily apparent that the law of iterated expectations implies that this objective function is equivalent to (4).

Constraint (5) is the delegation constraint reflecting the fact that the choice of report has necessarily been delegated to the manager, who then selects the announcement that maximizes his expected utility conditional on his private information, $x$. The delegation of $y$ implies a reporting strategy for the manager $y(x)$ that is an optimal response to $R$, as well as the manager’s private information regarding firm value. Constraint (6) is the participation constraint guaranteeing that the manager receives his reservation level of utility, $\overline{U}$, for any possible realization of firm value, $x$.

The shareholder seeks to minimize the sum the socially-wasteful effort directed at distortion, $g(x - y)$, and the information rents above the manager’s reservation wage integrated over all types of managers. Absent private information about $x$, so that all
reports are necessarily truthful (i.e., \( y = x \)), the optimal contract is \( R(y) = U \) and the expected profit of the shareholder would be \( E(x) - U \). Alternatively, if \( x \) is private information (so that (5) applies) but the participation constraint is relaxed to an ex ante constraint that holds in expected value, \( \int_x^x V f dx \geq U \), then the manager agrees to the compensation contract if the expected utility it provides is at least his reservation wage, though there may be some states in which the manager must pay the shareholder. As we analyze the problem, (6) is an ex interim constraint (i.e., it applies after the manager learns \( x \)) so that the manager agrees to the compensation contract if for every realization of \( x \) the contract provides more than his reservation wage. Structuring the constraint in this way implies the manager never prefers to leave the firm after learning \( x \).

Note that a linear contract with slope \( \beta \) is feasible and would motivate the manager report \( x \) without distortions. Choosing the constant term of this contract to satisfy the manager’s participation constraint assures that the contract is feasible in the optimization program. In particular, the intercept of this linear contract must be fixed so that the participation constraint of the lowest-type manager is satisfied. As a consequence, higher-type managers receive payments in excess of their reservation wage. From the shareholder’s perspective, an offsetting benefit is that no hidden reserves (which cost the shareholder \( \beta \) per unit) are created. While preventing hidden reserves is feasible, it is not optimal. The optimal contract induces all manager types to work for the firm at the lowest combined cost to the shareholder of distortions \( g(x - y) \) and the expected cost of future earnings manipulation \( \beta(x - y) \).

\[ \text{Relative to an ex interim participation constraint, an ex ante participation constraint implies lower payments to the manager: The contract } R(y) = \beta(y - E(x)) + U \text{ induces truthful reporting of earnings for every } x, \text{ and results in expected profit to the shareholder of } E(x) - U. \text{ To see this, note that, if } V = \beta(y - E(x)) + U - g(x - y) + \beta(x - y), \text{ then setting } dV/dy = 0 \text{ implies } x = y. \]
2.1 Optimal allocation without a solvency constraint

Since this is an environment of private information, Myerson’s (1979) Revelation Principle applies, and we may use Guesnerie and Laffont’s (1984) solution technique to characterize an optimal contract \( R(y) \). This approach recognizes explicitly the constraints on the implementation of contracts imposed by the information asymmetry. To reframe the problem described above so that Guesnerie and Laffont’s technique may be used, we define an allocation to be an action, \( y \), and a monetary transfer, \( R \). The solution technique is to condition the allocation on the private information, \( x \), so that the allocation is \( \{ R(x), y(x) \} \), and to recognize that any implementable contract must satisfy the incentive compatibility constraint.

Recasting (4), (5), and (6), an efficient allocation is a solution to the problem that maximizes the expected payoff to the risk-neutral owner,

\[
\max_{R(x), y(x)} \int_{x}^{\bar{x}} \Pi(R(x), y(x) \mid x) f(x) dx, \tag{7}
\]

subject to the incentive constraint

\[
V(R(x), y(x) \mid x) \geq V(R(\hat{x}), y(\hat{x}) \mid x) \quad \text{for every } x, \hat{x} \in [x, \bar{x}], \tag{8}
\]

and the participation constraint,

\[
V(R(x), y(x)) \geq \bar{U} \quad \text{for every } x \in [\underline{x}, \bar{x}]. \tag{9}
\]

We will, for the moment, ignore the participation constraint (9), but will return to this matter later.

When the incentive constraint (8) is satisfied, a manager who possesses the private information \( x \) prefers the allocation \( \{ R(x), y(x) \} \) over the alternatives for every \( \hat{x} \neq x \). The compensation contract \( R(y) \) can be recovered from the allocation \( \{ R(x), y(x) \} \) by inverting \( y(x) \) and substituting the resulting expression into \( R(x) \). When presented with the compensation contract \( R(y) \), the manager selects the report, \( y \), that maximizes his expected utility. As depicted in Figure 2, for a manager with private information \( x \), the report \( y \) is given by the tangency of the manager’s indifference curve, denoted as \( \nabla(x) \),
and the compensation contract, $R(y)$. Because of (8), the managers’ optimal choice is necessarily \{$R(x), y(x)$\}.

**[Figure 2]**

The incentive constraint (8) indicates that, of all the allocations \{$R(\hat{x}), y(\hat{x})$\} for $x \in [x, \overline{x}]$, the one most preferred by the manager who has observed $x$ is the allocation \{$R(x), y(x)$\}. The first-order necessary condition implied by (8) therefore is

$$
\frac{dV (R(\hat{x}), y(\hat{x}) | x)}{d\hat{x}} = V_R R' + V_y y' = 0
$$

(10)

at $\hat{x} = x$ where $V_i$ is the partial derivative with respect to the argument $i$, and the primes denote derivatives with respect to $x$.\(^4\) Totally differentiating $V$ with respect to $x$ yields

$$
\frac{dV}{dx} = V_R R' + V_y y' + V_x
$$

(11)

which, after substitution from (10), gives the result that

$$
\frac{dV}{dx} = V_x.
$$

(12)

Thus, the incentive constraint (8) can be recast as (12), which is a restriction on the utility of the manager conditional on his private information (or type), $x$.

The Hamiltonian expression associated with the problem of maximizing (7) subject to (12) may be written as

$$
H = \Pi f + \phi(x)V_x,
$$

(13)

where $y$ is the control variable, $V$ is the state variable, and $\phi(x)$ is the costate variable associated with the equation of motion, $V_x$.

**Proposition 1:** An efficient allocation \{$R(x), y(x)$\} for $x \in [x, \overline{x}]$ satisfies the following necessary conditions

(i) \[ \frac{g'(x - y)}{g''(x - y)} = \frac{1 - F(x)}{f(x)} \]

and

(ii) \[ R(x) = -\beta(x - y) + g(x - y) + \int_x^x \beta - g'(\alpha - y(\alpha))d\alpha + U. \]

\(^4\) We will address the second-order condition in the discussion below.
All proofs are in the Appendix. There are at this point two pieces of unfinished business that must be addressed to characterize a solution to the maximization problem (7). The first is that (10) is a first-order representation of the incentive constraint (8). To be assured that (10) characterizes a maximum, a second-order condition must also be satisfied. In this setting, the second-order condition is satisfied if the function \( y(x) \) satisfying part (i) of the proposition satisfies \( y'(x) > 0. \)

The second involves the participation constraint (9), an issue to which we now return. By part (ii) of Proposition 1, the utility of the \( x \)-type manager is \( U \), so that the participation constraint binds for him. As long as \( V_x \geq 0 \), the solution characterized by the proposition will result in the participation constraint being slack, and therefore satisfied, for \( x \geq x^* \). To guarantee this result, we make the following assumption.

**Assumption:** \(|g'| < \beta|.

Next, we present three examples that illustrate the nature of earnings management in the model. Examples 1 and 2 build intuition for Example 3, which combines and illustrates all the model tensions.

### 2.2 Example 1: Optimal contract without solvency constraint

To derive a closed-form solution for an optimal contract, we assume for the purposes of this section that \( x \) is distributed uniformly on the interval \([1, 1 + \beta] \), so that \( F(x) = (x - 1)/\beta \) and \( f(x) = 1/\beta \). We will also assume that the report manipulation costs are quadratic and of the form \( g(x - y) = (x - y)^2/2 \). Then, from part (i) of Proposition 1, we know that

\[
y(x) = 2x - \beta - 1 \tag{14}
\]

\(^5\) The profile \( y(x) \) is implementable only if

\[
\frac{\partial}{\partial x}\left(\frac{V_y}{V_R}\right) \cdot \frac{dy}{dx} \geq 0
\]

(Guesnerie and Laffont, 1984, Theorem 1). The first term is \( g'' \), which is assumed to be positive in our setting.

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and from part (ii) we derive the result that

\[ R(x) = \frac{\beta^2}{2} - 2x + x^2 + 1 + \bar{U}. \]  

(15)

Since \( y'(x) > 0 \), we are assured that the first-order condition for the incentive constraint (10) is sufficient for a maximum. Moreover, \( V_x = \beta - (x - y) \) which, upon substitution for \( y \), reduces to \( V_x = x - 1 \), is positive for every \( x > 1 \). Thus, the participation constraint (9) holds with equality at \( x = \bar{x} \), and is slack for higher values of \( x \). To see this, substitute for \( y \) and \( R \) to obtain

\[ V(R(x), y(x) \mid x) = \frac{x^2}{2} - x + \frac{1}{2} + \bar{U} \]  

(16)

which equals \( \bar{U} \) at \( x = 1 \).

Finally, to recover the efficient compensation contract, note that (14) implies that

\[ x = \frac{y + \beta + 1}{2} \]  

(17)

which, upon substitution into (15) yields

\[ R(y) = \frac{y^2}{4} + y \left( \frac{\beta - 1}{2} \right) + \frac{1 - \beta^2}{4} - \frac{\beta}{2} + \bar{U}. \]  

(18)

Correspondingly, the optimal reporting strategy for the manager then is

\[ y(x) = 2x - 1 - \beta. \]  

(19)

The efficient managerial compensation as a function of reported earnings is depicted in Figure 3.

[Figure 3]

Three features of this contract are objectionable. First, \( x \) is an invertible function of \( y \), so that firm value can be inferred from the manager’s report for all realizations of \( x \). This is at odds with the intuition that managers’ manipulation of accounting reports conceals some of managers’ private information from outsiders.
Second, this contract induces the manager to underreport value everywhere except when \( x = \bar{x} = 1 + \beta \). For instance, when \( x = \bar{x} \), the manager reports \( y(x) = 1 - \beta \) which is outside the support of \( X \). This distortion results in the resource cost \( g(x - y) \), which reduces the total surplus \( \Pi + V \) in the relationship between the manager and the shareholder. Because the manager chooses to understate value regardless of the realization of \( x \), there is never a situation in which the manager draws down the reserve he creates by understating value. This causes an inconsistency in the motivation for the form of the manager’s preference function. Recall that the benefit to the manager of adding to the reserve is \( \beta(x - y) \) where \( \beta \in (0, 1) \). The benefit of adding to reserves is that they are available to be drawn down in future periods and so can be used to increase the manager’s pay in a future period. If, however, there is no scenario in which reserves are drawn down, then it is inconsistent to assume \( \beta > 0 \) in the first place.

Third, there are some values of reported earnings, \( y \), for which the promised managerial compensation exceeds the reported value of the firm. Since the optimal contract \( R(y) \) has a slope less than \( \beta \) for every \( x < 1 + \beta \), a sufficient condition for \( R > y \) for some \( y \) is that this hold at the minimum value of \( x \). From (17) and (18), it is straightforward to demonstrate that \( R(1) - y(1) > 0 \) implies that \( \overline{U} > 1 - \beta + \beta^2/2 \). It does not conform well to intuition to suppose that the manager’s pay exceeds the reported value of the firm since shareholders generally enjoy limited liability. As it turns out, imposing the constraint that the manager’s pay cannot exceed reported value also addresses the first and second objections since it produces a non-invertible reporting function \( y(x) \) and can result in reserves being increased in some states and drawn down in others, as we illustrate next.

### 2.3 Optimal contract with a solvency constraint

In this section, we impose an additional constraint: the compensation paid to the manager can be no more than the reported value of the firm. This constraint, which we label the solvency constraint, may be expressed as

\[
y - R(y) \geq 0 \quad \text{for all } y. \tag{20}
\]
As we explain below, this additional contracting friction importantly changes the form of the optimal contract and the nature of reporting distortions. Formally, this constraint adds an additional term to the Hamiltonian, which becomes

$$ H = \Pi f + \phi(x) V_x + \mu(x) [y - R], $$

where $\mu(x)$ is a Lagrangean multiplier function. The following proposition characterizes the necessary conditions for a maximum.

**Proposition 2:** An efficient allocation $\{R(x), y(x)\}$ for $x \in [x, \bar{x}]$ in the presence of constraint (20) satisfies part (i) of Proposition 1 when $\mu(x) = 0$ and $R'(x) = y'(x) = 0$ when $\mu(x) > 0$.

Again, the principal seeks to minimize the sum of the socially-wasteful efforts of distortion, $g(x - y) = (x - y)^2/2$ and the information rent earned by the manager integrated over all types of managers. Although a linear contract with slope $\beta$ delivers the right incentives to the manager to reveal without distortion the condition of the firm, the optimal contract results in less surplus being paid to the manager yet induces every type of manager to work for the firm. A consequence of this optimal contract is a distortion of reported value away from raw earnings.

The import of Proposition 2 is that an efficient allocation entails “bunching” at the left end of the support of $x$, so that managers who have observed $x \in [x, \bar{x}]$ report $\tilde{y}$ and receive $\tilde{R}$, while those observing a higher value of $x$ select a report $y$ satisfying part (i) of Proposition 1.

We now turn to two examples that illustrate the form of the optimal contract and the optimal distortion of the report in each of two distinct cases. In Example 2, all types of managers distort their reports downwards from the realization of $x$; however, in Example 3, some types distort down and some types distort up. In both cases, each manager-type in a compact subset of the types makes the same report, so that the report imperfectly reveals the manager’s private information, which has implications for the price response to the report.

**Example 2:** Solvency constraint with reported value distorted down
For the purposes of this example, we will assume that $x$ is uniformly distributed on $[1, 1 + \beta]$, $g(x - y) = \frac{1}{2}(x - y)^2$, and $U = 1$. In the absence of the solvency constraint (20), the efficient compensation contract is given by (18).

For the participation constraint (9) to bind at $x = 1$, it must be that $V(R, y \mid 1) = 1$ when $R = y$. This implies that

$$y - \frac{(1 - y)^2}{2} + \beta(1 - y) = 1,$$

which is solved by $y = 1$. Accordingly, the bunching occurs at $\{R, y\} = \{1, 1\}$.

The value of $\tilde{x}$ is that for which part (i) of Proposition 1 would generate a report of $y(\tilde{x}) = 1$. Since in this example $y(x) = 2x - \beta - 1$ when $\mu = 0$, it follows that $\tilde{x} = 1 + \beta/2$. Accordingly,

$$y(x) = \begin{cases} 
1 & \text{if } x \in [1, 1 + \beta/2], \\
2x - \beta - 1 & \text{if } x \in (1 + \beta/2, 1 + \beta].
\end{cases}$$

This situation is depicted in Figure 4. All managers who observe $x \in [1, 1 + \beta/2]$ report $1 + \beta/2$, while all managers who observe $x \in [1 + \beta/2, 1 + \beta]$ report $2x - 1 - \beta$. In the latter case, the report can be inverted to learn firm value, $x$. In this region, note that a small increase $\delta$ in $y$ in this region implies a smaller increase, $1/2\delta$, in firm value. In the former case, firm value is the expected value of the firm conditional on knowing $x \in [1, 1 + \beta/2]$, which is $1 + \beta/4$. An increase $\delta$ in the report, $y$, implies a sharp jump in firm value.

[Figure 4]

Now we turn to a characterization of $R(x)$. For the manager with private information $\tilde{x}$, the payoff at $\{R = 1, y = 1\}$ is

$$V(1, 1 \mid \tilde{x}) = 1 - \frac{(\tilde{x} - 1)^2}{2} + \beta(\tilde{x} - 1)$$

which, after substituting for $\tilde{x}$, reduces to $1 + 3\beta^2/8$.

To maintain the incentive constraint, the payoff to a manager with private information above $\tilde{x}$ must satisfy

$$R = -\beta (x - y) + \frac{(x - y)^2}{2} - \int_{\tilde{x}}^x V_x(t) dt + \left[1 + \frac{3\beta^2}{8}\right],$$
where the term in brackets is the utility level required for the \( \tilde{x} \)-type manager. Substituting for \( y \) and \( V_x \), and integrating yields

\[
R(x) = x^2 - 2x + 2 - \frac{\beta^2}{4}.
\]

Substituting for \( y \) gives the optimal compensation contract,

\[
R(y) = \frac{y^2}{4} + y \left[ \frac{\beta - 1}{2} \right] - \frac{\beta}{2} + \frac{5}{4},
\]

which is a parallel shift upward by the amount \( \beta^2 / 4 \) of the optimal contract (18) in the absence of the solvency constraint (20). These contracts are depicted as \( \tilde{R}(y) \) and \( R(y) \), respectively, in Figure 5.

[Figure 5]

In Figure 5, the indifference curve of the manager with the lowest valuation, \( V_{\tilde{x}} \), is tangent to \( R(y) \) at \( y = 1 - \beta \). Higher-type managers make higher reports. The indifference curve of an \( x \)-type manager is tangent to \( R(y) \) at \( y(x) \) defined by (19).

Since the payment to the lowest-type manager, \( R(1 - \beta) = 1 - \beta^2 \) exceeds this manager’s reported value, \( 1 - \beta \), the solvency constraint, represented by the 45° line, is violated. The optimal contract given the solvency constraint is \( \tilde{R}(y) \), which is tangent to \( V_{\tilde{x}} \) at \( y = 1 \). Contract \( \tilde{R}(y) \) induces the \( \tilde{x} \)-type manager to report \( y = 1 \). A knot of higher-type managers also report \( y = 1 \). The indifference curve of an \( x \)-type manager is tangent to \( R(y) \) at \( y(x) \) defined by (23). Given \( U = 1 \), (23) implies there is a knot of manager types who report \( y = 1 = \tilde{x} \), but there are no managers who distort their report upward, so an objection raised earlier remains unaddressed; however, it is possible to parameterize this problem so that the manager’s optimal strategy involves overstating value for some realizations and understating value for other realizations, as we explain next.

Example 3: Solvency constraint with reported value distorted up and down

[Figure 6]
In Example 3, as in Example 2, firm value cannot be inferred perfectly from reported value because the manager optimally chooses to report a single value for a mass of firm values. Through this example, we illustrate how raising the manager’s reservation wage leads to a situation in which the reports of the lowest-type managers are distorted upward, and the reports of all other types of managers are distorted downward. Specifically, as depicted in Figure 6, all managers observing \( x \in [1, \bar{x}) \) report \( \tilde{y} \). If the manager observes firm value \( x \in [1, \tilde{y}] \), the manager’s report overstates firm value. If the manager observes firm value \( x \in (\tilde{y}, \bar{x}) \), the manager’s report understates firm value. Thus, in the second-best contract, there is hole in the distribution of reported earnings: the lowest earnings reported \( \hat{y} \) is greater than the lowest value of raw earnings, \( x \). It is still true that the manager with the highest earnings, \( \bar{x} \), reports earnings truthfully, \( y(\bar{x}) = \bar{x} \). At this point, the compensation contract has slope \( \beta \), like the optimal contract for the case without the a solvency constraint. For \( x < \bar{x} \), the compensation contract has slope less than \( \beta \) and becomes flatter for lower values of \( x \). The overall shape of the contract is convex.

Example 3 differs from Example 2 in that the manager’s reservation wage is higher than 1, \( \bar{U} > 1 \). Table 1 presents the critical values \( \bar{x} \) and \( \tilde{y} \) for various values of \( \bar{U} \) and \( \beta \).

<table>
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3. S-shaped price response to reports

Studies of financial disclosure in capital markets identify two strong empirical regularities: a divot in the distribution of firms’ reported earnings just below a variety of benchmarks and an S-shaped relationship between the unexpected component of earnings and the contemporaneous price response. The divot, first described by Hayn (1995), has been documented extensively in a stream of papers including Burgstahler and Dichev (1997) and Degeorge, Patel and Zeckhauser (1999). It is plain in histograms of earnings and highly significant in statistical tests premised on the notion that the distribution of unmanaged or raw earnings ought to change smoothly over small intervals. For instance, the tests indicate unusually low frequencies of small decreases in earnings.
and small losses and unusually high frequencies of small increases in earnings and small positive income and strongly suggest that earnings are managed or manipulated by the firm’s managers—Schipper (1989, 92) defines earnings management as “purposeful intervention in the external financial reporting process, with the intent of obtaining some private gain (as opposed to, say, merely facilitating the neutral operation of the process).”

Naturally, the question of how stock price changes when earnings are announced is complicated when the report is not the result of the neutral operation of the financial reporting process. A stock’s price after financial results are reported ought to reflect market participants’ beliefs about firm value conditioned on whatever information can be gleaned from potentially distorted earnings reports. The stock return contemporaneous with earnings announcements is non-linear. Freeman and Tse (1992), Das and Lev (1994), and Skinner and Sloan (2001) document that stock price reactions to earnings announcements, measured relative to earnings forecasts, exhibit an S-shaped pattern: stock return is a monotone increasing function of the forecast error, but stock returns increase most sharply when earnings are in the neighborhood of the forecast. Stock returns increase less sharply when earnings are either far above or far below the forecast so that stock returns are a convex function of earnings below the forecast and a concave function of earnings above the forecast.\(^6\)

Stock price is the expected value of the firm conditional on whatever report is available. Interpreting the report as accounting information implies that it might be decomposed into elements of value (in particular, book value and earnings). For simplicity, we require that these elements have the same price multiple. Liu and Thomas (2000), who suppress earnings management considerations, make the observation that the price multiple on unexpected earnings should be one, but whether this result obtains in empirical tests depends critically on the adequacy with which one is able to measure changes in expected future earnings that occur simultaneously with the announcement of current period earnings. Our analysis can be related to their Ohlson-esque set-up by considering the private information of the manager in our model, \(x\), to be information about the

\(^6\) Skinner and Sloan (2001, Figure 4) plot quarterly abnormal stock returns as a function of the quarterly earnings forecast error. A striking S-shaped pattern emerges.
current period change in firm value, which is entirely transitory. We thus side-step issues related to how innovations in current period earnings are related to expectations of future earnings.

Returning to Example 2 and figure 4, it is apparent that reported values of $y > 1$ can be inverted by investors who therefore learn firm value exactly from such reports. A report of $y \in (1, 1 + \beta]$ leads to an inference of firm value of $1 + (\beta + y)/2$. A report of $y = 1$ is made a manager whose private information is that firm value is anywhere between 1 and $1 + \beta/2$. Investors rationally update their beliefs of firm value conditional on observing $y = 1$ to the mean of the types who make this report, namely $1 + \beta/4$. Labeling $y = 1$ as an earnings benchmark, this pattern of posterior prices reveals that price changes abruptly as $y$ increases above 1 and more slowly afterwards. This analytical result approximates the right half of the observed S-shaped response of stock price revisions to announced earnings measured relative to earnings forecasts.

4. Extensions

There are several directions in which this research could be extended:

- We could develop comparative statics on the impact of increasing or decreasing the cost of earnings management.

- An issue (briefly mentioned above) is whether the costs of distortions are borne entirely by the manager (and are represented by $g(x - y)$), or whether there are some additional costs related to capital market inefficiency that are different from the manager’s cost of distortion and that are borne by persons other than the manager—this brings up the issue of why the shareholder wants the manager to report earnings at all and what the costs are of distorted earnings.

- We could add a new random variable $\theta$ to the analysis which is unknown to the market at large when stock price is set (and so $\theta$ affects returns), but which is known to the manager and shareholder when contracts are written so that it does not constitute an additional piece of private information that would foul up the contracting between the manager and shareholder. The goal is to turn the knot into a divot.
5. Discussion and conclusions

Why is it rational for managers to distort earnings? One explanation is that compensation contracts have caps and floors or other kinks, so that the marginal value to a manager of reporting an extra dollar of earnings to the left of the kink differs from the marginal value to the left of the kink. For instance, suppose the manager’s annual bonus is 10% of earnings, but is capped at $1 million. If raw earnings in the current period are above $1 million, the value of the last $1 of earnings to the manager in the current period is zero, but if earnings in the next period are below $1 million, then the value of an incremental dollar of earnings next period is 10¢. This suggests that the manager would benefit from shifting earnings from the current period to the next period. As a result, a cap could create a divot in the distribution of earnings since managers benefit from manipulating earnings down when raw earnings are above the cap and manipulate earnings up when raw earnings are below the cap. Healy (1985) explores the distribution of earnings around caps in compensation contracts containing such a provision.

While kinks may indeed induce divots, this line of research begs the questions of when and why kinked compensation contracts are optimal and the empirical issue of whether kinks are prevalent enough to drive the divot. On the latter point, Gaver, Gaver, and Austin (1993) note that in a sample of 1,588 proxy statements for 1986, only 7.9% of firms operate a bonus plan and explicitly state the bonus formula. Of these firms, only 31.4% impose a cap. Note also that stock-based compensation, including restricted stock and stock options, is quite common and obviously has no cap. Another difficulty that arises in trying to explain the divot by reference to explicit caps on managers’ compensation is that caps, where they exist, are unlikely to coincide with the earnings benchmarks (i.e., zero earnings, the prior year’s earnings, and analysts’ consensus forecast of earnings) around which divots have been observed. This suggests a need for an alternative theory that rationalizes both the divot and the S-shaped response of price to announced earnings that does not rely on the explanation that compensation contracts are kinked for exogenous reasons.

The prevalence of caps is hard to assess in part because some terms of compensation arrangements are either non-public or implicit.
This paper presents such a theory. In our analysis, it is notable that the optimal (and hence endogenously determined) compensation contract that produces these outcomes does not contain a cap (or ceiling) of the type studied by Healy (1985). This is important because there are reasons to believe that the caps on executive compensation are infrequent, which argues against caps driving divots. Instead, the managers’ pay is a smooth, increasing, and convex function of reported earnings.

A by-product of a contract designed with a sole focus on the employment relationship between the manager and shareholder is that inferences about the value of the firm conditioned on publicly-available information (and hence the market-determined price of the firm’s stock) are affected by the reporting distortions. In particular, since there is a mass of value reports at a threshold, it is not possible to infer firm value perfectly from reported value. Within the model we explore, it is feasible for the shareholder to write a contract that reduces or eliminates the the mass of types making the same report. This is not optimal because the shareholder would derive no net benefit from a more revealing accounting report. If the shareholder (or some other economic actor) valued more informative reports per se, the contract could be revised to induce more informative reporting from the manager—of course at a cost to the shareholder. Thus, there is a tradeoff between providing full information about firm value to the capital market because it require a less efficient compensation contract that lowers firm value. Future research might address an augmented model where the accounting report had both an explicit valuation purpose and a contracting purpose.
Appendix A: Proofs

Proof of Proposition 1: We may rewrite (1) as \( R = V + g - \beta(x - y) \) which, upon substitution into (13), yields

\[
H = [x - \beta(x - y) - [V + g - \beta(x - y)]]f + \phi[\beta - g']
\]

since \( V_x = \beta - g' \). The Pontryagin (necessary) conditions for a maximum are \( \frac{d\phi}{dx} = -\frac{\partial H}{\partial V} \) and \( H_y = 0 \), which may be written as

\[
\frac{d\phi}{dx} = f \tag{A.1}
\]

and

\[
fg' + \phi g'' = 0, \tag{A.2}
\]

respectively. Integrating both sides of (A.1) and noting the transversality condition \( \phi(x) = 0 \), implies that \( \phi(x) = F - 1 \). Substitution of this result into (A.2) yields part (i) of the proposition, which implicitly characterizes the function \( y(x) \).

We may use knowledge of the function \( y(x) \) and of the equation of motion \( V_x \) to recover the transfer function \( R(x) \). First, note that the total surplus, \( \Pi + V = x - g \), is divided between the shareholders and the managers. As a result, we may write

\[
x - R - \beta(x - y) = x - g - \left[ \int_{\underline{x}}^{x} V_x(\alpha) d\alpha + U \right], \tag{A.3}
\]

where the term on the left hand side is shareholder utility, and the term in brackets on the right is the utility of the manager. Noting that \( V_x = \beta - g' \) and solving for \( R \) yields part (ii) of the proposition. \( \square \)

Proof of Proposition 2: The first part of the theorem is a straightforward extension of the approach used in the proof of Proposition 1. With respect to the second part, note that, when \( \mu > 0 \), then \( R = y \) and the incentive compatibility condition (10) may be written as

\[
(V_r + V_y)y' = 0,
\]
so that either $y' = 0$ or $V_r + V_y = 0$. We will demonstrate that $y' \neq 0$ leads to a contradiction.

Assume that $y' \neq 0$ for the value of $x$ where $\mu(x) > 0$. Then, after taking the appropriate derivatives, (20) implies

$$g'(x - y) = \beta - 1$$

from which it follows that value is overstated by a constant amount. Since the manager could always earn positive profit of $x$ by reporting value truthfully, it follows that the profit is higher than $x$. Thus, $V(x) > 0$ whenever $\mu(x) > 0$.

Also, we know that $V(x) > 0$ for all $x$ where $\mu(x) = 0$, since $V_x > 0$ and the incentive constraint (8) necessarily applies. Thus, $V(x) > 0$ for every $x$, and a lump sum reduction in $R$ would increase the expected payoff (7) to the manager without violating either the incentive (8) or the participation (9) constraints. Thus, a contract with $y' \neq 0$ when $\mu > 0$ cannot be a maximizing solution, and we must have $y' = 0$ when $\mu > 0$. □
References


Manager privately observes firm value, $x$. Manager and shareholder commit to a compensation agreement, $R$, that is a function of reported value, $y$. Manager incurs manipulation cost $g(y - x)$, reports value $y$, and receives compensation $R(y)$. Manager reaps a future benefit (or incurs an opportunity cost) from the change in the hidden reserve of $\beta(x - y)$.

Figure 1. Timeline.
Figure 2. When presented with the compensation contract $R(y)$, the manager selects the earnings report, $y$ that maximizes his expected utility. This choice occurs at the tangency of a manager’s indifference curve with private information $x$, denoted as $\overline{V}(x)$, and the compensation contract $R(y)$. 

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*Figure 2.* When presented with the compensation contract $R(y)$, the manager selects the earnings report, $y$ that maximizes his expected utility. This choice occurs at the tangency of a manager’s indifference curve with private information $x$, denoted as $\overline{V}(x)$, and the compensation contract $R(y)$. 

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Figure 3. This figure presents the optimal contract $R$ as a function of the manager’s report $y$ in the case where $zz$. 
Figure 4. Corresponding to Example 2, this figure presents the relationship between the manager’s private information about firm value, $x$, and the public report he makes of firm value, $y$. 
Figure 5. This figure presents the optimal contracts $R$ and $\tilde{R}$, corresponding to the Example 1 (without the solvency constraint) and Example 2 (with and without a solvency constraint), respectively, as a function of the manager’s report, $y$ in the case where $U = 1$. 
Figure 6. Corresponding to Example 3, this figure presents the relationship between the manager’s private information about firm value, $x$, and the public report he makes of firm value, $y$. 