Market liquidity and volume around earnings announcements*

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This paper suggests that earnings announcements provide information that allows certain traders to make judgments about a firm's performance that are superior to the judgments of other traders. As a result, there may be more information asymmetry at the time of an announcement than in nonannouncement periods. More information asymmetry implies that bid-ask spreads increase, suggesting that market liquidity decreases at the time of an earnings announcement. Furthermore, informed opinions resulting from public disclosure may lead to an increase in trading volume, despite the reduction in liquidity that accompanies announcements.

1. Introduction

This paper suggests a model of trade in which financial accounting disclosures simultaneously induce increased information asymmetry, less liquidity, and more trading volume. In particular, in our analysis we are concerned with disclosures that have two salient characteristics. First, they disseminate data for which there may be no alternative sources of private information. Second, they

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See Kim and Verrecchia (1991b) for a discussion of announcements that are anticipated through private information gathering in advance of the announcements. This approach assumes that there exist alternative sources of private information about a public announcement. Liquidity is not an issue in Kim and Verrecchia (1991b) because markets are assumed to be perfectly competitive.
provide information that may lead to different interpretations of a firm's performance. This characterization is sufficiently broad to include earnings announcements, management and analysts' forecasts, 10-K filings, and other summaries of detailed financial accounting statistics. For convenience, however, we imagine these disclosures to be specifically earnings announcements throughout the remainder of the paper.

In our model some market participants process earnings announcements into private, and possibly diverse, information about a firm's performance at some cost (e.g., time and effort). This private information can be thought of as informed judgments or opinions. Market participants who provide informed judgments are those traders willing to bear the cost for engaging in this activity. Institutionally, traders capable of informed judgments from public sources can be thought of as market experts who follow a firm closely (e.g., large shareholders, financial analysts, managers at competing firms). Through their activities, price impounds opinions of a firm's performance. In the absence of announcements there are no opportunities for traders capable of informed judgments to exploit their ability to process public information. This lessens the possibility of information asymmetries arising. Alternatively, earnings announcements stimulate informed judgments. These informed judgments, in turn, create or exacerbate information asymmetries between traders and market makers. This implies that the market becomes less liquid as a direct consequence of more disclosure.

One feature of our model is that less liquidity does not, in and of itself, imply less trading activity around public announcements. While discretionary liquidity traders will avoid these periods, traders who process public information face a more subtle trade-off. They choose between being relatively well-informed and trading in relatively illiquid markets versus being relatively uninformed and trading in liquid markets. Investors capable of processing publicly available information prefer the former because the price quotations of market makers in illiquid markets only partially offset the advantage of being well-informed. Thus, information processors trade at the time of an announcement. As we show formally below, expected trading volume can be higher at the time of an earnings announcement than at nonannouncement dates. This follows if there is sufficiently little public information available at the time of an announcement or the cost of processing information is small. In effect, volume is driven up by informed trading at the time of an earnings announcement. These results imply that trading volume and market liquidity may be negatively associated.3

2 Other papers suggesting diverse interpretations of a firm's performance from a common source of public information include Pfleiderer (1984), Holthausen and Verrecchia (1990), Indjejikian (1991), and Harris and Raviv (1991).

3 This is contrary to the intuition offered in Admati and Pfleiderer (1988), for example. They suggest that the impetus of discretionary liquidity traders to trade in periods of greatest liquidity
In addition to showing that the market becomes less liquid at the time of an earnings announcement, with the possibility of more volume, we discuss the behavior of prices around an earnings announcement. An earnings announcement introduces variance into price changes and information into price, consistent with the intuition that disclosure increases the flow of information into the market.\footnote{Kim and R.E. Verrecchia, Liquidity and volume around earnings announcements}

Public disclosure has received little explicit attention in theoretical models whose major focus is understanding market liquidity.\footnote{Public disclosure has received little explicit attention in theoretical models whose major focus is understanding market liquidity. One difficulty with analyzing this issue is that there are two ways to characterize public disclosure, and these characterizations have different empirical implications. In addition to the one suggested in this paper, an alternative is to consider an economy in which there exist informed traders who are endowed with superior knowledge of the underlying performance of a security. An interpretation of this approach is that there exist shareholders affiliated with a firm who have superior information about the firm's performance based on their affiliation (e.g., managers, creditors, large shareholders). The presence of informed traders requires that prices quoted by market makers to buy or deliver shares of a firm's securities be conditioned over the quantity of shares requested. In effect, price quotations address the adverse selection problem faced by a market maker by taking into consideration the (potentially superior) information implied by a trader's request.}

In this alternative characterization, public disclosure ameliorates the adverse selection problem by partially, or fully, revealing to market makers information known by informed traders. Consequently, disclosure makes transaction prices quoted by market makers less sensitive to buy and sell orders. This implies that the market becomes more liquid at the time of a public announcement.\footnote{In this alternative characterization, public disclosure ameliorates the adverse selection problem by partially, or fully, revealing to market makers information known by informed traders. Consequently, disclosure makes transaction prices quoted by market makers less sensitive to buy and sell orders. This implies that the market becomes more liquid at the time of a public announcement.}

This characterization has the following empirical implication. Bid–ask spreads are a measure of the degree of liquidity for firms' securities. Assuming that public announcements reduce information asymmetry, bid–ask spreads should be wider for an extended period leading up to a public announcement.\footnote{This characterization has the following empirical implication. Bid–ask spreads are a measure of the degree of liquidity for firms' securities. Assuming that public announcements reduce information asymmetry, bid–ask spreads should be wider for an extended period leading up to a public announcement. But Admati and Pfleiderer (1988) do not consider public disclosure. Public disclosure changes investors' (private) incentives to become informed and trade at announcement versus nonannouncement dates. Though price quotations offer only partial protection against the activities of traders with superior information, this is typically sufficient to sustain market making. This is because losses to informed traders offset gains from servicing investors trading for noninformation reasons.}

\footnote{Similar results are offered in models of pure competition in papers by Holthausen and Verrecchia (1988) and Kim and Verrecchia (1991a,b).}

\footnote{Examples of papers of this genre include Kyle (1985), Glosten and Milgrom (1985), Diamond and Verrecchia (1987), Easley and O'Hara (1987), and Admati and Pfleiderer (1988). Papers dealing explicitly with public disclosure and liquidity include Diamond and Verrecchia (1991) and McNichols and Trueman (1994).}

\footnote{Although price quotations offer only partial protection against the activities of traders with superior information, this is typically sufficient to sustain market making. This is because losses to informed traders offset gains from servicing investors trading for noninformation reasons.}

\footnote{See, for example, the discussion in Diamond and Verrecchia (1991).}
(perhaps weeks or months) than they are as soon as the announcement takes place. The intuition here is that market makers set a large bid-ask spread over periods of greatest information asymmetry, and then lower the spread when an announcement occurs that reduces information asymmetry.

While this alternative characterization appears valid for interpreting certain types of disclosures (such as, perhaps, announcements that reveal 'inside information') and clearly describes one effect of more disclosure, it may fail to capture a second effect of a public announcement. Earnings announcements stimulate informed judgments among traders who process public disclosure into private information. Traders may possess no alternative source of private information. The ability of information processors to produce superior assessments of a firm's performance on the basis of an earnings announcement provides them with a comparative information advantage over market makers. For example, specialists are not thought to do any fundamental analysis, such as analyzing in great detail the annual reports of the companies whose shares they trade. They are in the business to maximize the turnover of their capital, and not the value of their inventory. Therefore, earnings announcements prompt market makers to increase the bid-ask spread during the brief window (perhaps one or two days) surrounding their release. This protects market makers against the temporary information advantage held by processors of public information. When information processors are significantly active, more trading volume may also result despite the reduction in liquidity.

While the empirical evidence concerning this issue is not unequivocal, there is at least a suggestion that bid-ask spreads increase in response to earnings announcements. For example, Morse and Ushman (1983) and Skinner (1991), using samples of Over-The-Counter (OTC) securities, find no clear evidence that bid-ask spreads change around earnings announcements. However, Skinner (1991) does note that spreads increase immediately after announcements that convey relatively large earnings surprises. Examining a sample of New York Stock Exchange (NYSE) firms, Venkatesh and Chiang (1986) document an increase in spreads for irregular patterns of earnings and dividend announcements, but not otherwise. Patel (1991) offers evidence that spreads increase after earnings announcements, implying an increase in information asymmetry after these disclosures. Lee et al. (1991), using NYSE specialist quotes, find a significant increase in spreads surrounding earnings announcements. In the context of

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8 This notion is supported by empirical work in Lys and Sohn (1990). They find evidence indicating that analyst forecasts are informative even when they are preceded by corporate accounting disclosures. This suggests that analysts process public announcements into private informed judgments.

9 See, for example, the discussion on p. 211 of Mayer (1988). He states: 'In general, NYSE (New York Stock Exchange) specialists do not take a view of where a stock is going over time... Some of them do not read the annual reports of companies whose shares they trade.'
our model, one interpretation of their results is that market makers increase spreads \textit{in anticipation} of an earnings announcement, as opposed to waiting until the announcement actually occurs. This guards against 'leaks' and the possibility that some traders have the opportunity to process earnings announcements before they are generally made public. Increasing spreads in anticipation of an announcement is facilitated by the fact that earnings announcement dates are highly predictable. Lee et al. (1991) also show that the announcement effect in bid–ask spreads rapidly dissipates. This last observation suggests that earnings announcements provide information processors with only a temporary advantage over market makers.

In section 2 of the paper we provide a detailed description of our model. In section 3 we characterize an equilibrium to a market comprised of a market maker, potential information processors, and discretionary and nondiscretionary traders. The incentives of (potential) traders to process information at some cost is analyzed in section 4. This leads to a characterization of the equilibrium number of information processors at the time of an earnings announcement. Finally, liquidity and volume at the time of an earnings announcement are discussed in section 5. Our results here center around the fact that in comparing earnings announcement to nonearnings announcement dates, expected volume can be higher despite the fact that the market is less liquid. We conclude the paper in section 6 with summary remarks.

2. The model

We use a variation of the models of Kyle (1985) and Admati and Pfleiderer (1988) with \( T \) periods, \( s = 1, 2, \ldots, T \). There are four types of risk-neutral agents: a market maker, potential information processors, \( L \) nondiscretionary liquidity traders, and \( T \cdot M \) discretionary liquidity traders. There is one risky asset, which we call the firm, and riskless bonds. One bond pays off one unit of consumption good in period \( T \). The firm generates cash flow of \( u_s \) in period \( s \). Each \( u_s \) is an independent, normally distributed random variable with mean 0 and variance \( a_s \).\(^{10}\) The liquidating value of the firm, denoted by \( \bar{U} \), is a random variable defined by

\[
\bar{U} = \sum_{s=1}^{T} \bar{u}_s.
\]

At the end of period \( s \), the realization of \( \bar{u}_s \) becomes common knowledge. Institutionally, this arrangement can be thought of as one in which the firm

\(^{10}\)It is assumed without loss of generality that all random variables have zero means.
earns revenue by completing a series of $T$ independent contracts (e.g., the firm builds customized homes under contract, one at a time). As each contract is completed, the cash flow generated by that contract becomes known (by the firm reporting this information or otherwise). In this context, a period is not a fixed length of time or a cycle over which a firm must report, but rather the length of time it takes to complete a contract. Before the $t$th contract is completed, at time $t$, the firm publicly discloses a signal, $Y$, that contains information about the firm’s (anticipated) cash flow for period $t$. The signal is of the form of

$$Y = u_t + \varepsilon,$$

where the variance of $u_t$ is $a_t$, and $\varepsilon$ is normally distributed with mean 0 and variance $d$. For notational convenience, define $\sigma = a_t$. Institutionally, $Y$ can be thought of as an earnings announcement that imperfectly communicates, or forecasts, a cash flow whose realization ultimately becomes known.

In anticipation of $Y$ at time $t$, those with the ability to process announcements decide whether or not to do so at a fixed cost $C$ and become information processors. We denote this endogenously determined number of information processors at date $t$ by $N_t$. Simultaneous with the dissemination of $Y$, an information processor $i$ observes (at a cost $C$)

$$\tilde{Y}_i = \tilde{\delta} + \tilde{\varepsilon}_i,$$

where $\tilde{\varepsilon}_i$ is normally distributed with mean 0 and variance $e$ for all $i$. Note that $\tilde{\delta}_i$ alone is not an informative signal about the firm’s liquidating value $U$, since both $\tilde{\delta}$ and $\tilde{\varepsilon}_i$ are independent of $U$. Combined with the announced signal $Y = u_t + \varepsilon$, however, $\tilde{\delta}_i$ generates a signal $\tilde{Y} - \tilde{\delta}_i = u_t - \varepsilon$ and this provides information about the firm’s performance.

Institutionally, $\tilde{\delta}_i$ can be thought of as the information a trader gleaned about the random error in financial reports by studying the firm. For example, in the case of an earnings announcement characterized by $Y = u_t + \varepsilon$, the random error $\tilde{\delta}$ represents the discrepancy between cash flow in period $t$ ($u_t$) and the forecast of that cash flow implicit in current accounting profits ($\tilde{\delta}$). This discrepancy arises, perhaps, from the failure of accounting profits to recognize unrealized gains, use consistent accounting procedures, or avoid questionable levels of capitalization. By studying data issued by the firm (at some cost $C$), traders who process public disclosures are better prepared to translate current accounting profit figures into superior assessments of the firm’s cash flow realization. For convenience we assume that the market maker observes only $\tilde{Y}$, consistent with the notion that specialists do no fundamental analysis. One

\[11\] Variables without time subscripts are those associated with the disclosure at $t$ unless otherwise noted.
could have assumed the market maker, along with information processors, observes \( \delta \) with idiosyncratic error. However, provided that the market maker's information is not a sufficient statistic for interpreting \( \tilde{u}_t \) conditional on the observations of all information processors, the disclosure of \( \tilde{Y} \) creates information asymmetry between the market maker and other market participants. This is all that is required for the subsequent results of the paper to hold.\(^{12}\)

The firm makes an earnings announcement only at date \( t_1 \). Therefore, we refer to \( T \) as the 'earnings announcement' date to distinguish it from all other time periods or reference points in time. These other time periods (or reference points) are referred to as 'nonearnings announcement' or 'nonannouncement' dates. Note that we allow for the possibility that a firm announces cash flow when a contract is completed (as well as it being revealed in some other fashion). However, in our model the distinction between an earnings announcement and other-type disclosures is that the former can be processed into a superior assessment of the firm's performance, whereas the latter cannot. Consistent with this idea, we assume that the revelation of a cash flow, \( \tilde{u}_t \), however it occurs, cannot be processed into a superior assessment of the firm's performance. The intuition here is that valuation of the firm's performance based on knowledge of cash flows cannot be improved upon through information processing. This distinction between an earnings announcement and other-type disclosures is sufficient to motivate the results of this paper.

We assume that the covariance between the error terms \( \tilde{\epsilon}_i \) and \( \tilde{\epsilon}_{i'} \), for any two information processors \( i \) and \( i' \), is \( \rho \epsilon \) where \( \rho \) is the correlation coefficient and is assumed to lie between (and including) 0 and 1. When \( \rho = 0 \), the error terms among different processors are independent of one another. When \( \rho = 1 \), error terms are identical and all information processors observe the same information. Therefore, diversity of opinion among information processors is measured by \( 1 - \rho \). In the analysis below, we treat \( \rho \) as an exogenous variable that can vary between 0 and 1. The important point is that our results below do not depend on whether information processors have homogeneous beliefs (\( \rho = 1 \)), fully heterogeneous beliefs (\( \rho = 0 \)), or some combination of common and idiosyncratic beliefs (\( 1 > \rho > 0 \)).

Note that to keep the analysis facile, we have assumed that information processors have homogeneous priors, whereas posteriors can be either homogenous or heterogeneous, depending upon \( \rho \). However, in principle it makes no difference whether the beliefs of information processors diverge or

\(^{12}\text{An alternative way to model information processing would be to assume that an earnings announcement provides each information processor with a signal } \tilde{u}_t + \delta + \epsilon_i \text{ [see, e.g., the discussion in Holthausen and Verrecchia (1990)]. In this interpretation the market maker would be required to be provided with a signal inferior (in a statistical sense) to the signal available to information processors.}\)

\(^{13}\text{Assuming multiple earnings announcements does not alter the basic analysis or intuition.}\)
converge on the basis of an earnings announcement. For example, this analysis could be done with a single information processor, thereby making this issue irrelevant. Instead, what is critical is whether an earnings announcement creates information asymmetries between the market maker and information processors. Information asymmetry affects liquidity, which, in turn, influences other aspects of the analysis (such as volume), and liquidity is the focus of our discussion.

While information processors trade in order to take advantage of their private information, liquidity traders trade for liquidity reasons. We assume that \( L \) non-discretionary liquidity traders trade at all dates, whereas each of the \( T \cdot M \) discretionary liquidity traders can choose to trade at one date. It is assumed that each liquidity trader's net demand when he trades is a normally distributed random variable with mean 0 and variance 1. All random variables in this model are mutually independent unless assumed otherwise.

The market operates as follows. At dates \( s = 1, \ldots, t - 1, \tau, t, \ldots, T \), information processors and liquidity traders submit their market orders for the risky asset to the market maker. Denote the \( i \)th processor's net demand by \( \tilde{x}_{is} \), and the aggregate processors', nondiscretionary, and discretionary liquidity demand by \( \tilde{z}_{ns}, \tilde{z}_{ns}, \) and \( \tilde{z}_{ds} \), respectively. The market maker cannot distinguish among orders from different types of agents, but makes use of the total demand order,

\[
\tilde{\omega}_{s} = \tilde{x}_{s} + \tilde{z}_{ns} + \tilde{z}_{ds},
\]

to infer the existing private information to a certain degree. She then sets the price so that her expected profit is 0 at each date.\(^{14}\)

3. Market equilibrium

In this section we derive a market equilibrium with an exogenously given number of information processors, \( N \), and a given variance of liquidity trading, \( \sigma \). The decisions of potential processors and discretionary liquidity traders are analyzed in the next section. We seek an equilibrium in which the market participants make conjectures about the actions of others and the conjectures turn out to be correct.

We begin with the analysis at the time of an earnings announcement. At date \( \tau \), information processors choose their net demand orders, \( \tilde{x}_{i} \) 's, and the market maker offers a price, \( \tilde{P}_{\tau} \), based on their respective information. Let the linear conjectures about their action–information relations be written as

\[
\tilde{x}_{i} = \beta \tilde{Y} + \gamma \tilde{O}_{i}, \quad \tilde{P}_{\tau} = \tilde{U}_{\tau-1} + \alpha \tilde{Y} + \lambda \tilde{\omega}.
\] \(^{(1)}\)

\(^{14}\)This zero-profit assumption is commonly used to reflect the fact that market making is competitive and closely regulated.
where $U_{t-1} = \sum_{k=1}^{t-1} u_k$. Given eq. (1) we analyze the profit-maximizing choice of each type. Let $\Pi_i^x$ denote the expected profit of the $i$th processor from trades conditional on his available information.

The $i$th information processor's problem is to

$$\max_{\tilde{x}_i} \Pi_i^x = \mathbb{E}[\tilde{x}_i(\tilde{U} - \tilde{P}_i)|\tilde{U}_{t-1}, \tilde{Y}, \tilde{O}_i]$$

$$= \mathbb{E}[\tilde{x}_i(\tilde{U} - \tilde{U}_{t-1} - \alpha \tilde{Y} - \lambda \tilde{w})|\tilde{U}_{t-1}, \tilde{Y}, \tilde{O}_i]$$

$$= \mathbb{E}[\tilde{x}_i(\tilde{U} - \alpha \tilde{Y} - \lambda (\tilde{x} + \tilde{z}_n + \tilde{z}_d)|\tilde{Y}, \tilde{O}_i]$$

$$= \mathbb{E} \left[ \tilde{x}_i \left\{ \tilde{u}_i - \alpha \tilde{Y} - \lambda \tilde{x}_i - (N - 1)\lambda \beta \tilde{Y} - \lambda \sum_{k \neq i} \tilde{O}_k \right\} |\tilde{Y}, \tilde{O}_i \right]$$

$$= \tilde{x}_i \left[ \frac{a(d + e) \tilde{Y} - ad \tilde{O}_i}{ad + ae + de} - (N - 1)\lambda \beta \tilde{Y} - \lambda \tilde{x}_i \right.$$ \left.
\right. \left. \right.$$ \left. \right.$$- (N - 1)\lambda \tilde{y} - \frac{(1 - \rho)de \tilde{Y} + \{ad + pe(a + d)\}\tilde{O}_i}{ad + ae + de} \right].$$

Note that the net demand of information processors is independent of the endowment of the risky asset because information processors are risk-neutral. This implies that price, liquidity, and volume are also independent of endowment in our discussion below. Therefore, we ignore it without loss of generality throughout this paper. By solving the first-order condition the optimal net demand is obtained as

$$\tilde{x}_i = \frac{1}{2\lambda} \left\{ \left[ -\{\alpha + (N - 1)\lambda \beta\} + \frac{a(d + e) - (N - 1)\lambda \tilde{y}(1 - \rho)de}{ad + ae + de} \right] \tilde{Y} \right.$$ \left. \right.$$ \left. \right.$$- \frac{ad + (N - 1)\lambda \tilde{y}\{ad + pe(a + d)\}}{ad + ae + de} \tilde{O}_i \right\}.$$

By using the condition of self-fulfilling expectations with respect to eq. (1), we get

$$\gamma = -\frac{ad + (N - 1)\lambda \gamma \{ad + pe(a + d)\}}{2\lambda \{ad + ae + de\}},$$

and $\gamma$ is solved as

$$\gamma = \frac{-ad}{\lambda [(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]}. \quad (2)$$
Also,
\[ \beta = \frac{1}{\lambda} \left[ -\{\alpha + (N - 1)\lambda\beta\} + \frac{a(d + e) - (N - 1)\lambda(1 - \rho)de}{ad + ae + de} \right], \]
and \( \beta \) is solved as
\[ \beta = \frac{1}{(N + 1)\lambda} \left[ -\alpha + \frac{a[(N + 1)d + \{2 + (N - 1)\rho\}e]}{(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)} \right]. \] (3)

Given any \( \alpha \) and \( \lambda \), eqs. (1), (2), and (3) determine information processor \( i \)'s equilibrium demand order as a function of the two observed signals \( \hat{Y} \) and \( \hat{O}_1 \).

The market maker sets the price equal to her expectation of \( \hat{U} \) conditional on her information, i.e.,
\[ \hat{P}_i = E[\hat{U} | \hat{U}_{t-1}, \hat{Y}, \hat{O}] = \hat{U}_{t-1} + E[\hat{u}_i | \hat{Y}, \hat{O}]. \]

In the appendix the conditional expectation is calculated, and by using the equivalence of the above price relation to eq. (1), \( \alpha \) and \( \lambda \) are solved as
\[ \alpha = \frac{a}{a + d}, \] (4)
\[ \lambda = \frac{N a^2 d^2 (ad + ae + de)}{\sqrt{v(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]^2}}. \] (5)

Eqs. (4) and (5) can be substituted back into eqs. (2) and (3) to give
\[ \beta = \frac{vd^2}{\sqrt{N(a + d)(ad + ae + de)}}, \] (6)
\[ \gamma = -\frac{v(a + d)}{\sqrt{N(ad + ae + de)}}. \] (7)

Eqs. (1), (4), (5), (6), and (7) provide a complete characterization of the unique market equilibrium at the time of an earnings announcement for any given numbers of information processors and liquidity traders.

At a nonearnings announcement date, \( t' \), say, the absence of an announcement such as \( \hat{Y} \) that can be processed into a superior assessment of the firm's performance implies that there is no private information in the market, and consequently there is no informed trading, i.e., \( x_{it'} = 0 \). Here, all demand orders
are from liquidity traders. Therefore, the market price is insensitive to the total demand order ($\lambda_t = 0$) and simply reflects all available public information, i.e., $\hat{P}_t = \hat{U}_t$.

In eq. (5) $\lambda$ is the inverse of Kyle's (1985) market depth measure. The market depth parameter, $1/\lambda$, is the order flow necessary to induce prices to rise or fall by one dollar. A small $\lambda$ implies that a trader can buy or sell a large amount of stock for a price not very different, on average, from the current market price, and the market can be said to be liquid. Similarly, a large $\lambda$ implies an illiquid market. At a nonearnings announcement date, $\lambda = 0$ and the market is infinitely deep.

4. Information acquisition and discretionary trade

In this section we analyze how the number of information processors is determined and at which date the discretionary liquidity traders choose to trade. A discretionary liquidity trader's expected profit is

$$E[\Pi'_k] = E[\tilde{z}_{dk}(\hat{U} - \hat{P}_t)] = E[\tilde{z}_{dk}(\hat{U} - \hat{U}_{t-1} - \alpha\tilde{Y} - \lambda(\tilde{x} + \tilde{z}_n + \tilde{z}_d))] = -\lambda E[\tilde{z}_{dk}^2] = -\lambda \text{var}(\tilde{z}_{dk}) = -\lambda,$$

and he always chooses to trade at a date when $\lambda$ is lower. Note that $\lambda$ is positive for any positive $a$, $d$, and $N$, which will be the case at the time of an earnings announcement. Therefore all $T \cdot M$ discretionary liquidity traders trade at non-earnings announcement dates and have zero expected profits. Assuming that their trades are evenly spread out over $T$ nonearnings announcement dates (thus $M$ at each nonannouncement date), this implies that the variance of the total liquidity demand, denoted by $v$ in section 3, is $L + M$ at each nonannouncement date and $L$ at the earnings announcement date.

Note first that potential information processors do not engage in information processing at nonannouncement dates because there is no public information to process. At time $t$, $\lambda$ can be rewritten by using eq. (5) and $v = L$ as

$$\lambda = \frac{N a^2 d^2 (a d + a e + d e)}{\sqrt{L(a + d)[(N + 1) a d + \{2 + (N - 1) \rho\} e(a + d)]^2}}.$$
The number of information processors at the time of an earnings announcement, \( N \), in eq. (8) is endogenous and can be solved for from the equilibrium condition (the expected profit is calculated in the appendix):

\[
E[\Pi^*_t] - C = \sqrt{\frac{La^2d^2(ad + ae + de)}{N(a + d)[(N + 1)ad + [2 + (N - 1)\rho]e(a + d)]^2}} - C
\]

\[
= \frac{L\lambda}{N} - C
\]

\[
= 0.
\]  

(9)

This condition of zero overall profit for each processor must be satisfied in equilibrium. Note that the overall profit is decreasing in \( N \), approaches infinity as \( N \) approaches zero, and becomes negative for a sufficiently large \( N \). If the profit from information processing is positive, \( N \) will increase until the profit becomes zero. If it is negative, \( N \) will decrease until the profit becomes zero. For convenience, we treat \( N \) as a number that can take any nonnegative value. Then, the solution for \( N \geq 0 \) that satisfies eq. (9) always exists and is unique for any nonnegative \( a, d, e, \) and \( \rho \).

In order to see how the equilibrium \( N \) is affected by changes in the parameters, we rewrite eq. (9) as

\[
\Phi \equiv Nh[N + 1 + (2 + (N - 1)\rho)eh] - \frac{L(1 + eh)}{C^2}
\]

\[
= 0,
\]  

(10)

where \( h \equiv \text{var}^{-1}(\bar{U} | \bar{U}_{t-1}, \bar{Y}) = (a + d)/ad \) measures the precision of all available public information at the time of an earnings announcement. Public precision, \( h \), is decreasing in either the variance of the prior information at the time of an earnings announcement, \( a \), or the variance of the error in the announcement, \( d \). The following lemma is obtained from eq. (10). All proofs are provided in the appendix.

**Lemma 1.** The equilibrium number of information processors at the time of an earnings announcement is decreasing in the precision of public information (increasing in either \( a \) or \( d \)), increasing in the diversity of opinion among information processors, increasing in the number of nondiscretionary liquidity traders, and

\(^{15}\text{Requiring that } N \text{ be an integer does not change qualitatively the results of this paper.}\)
decreasing in processing cost. That is, \( \frac{dN}{dh} < 0 \) (\( \frac{dN}{da} > 0 \) and \( \frac{dN}{dd} > 0 \)), \( \frac{dN}{d(1 - \rho)} > 0 \), \( \frac{dN}{dL} > 0 \), and \( \frac{dN}{dC} < 0 \). The sign of \( \frac{dN}{de} \) is ambiguous.

Lemma 1 is intuitive. It says that the number of information processors, or the intensity of information processing activities, increases as there is less public information (less \( h \)), i.e., either less-precise priors at the time of an earnings announcement (more \( a \)) and/or a less-precise earnings announcement (more \( d \)). This is because the informational advantage of becoming an information processor increases. It is increasing in diversity because a greater diversity of opinion of fixed quality (fixed \( e \)) means that more things can be learned through information processing. This, in turn, allows for a greater number of information processors in equilibrium. The number of information processors is increasing in the number of nondiscretionary liquidity traders (\( L \)) and decreasing in the processing cost (\( C \)), because the net profit from information processing activities is equal to the gross profit (\( E \llbracket \Pi \rrbracket \)) arising through the trading losses of liquidity traders, minus the processing cost.

One comparative static we ignore in this analysis is the change in processors’ individual information processing skills (\( e \)). This is because an increase in processors’ individual information processing skills (less \( e \)) can either increase or decrease the number of processors depending on the relative magnitude of its two effects. The first effect implies a higher quality of processed information for all information processors which works to increase their expected trading profits for a given \( \lambda \). The second effect implies less liquidity (higher \( \lambda \)), because the market maker now has to protect herself against the higher quality of processed information, which works to decrease their profits. If the combined effect is to increase (decrease) profits, then the number of information processors increases (decreases) until a new equilibrium is attained. When processed information is identical among information processors (\( \rho = 1 \)), it can be shown that the first effect dominates, and an increase in individual processing skills always increases the number of information processors. When processed information is diverse (\( \rho < 1 \)), however, the combined effect is ambiguous.

5. Liquidity and volume around earnings announcements

In this section we first examine how our measure of liquidity, \( 1/\lambda \), is different at the earnings announcement date than at nonannouncement dates. In addition, we perform comparative statics over \( 1/\lambda \) for various exogenous parameters. The following proposition states the results.

*Proposition 1.* The market is less liquid at the earnings announcement date than at nonearnings announcement dates. Moreover, market liquidity is increasing in the precision of public information (decreasing in either \( a \) or \( d \)), decreasing in the
diversity of opinion among information processors, and increasing in the number of nondiscretionary liquidity traders. That is, \( \frac{d(1/\lambda)}{dh} > 0 \) (\( \frac{d(1/\lambda)}{da} < 0 \) and \( \frac{d(1/\lambda)}{dd} < 0 \)), \( \frac{d(1/\lambda)}{d(1 - \rho)} < 0 \), and \( \frac{d(1/\lambda)}{dL} > 0 \). The sign of \( \frac{d\lambda}{dC} \) is indeterminate.

Liquidity increases as there is more public information or as there is less diversity, because the number of information processors decreases in both cases and because \( \lambda = NC/L \) in equilibrium. An increase in the number of nondiscretionary liquidity traders directly increases liquidity. It also increases the number of traders inclined to process information, which, in turn, affects liquidity. The fourth result of Proposition 1 shows that the combined effect of more liquidity traders and more information processors nonetheless increases liquidity. Finally, an increase in processing cost has an ambiguous effect on liquidity. This is because liquidity is influenced by the aggregate processing cost \( NC \), i.e., \( \lambda = NC/L \). An increase in individual processing cost increases \( C \), but also reduces the number of information processors \( N \) as shown in Lemma 1. This implies that it has an ambiguous effect on the aggregate processing cost and, in turn, on liquidity.

Recall that in the introduction we pointed out an alternative characterization of public disclosure predicated on disclosure reducing information asymmetry. The model analyzed here could be expanded to capture this effect by imagining that a few market participants learn \( \tilde{Y} \), or perhaps information superior to \( \tilde{Y} \), regardless of whether \( \tilde{Y} \) is disclosed at time \( t \). In this characterization disclosing \( \tilde{Y} \) has two effects: as described in our analysis, it creates information asymmetry through the activities of information processors, and, as a second effect, it reduces information asymmetry by disclosing information \( (\tilde{Y}) \) that would otherwise be known by only a few participants in the market. Consequently, in a more general model, whether liquidity increases or decreases depends on whether the announcement effect associated with creating information asymmetry dominates the effect associated with reducing it (or vice versa). Nonetheless, Proposition 1 establishes that even in a model dealing exclusively with announcements that create information asymmetry, more public information increases liquidity. In this sense Proposition 1 offers a result similar to those found in analyses dealing exclusively with disclosures that reduce information asymmetry directly [e.g., Diamond and Verrecchia (1991)].

We now turn to volume. Volume at a nonearnings announcement date \( t' \), denoted by \( V_{t'} \), is simply

\[
V_{t'} = \frac{1}{2} \left[ \sum_{j=1}^{L} |\tilde{z}_{njt'}| + \sum_{k=1}^{M} |\tilde{z}_{dkt'}| \right].
\]
The expected volume at a nonannouncement date \( t' \), denoted by \( \bar{V}_0 \), is

\[
\bar{V}_0 = \frac{1}{2} \sqrt{\frac{2}{\pi}} \left[ L \sqrt{\text{var}(\tilde{z}_{n_{k'}})} + M \sqrt{\text{var}(\tilde{z}_{d_{k'}})} \right] = \frac{L + M}{\sqrt{2\pi}} .
\] (11)

Volume at the time of an earnings announcement, denoted by \( V \), is

\[
V = \frac{1}{2} \left[ \sum_{i=1}^{N} |\tilde{x}_i| + \sum_{j=1}^{L} |\tilde{z}_{nj}| \right].
\]

The expected volume at the time of an earnings announcement, denoted by \( \bar{V} \), is

\[
\bar{V} = \frac{1}{2} \sqrt{\frac{2}{\pi}} \left[ N \sqrt{\text{var}(\tilde{x}_i)} + L \sqrt{\text{var}(\tilde{z}_{nj})} \right] = \frac{\sqrt{LN} + L}{\sqrt{2\pi}} .
\] (12)

The following lemma summarizes the above results.

**Lemma 2.** The expected trading volume is higher at the earnings announcement date than at nonearnings announcement dates if and only if \( \sqrt{LN} > M \).

Note that while discretionary liquidity traders avoid the earnings announcement date because the market is less liquid, information processors do not trade at nonannouncement dates because they have no incentive to do so. At nonannouncement dates the valuation of the firm's performance cannot be improved upon through information processing. Thus, nonearnings announcement dates provide no opportunity to process public information into superior assessments of a firm's performance and hence no incentive to trade. Lemma 2 says that higher average volume accompanies public disclosure if there are sufficiently many information processors and/or nondiscretionary liquidity traders relative to the number of discretionary liquidity traders. This is because the amount of trading at the time of an earnings announcement \( (\sqrt{LN}/2\pi) \) depends not only on the number of information processors \( (N) \) but also on the number of (nondiscretionary) liquidity traders trading at that time.

The following proposition is now obtained from Lemmas 1 and 2.

**Proposition 2.** Expected trading volume at the time of an earnings announcement is decreasing in the precision of public information (increasing in either \( a \) or \( d \)), increasing in the diversity of opinion among information processors, increasing in the number of nondiscretionary liquidity traders, and decreasing in the information processing cost. That is, \( d\bar{V}/dh < 0 \) (\( d\bar{V}/da > 0 \) and \( d\bar{V}/dd > 0 \)), \( d\bar{V}/d(1 - \rho) > 0 \), \( d\bar{V}/dL > 0 \), and \( d\bar{V}/dC < 0 \). Moreover, expected volume is higher at the earnings
announcement date than at nonannouncement dates if sufficiently little public information is available (h is small, i.e., both a and d are large), or if there is a sufficiently large number of nondiscretionary liquidity traders, or if the processing cost is sufficiently small.

The first part of Proposition 2 is true because all of the following stimulate information processing activities and increase informed trading: a decrease in the precision of public information (less-precise priors at the time of the earnings announcement or a less-precise earnings announcement), an increase in the diversity of opinion among information processors, and/or a decrease in processing cost. An increase in the number of liquidity traders directly increases volume, but also indirectly increases it by attracting more information processors.

The second part of Proposition 2 (especially the first result) is important in this paper. It suggests that high volume at the time of an earnings announcement results from a large amount of informed trading, because liquidity trading is less at the earnings announcement date than at nonannouncement dates. This result highlights the difference of our model from that of Admati and Pfleiderer (1988) in which high volume arises from large amounts of both informed and liquidity trading.

We now consider the effect of an earnings announcement on the movement of the market price. The price change at the time of an earnings announcement is

\[
\tilde{P}_t - \tilde{P}_{t-1} = \tilde{U}_{t-1} + \alpha \tilde{Y} + \lambda \tilde{\omega} - \tilde{U}_{t-1} \\
= \alpha \tilde{Y} + \lambda (\tilde{x} + \tilde{z}_n). 
\]  

As suggested by eq. (13), the earnings announcement causes information processing activities (positive \(N\)) and, in turn, the market maker protects herself against the informational advantage held by information processors (positive \(\lambda\)). Trivially, any announcement that shifts price creates variance in price change. The variance of the price change at the time of an earnings announcement, denoted by \(\Delta\), is calculated (in the appendix) as

\[
\Delta \equiv \text{var}(\tilde{P}_t - \tilde{P}_{t-1}) \\
= \frac{a^2}{a + d} + \frac{Na^2d^2}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]}. 
\]  

At the post-earnings announcement date \(t\), \(\tilde{u}\) becomes common knowledge, and market participants no longer use \(\tilde{Y}\) or \(\tilde{\omega}\) to assess the firm's future value \(\tilde{U}\).
The price simply becomes $\tilde{P}_t = \tilde{U}_t$ and

$$\tilde{P}_t - \tilde{P}_* = \tilde{U}_t - \tilde{U}_{t-1} - \alpha \tilde{Y} - \lambda \tilde{\omega}$$

$$= \tilde{u}_t - \alpha \tilde{Y} - \lambda (\tilde{x} + \tilde{z}_n).$$

(15)

Its variance can be calculated (in the appendix) as

$$\text{var}(\tilde{P}_t - \tilde{P}_*) = a - \frac{a^2}{a + d} - \frac{Na^2d^2}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]}.$$

It is clear from this expression that the variance of the post-announcement price change is less than $a$, which is the variance of $\tilde{u}_t$. This is because price at the time of the earnings announcement, $\tilde{P}_t$, partially reflects $\tilde{u}_t$, i.e., $\tilde{P}_t$ is an increasing function of the announced signal $\tilde{Y}$ about $\tilde{u}_t$.

The next result explores how the variance of price change at the time of the earnings announcement is affected by shifts in various exogenous parameters.

**Proposition 3.** The variance of price change at the time of an earnings announcement is decreasing in both the precision of the prior information at the time of the announcement and the precision of the error in the announcement, increasing in the diversity of opinion among information processors, increasing in the number of liquidity traders, and decreasing in the information processing cost. That is, $\frac{dA}{da} > 0$, $\frac{dA}{dd} > 0$, $\frac{dA}{d(1 - \rho)} > 0$, $\frac{dA}{dL} > 0$, and $\frac{dA}{dC} < 0$.

Propositions 2 and 3 imply that shifts in the (exogenous) parameters $a, d, \rho, L$, and $C$ move expected trading volume and the variance of price change in the same direction at the time of an earnings announcement. This suggests that volume and the absolute value of price change are positively associated at the time of an earnings announcement, an observation commonly made in both the empirical literature and analyses involving perfectly competitive markets.\(^{16}\)

In general, the magnitude of price change is closely related to how informative the price is. The informativeness of the price at the time of an earnings announcement can be measured by the reduction of uncertainty due to

\(^{16}\)See Karpoff (1987) for a review of studies of the relation between volume and price changes. See Kim and Verrecchia (1991a,b) for discussions of the relation between volume and price change around an announcement in perfectly competitive markets.
the price. The reduction, denoted by $F$, can be expressed as (calculated in the appendix)

$$F \equiv \text{var}(\tilde{U} | \tilde{U}_{t-1}, \tilde{Y}) - \text{var}(\tilde{U} | \tilde{U}_{t-1}, \tilde{Y}, \tilde{P}_t)$$

$$= \left[ \text{var}(\tilde{U} - \tilde{U}_t) + \text{var}(\tilde{u}_t | \tilde{Y}) \right] - \left[ \text{var}(\tilde{U} - \tilde{U}_t) + \text{var}(\tilde{u}_t | \tilde{Y}, \tilde{P}_t) \right]$$

$$= \text{var}(\tilde{u}_t | \tilde{Y}) - \text{var}(\tilde{u}_t | \tilde{Y}, \tilde{P}_t)$$

$$= \frac{Na^2d^2}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\} e(a + d)]}. \quad (16)$$

Note that since the price at a nonearnings announcement date simply reflects all public information, i.e., $\tilde{P}_t = \tilde{U}_t$, the reduction of uncertainty due to a nonearnings announcement date price is zero. Eq. (16) implies the following proposition.

**Proposition 4.** The informativeness of price at the time of an earnings announcement is decreasing in both the precision of the prior information at the time of the announcement and the precision of the error in the announcement, increasing in the diversity of opinion among information processors, increasing in the number of liquidity traders, and decreasing in the information processing cost. That is, $dF/da > 0$, $dF/dd > 0$, $dF/d(1 - \rho) > 0$, $dF/dL > 0$, and $dF/dC < 0$.

Price is informative at the time of an earnings announcement because it aggregates (with noise) the private information gathered in conjunction with the announcement. Consequently, expectations conditioned over all of price, the earnings announcement, and the realization of cash flow through to the earnings announcement (i.e., through to date $\tau$) are superior to those conditioned over only the announcement and cash flow. At a nonearnings announcement date, there is no private information gathering. Hence, price contains no information beyond that which is commonly known (i.e., the realization of cash flows up to that date).

### 6. Conclusion

The crux of this paper is the suggestion that public disclosure of financial accounting data, particularly earnings announcements, provides a source of private information to certain traders through their information processing activities. This creates information asymmetry between these traders and
market makers. Institutionally, traders who process public announcements are those market participants most capable of making informed judgments about a firm's performance on the basis of publicly available information. Presumably, these are the traders who bear the lowest cost for engaging in this activity. We capture this phenomenon in our model by requiring that information processors absorb a fixed cost. This makes the number of information processors endogenous to the market. It also allows us to understand how public information influences the incentives to become informed. In short, rather than reducing information asymmetry, earnings announcements create asymmetry through the (endogenous) activities of traders who process public announcements into private information.

The contribution of our model as a theoretical exercise is that it captures the role of an earnings announcement in markets that are not perfectly competitive (i.e., illiquid). Its advantage vis-à-vis other alternative formulations is that the results presented here appear consistent with casual empiricism. Institutionally, market makers are not thought to be as capable of processing published reports about firms' activities as traders who follow a firm closely. Therefore, a public announcement places a market maker at some (perhaps brief) informational disadvantage. Our model only characterizes increases in bid–ask spreads at the time of an earnings announcement, and not before. However, logically market makers are likely to increase spreads in anticipation of an earnings announcement, to guard against investors acting on the information before it is disclosed publicly (e.g., 'leaks'). This suggests that spreads temporarily widen around announcements. As this advantage dissipates, spreads fall back to the level that prevailed before the announcement was anticipated. There is no element of elapsed time in our model, however, and so how long this advantage remains is an empirical issue not addressed in this paper. While our results only suggest an increase in expected volume at the time of an earnings announcement under certain circumstances, typically increased volume is observed [see, e.g., Skinner (1991) and Lee et al. (1991)]. In the context of our model, this suggests that the public information available at the time of an earnings announcement is not sufficient to preclude the possibility of processing the information into superior, private assessments. When volume does arise, one advantage of our explanation is that it suggests increased trading activity from informed trading, and not for liquidity reasons.

The results of our analysis feature the role of five exogenous variables: the precision of the prior information at the time of the earnings announcement, the precision of the error in the announcement, the diversity of opinion among individual information processors, the number of nondiscretionary liquidity traders in the market, and the cost of processing information. However, the results involving the first two are particularly important in that they lead to an inescapable policy implication: more public information increases liquidity (see, for example, Proposition 1). This implication is straightforward in a model in
which announcements eliminate, or reduce, information asymmetries directly [e.g., Diamond and Verrecchia (1991)]. This implication is more subtle in the context of the analysis done here because earnings announcements create information asymmetry through the activities of information processors. Consequently, one might infer that public disclosure, as manifest in an earnings announcement, makes markets less liquid. But this is not the correct interpretation of this fact. In our model, the precision of public information increases as either the variance of prior information at the time of the earnings announcement decreases or the variance of the error in the earnings announcement decreases. As the precision of public information increases, liquidity increases, and in this sense more public information implies more liquidity. In effect, more public information ameliorates the problem disclosure creates in offering some subset of traders (i.e., information processors) an opportunity at the expense of other market participants. The simple economic interpretation of this result is that more public information reduces or eliminates potential information asymmetries. This suggests that, if a firm is required to disclose (and thereby create potential information asymmetries) and if the firm is concerned about liquidity, then the firm should provide adequate disclosure to reduce opportunities for information processors. An ancillary implication of this result is that it suggests that liquidity may be a useful metric for evaluating the effects of disclosure requirements.

Although the idea that more public information reduces information asymmetry may seem obvious, this relation may not occur in richer models of private information gathering. For example, in Kim and Verrecchia (1991b), investors with diverse economic characteristics gather private information in anticipation of a public announcement. Investors’ diverse economic characteristics imply that they acquire private information of different precisions at costs that reflect the differential quality of the information acquired. There, information asymmetry among investors at the time of the announcement is unimodal in the precision of the announcement. Unimodal behavior is explained by the fact that, first, information asymmetry rises as the precision of the announcement rises, because it attracts (differential) private information gathering in anticipation of the announcement. However, after a certain point, further increases in the precision of an announcement reduce information asymmetries by providing an announcement that dominates investors’ beliefs vis-à-vis what each may know privately. This aspect of information asymmetry is not captured in the model presented in this paper because (potential) information processors are identical and acquire private observations of identical precision (at identical cost). (Note, however, that each information processor makes a different private observation.) Therefore, an interesting extension of the model presented here is to allow for private information gathering of differential quality, because this possibility is likely to have a material effect on the results of this paper.
Appendix

Calculation of $E[\bar{u}_t | \bar{Y}, \bar{\omega}]$, $\lambda$, and $\alpha$

By using the standard formula the conditional expectation can be expressed as

$$
E[\bar{u}_t | \bar{Y}, \bar{\omega}] = \frac{1}{\text{Det}} \left\{ \frac{N^2 a^2 d^2}{(N + 1) \lambda^2 [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]} \right\} \hat{Y} + \frac{N(N - 1)(1 - \rho)a^2 d^2 e}{(N + 1) \lambda^2 [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]^2} \hat{\omega},
$$

where the determinant, $\text{Det}$, of the variance-covariance matrix of $(\bar{Y}, \bar{\omega})$ is

$$
\text{Det} = v(a + d) + \frac{N a^2 d^2 [N a d + \{1 + (N - 1)\rho\} e(a + d)]}{\lambda^2 [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]^2}.
$$

Therefore,

$$
\lambda = \frac{1}{\text{Det}} \frac{N a^2 d^2}{\lambda [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]}.
$$

Eq. (5) directly follows from this expression. Similarly,

$$
\alpha = \frac{1}{\text{Det}} \left\{ \frac{N^2 a^2 d^2}{(N + 1) \lambda^2 [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]} \right\} + \frac{N(N - 1)(1 - \rho)a^2 d^2 e}{(N + 1) \lambda^2 [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]^2}.
$$

This can be simplified as

$$
\alpha = \frac{a}{(a + d)} \left[ v - \frac{N(N - 1)(1 - \rho)a^2 d^2 e}{(N + 1) \lambda^2 [(N + 1)\lambda d + 2 + (N - 1)\rho e(a + d)]^2} \right]
$$

Since the inside of the big square bracket is positive, $\alpha = a/(a + d)$. 
Calculation of expected profit

By using eqs. (4) to (7), the expected profit of the $i$th processor can be written as

$$E[\Pi_i^e] = E[\lambda \tilde{x}_i^2]$$

$$= \lambda \text{var}(\tilde{x}_i)$$

$$= \lambda [\beta^2 a + (\beta + \gamma)^2 d + \gamma^2 e].$$

This can be simplified as

$$E[\Pi_i^e] = \frac{a^2 d^2 (ad + ae + de)}{\lambda (a + d) [(N + 1)ad + \{2 + (N - 1)\rho\} e(a + d)]^2}$$

Proof of Lemma 1

From eq. (10)

$$\frac{\partial \Phi}{\partial N} = h[N + 1 + \{2 + (N - 1)\rho\} \epsilon h]$$

$$\times [N + 1 + \{2 + (N - 1)\rho\} \epsilon h + 2N(1 + \rho \epsilon h)]$$

$$= h[N + 1 + \{2 + (N - 1)\rho\} \epsilon h][3N + 1 + \{2 + (3N - 1)\rho\} \epsilon h]$$

$$> 0,$$

$$\frac{\partial \Phi}{\partial \epsilon} = N[N + 1 + \{2 + (N - 1)\rho\} \epsilon h]$$

$$\times [N + 1 + \{2 + (N - 1)\rho\} \epsilon h + 2\{2 + (N - 1)\rho\} \epsilon h] - \frac{Le}{C^2}$$

$$= N[N + 1 + \{2 + (N - 1)\rho\} \epsilon h][N + 1 + 3\{2 + (N - 1)\rho\} \epsilon h]$$

$$- \frac{e}{1 + \epsilon h} Nh[N + 1 + \{2 + (N - 1)\rho\} \epsilon h]^2$$
\begin{align*}
&= \frac{N}{1 + eh} \left[ N + 1 + \{2 + (N - 1)\rho\}eh \right] \\
&\quad \times \left[ N + 1 + \{2 + (N - 1)\rho\}eh(3 + 2eh) \right] \\
&> 0,
\end{align*}

\frac{\partial \Phi}{\partial \rho} > 0,

for \( N > 1, \)

\begin{align*}
\frac{\partial \Phi}{\partial L} &= -\frac{1 + eh}{C^2} < 0, \\
\frac{\partial \Phi}{\partial C} &= \frac{2L(1 + eh)}{C^3} > 0.
\end{align*}

The claims of the lemma follow by using the implicit function theorem and the signs of the above partial derivatives.

{
\textit{Proof of Proposition 1}}

The first statement is true because of the fact that \( N \) and \( \lambda \) are positive for \( d < \infty \) and are zero otherwise. In order to prove the rest, rewrite \( \lambda \) as

\[ \lambda = \frac{NC}{L}. \]

Since this condition must hold in equilibrium, by Lemma 1, \( \frac{d\lambda}{dh} = (C/L)(dN/dh) < 0 \) and \( \frac{d\lambda}{d\rho} = (C/L)(dN/d\rho) < 0. \) From the proof of Lemma 1,

\[ \frac{dN}{dL} = \binom{N}{L} \frac{N + 1 + \{2 + (N - 1)\rho\}eh}{3N + 1 + \{2 + (3N - 1)\rho\}eh}. \]

Therefore,

\[ \frac{d\lambda}{dL} = -\binom{NC}{L^2} \frac{2N(1 + \rho eh)}{3N + 1 + \{2 + (3N - 1)\rho\}eh} < 0. \]
Similarly, from the proof of Lemma 1,

\[
\frac{dN}{dC} = -\left(\frac{2N}{C}\right) \frac{N + 1 + \{2 + (N - 1)\rho\}e^h}{3N + 1 + \{2 + (3N - 1)\rho\}e^h},
\]

Therefore,

\[
\frac{d\lambda}{dC} = \left(\frac{C}{L}\right) \frac{dN}{dC} + \frac{N}{L}
\]

\[
= \left(\frac{N}{L}\right) \frac{N - 1 + \{-2 + (N + 1)\rho\}eh}{3N + 1 + \{2 + (3N - 1)\rho\}e^h},
\]

whose sign is indeterminate. This completes the proof.

**Proof of Proposition 2**

The first part of the proposition directly follows from Lemma 1 and eq. (12). In order to prove the second part, first note from eq. (10) that \(N\) approaches infinity as \(L\) approaches infinity, or as \(h\) or \(C\) approaches zero. Now the claims are true by eq. (12).

**Calculation of \(\text{var}(\tilde{P}_t - \tilde{P}_{t-1})\) and \(\text{var}(\tilde{P}_t - \tilde{P}_t)\)**

By using eqs. (13), (4), (5), (6), and (7) and simplifying, we get

\[
\tilde{P}_t - \tilde{P}_{t-1} = \alpha \tilde{Y} + \lambda (\tilde{x} + \tilde{z}_n)
\]

\[
= \frac{Nd^2 + (N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} a\tilde{u}_t,
\]

\[
+ \frac{ad + \{2 + (N - 1)\rho\}e(a + d)}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} a\tilde{\xi}
\]

\[
+ \frac{ad}{(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)} \sum \tilde{e}_i + \lambda \tilde{z}_n.
\]

It is now straightforward to calculate the variance as in eq. (14).
Similarly from eq. (15) and the above,

\[
\tilde{F}_t - \tilde{F}_t = \tilde{u}_t - \alpha \tilde{Y} - \lambda(\tilde{x} + \tilde{z}_n)
\]

\[
= \tilde{u}_t - \frac{Nd^2 + (N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} a\tilde{u}_t
\]

\[
- \frac{ad + \{2 + (N - 1)\rho\}e(a + d)}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} a\tilde{\delta}
\]

\[
- \frac{ad}{(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)} \sum \tilde{e}_i - \lambda \tilde{z}_n.
\]

Its variance is

\[
\text{var}(\tilde{F}_t - \tilde{F}_t) = a \left[ 1 - \frac{2a}{a + d} \frac{2Nd^2}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} \right]
\]

\[
+ \frac{a^2}{a + d} + \frac{Na^2d^2}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]},
\]

which can be rewritten as in the expression in the text.

**Calculation of F**

\[
F = \text{var}(\tilde{U} | \tilde{U}_{t-1}, \tilde{Y}) - \text{var}(\tilde{U} | \tilde{U}_{t-1}, \tilde{Y}, \tilde{F}_t)
\]

\[
= \left[ \text{var}(\tilde{U} - \tilde{U}_t) + \text{var}(\tilde{u}_t | \tilde{U}_{t-1}, \tilde{Y}) \right]
\]

\[- \left[ \text{var}(\tilde{U} - \tilde{U}_t) + \text{var}(\tilde{u}_t | \tilde{U}_{t-1}, \tilde{Y}, \tilde{F}_t) \right]
\]

\[
= \text{var}(\tilde{u}_t | \tilde{Y}) - \text{var}(\tilde{u}_t | \tilde{Y}, \alpha \tilde{Y} + \lambda \tilde{\omega}),
\]

where

\[
\alpha \tilde{Y} + \lambda \tilde{\omega} = \frac{Nd^2 + (N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} a\tilde{u}_t
\]

\[
+ \frac{ad + \{2 + (N - 1)\rho\}e(a + d)}{(a + d)[(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)]} a\tilde{\delta}
\]

\[
+ \frac{ad}{(N + 1)ad + \{2 + (N - 1)\rho\}e(a + d)} \sum \tilde{e}_i + \lambda \tilde{z}_n.
\]
as in the calculation of $\tilde{P}_t - \tilde{P}_{t-1}$ above. It is easy to calculate that $\text{cov}(\tilde{u}_t, \tilde{y}_t) = \text{cov}(\tilde{Y}, \alpha \tilde{Y} + \lambda \tilde{d}) = a$ and $\text{var}(\alpha \tilde{Y} + \lambda \tilde{d}) = \text{cov}(\tilde{u}_t, \alpha \tilde{Y} + \lambda \tilde{d}) = (a^2/(a + d)) + (Na^2d^2/(a + d))[\{2 + (N - 1)\rho\}e(a + d)]$. By using the formula for calculating the conditional variance of a normally distributed random variable, $F$ can be calculated and simplified as

$$ F = \left( a - \frac{a^2}{a + d} \right) $$

$$ - \left( a - \frac{a^2}{a + d} - \frac{Na^2d^2}{(a + d)[\{2 + (N - 1)\rho\}e(a + d)]} \right) $$

$$ = \frac{Na^2d^2}{(a + d)[\{2 + (N - 1)\rho\}e(a + d)]}. $$

**Proof of Propositions 3 and 4**

For Proposition 3, it is easy to check that $\partial A/\partial a > 0$, $\partial A/\partial d > 0$, $\partial A/\partial p < 0$, and $\partial A/\partial N > 0$. By Lemma 1 and the above,

$$ \frac{\partial A}{\partial a} = \frac{\partial A}{\partial a} + \frac{\partial A}{\partial N} \frac{dN}{da} > 0, $$

$$ \frac{\partial A}{\partial d} = \frac{\partial A}{\partial d} + \frac{\partial A}{\partial N} \frac{dN}{dd} > 0, $$

$$ \frac{\partial A}{\partial p} = \frac{\partial A}{\partial p} + \frac{\partial A}{\partial N} \frac{dN}{dp} < 0, $$

$$ \frac{\partial A}{\partial L} = \frac{\partial A}{\partial N} \frac{dN}{dL} > 0, $$

$$ \frac{\partial A}{\partial C} = \frac{\partial A}{\partial N} \frac{dN}{dC} < 0. $$

The proof for Proposition 4 proceeds in the same fashion as the above.

**References**


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