Some implications of the Braess Paradox for pricing and investment in electric power systems

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Abstract

Braess' Paradox describes a network configuration in which adding capacity to the network creates congestion. Although the paradox was first described in the context of traffic engineering, it has been shown to hold in a variety of other contexts including computer and piping networks. We illustrate how the Braess Paradox can hold in electric power networks, and discuss some of its implications for investment and pricing in the restructured electricity industry. Using a small-scale test network and assuming DC load flows, we find that the traditional optimal power flow algorithm fails to identify the active constraint in the system. We also find that systems which exhibit Braess' Paradox also fail to exhibit localized response; increasing capacity in a congested line does not lower the total cost of serving load unless all congested lines in the system are upgraded. In these systems, locational prices (as currently used) will not identify the source of network congestion.

Introduction: Wheatstone Networks and Braess' Paradox

Consider a simple network as shown in Figure 1a. Injection into the network occurs at node A and withdrawal from the network occurs at node B. There are two paths going from nodes A to B. Assuming that both paths are identical in every way, one-half of the network traffic will travel on each path from node A to B. However, if a third path is added to the network which intersects the other two paths (but not nodes A or B), this additional path causes congestion in the network rather than relieving congestion. This situation is shown in Figure 1b.

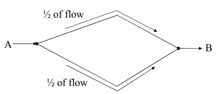

Figure 1a: Half of the flow between nodes A and B follows each path.

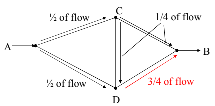

Figure 1b: The additional link reroutes some traffic exiting node C. As a result, congestion is created on the link between nodes D and B.

The network in Figure 1b is referred to as a Wheatstone Network (the link between C and D is called the Wheatstone bridge), and the result that adding capacity to a network can cause congestion is referred to as Braess' Paradox. Although originally described in the context of traffic flows (Arnott and Small 1992), the Braess Paradox has gained in popularity in the analysis of computer networks (Bean et. al. 1997) and piping systems (Calvert and Keady 1993). Although the paradox has been described in electric circuits (Cohen and Horowitz 1991), it appears not to have been applied to large-scale power systems.

Notation

Buses in the network are indexed by $i = \{1,\ldots,n\}$

Links in the network are denoted S_{ij}, $i,j = \{1,\ldots,n\}$

P_{Gi} = Real power output at bus i;

P_{Li} = Real power demanded at bus i;

F_{ij} = Real power flow between buses i and j;

π_i = Locational marginal price at bus i in \$/MW;

μ_{ij} = Shadow price of congestion on line S_{ij} in \$/MW;

All load-flow calculations were performed using the DC optimal power flow algorithm (neglecting losses and reactive power) embedded in Matpower, a package of Matlab files for solving power flow problems. Documentation is available at: http://www.pserc.cornell.edu/matpower.

Braess' Paradox in a simple electric power system

Figure 2 illustrates Braess' Paradox in a simple electric power network. The load at bus 4 is assumed to have a totally price-inelastic demand of 100 MW. The generator located at bus 1 has a capacity of 100 MW and the generator located at bus 4 has a capacity of 10 MW. The cost functions of the two generators are constructed so that generator G1 is always cheaper to operate than generator G2. All lines in the system have a capacity of 55 MW. Lines S_{12} and S_{34} have larger admittances than the other lines in the system.

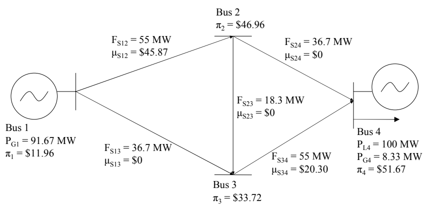

Figure 2: An illustration of the Braess Paradox in an electric power system. Without the Wheatstone bridge (line S_{23}) Generator G1 fills all 100 MW of load and there is no congestion in the system. Adding a Wheatstone bridge to the system causes congestion on lines S_{12} and S_{34}. Generator G1 can no longer produce 100 MW without violating the system security constraints, so Generator G2 must produce some power. This raises the cost of serving load in the system.

Without the Wheatstone bridge (line S_{23}), Generator G1 produces all 100 MW of power, and there is no congestion. The total system cost is \$1,622.20 per hour. Once line S_{23} is added to the system, lines S_{12} and S_{34} carry their maximum capacity of 55 MW and the system is thus congested. Generator G2 must be dispatched and the system cost rises to \$1,945.50 per hour.

Result 1: Localized investment fails to relieve constraints

Constrained lines have positive shadow prices, which are normally interpreted as the marginal value of relaxing a constraint. In Figure 2, lines S_{12} and S_{34} are the only ones with positive shadow prices; the results suggest that increasing capacity on line S_{12} by 1 MW will lower overall system cost by \$45.87 per hour, and a similar upgrade on line S_{34} will lower costs by \$20.30 per hour. Optimization theory would suggest upgrading line S_{12}, since the benefit for each investment dollar is more than double that of upgrading line S_{34}.

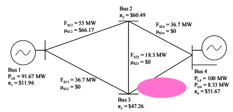

Figure 3a: Line S_{34} is upgraded to 56 MW while line S_{12} remains rated at 55 MW. The shadow price on line S_{34} falls to zero, but the total system cost remains at the pre-upgrade level of \$1,945.50 per hour.

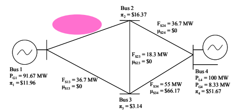

Figure 3b: Line S_{12} is upgraded to 56 MW while line S_{34} remains rated at 55 MW. The shadow price on line S_{12} falls to zero, but the total system cost remains at the pre-upgrade level of \$1,945.50 per hour.

However, as shown in Figures 3a and 3b, upgrading one of the two congested lines does not get rid of the global constraint. The upgrade yields no system benefits beyond reducing the relevant shadow price to zero; the total cost of serving the load remains at the pre-upgrade level of \$1,945.50. Only when both lines S_{12} and S_{34} are upgraded does the system see benefits, with total and locational-marginal costs falling as shown in Figure 4. The total cost of serving the 100 MW of load falls to \$1,879.63 per hour, while the locational marginal price at bus 4 falls to \$51.33. The price at bus 1 is the only one that rises; this is because expanding capacity on both lines S_{12} and S_{34} allows Generator G1 to produce more power.

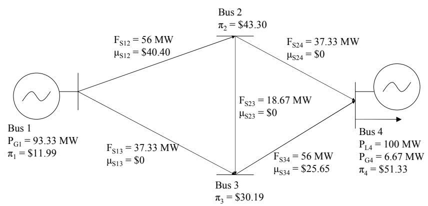

Figure 4: After upgrading both lines S_{12} and S_{34}, the total cost of serving load in the system has fallen to \$1,879.63 per hour. The locational price at bus 4 has also fallen, to \$51.33. Note that the price at bus 1 rises because expanding both lines allows Generator G1 to produce more power.

Result 2: Shadow prices identify congestion, not constraints

Related to the investment issue is a secondary result that the shadow prices identify congestion, but not active constraints. The OPF performed on the systems in Figures 2 and 4 would tell operators that lines S_{12} and S_{34} are congested, but only tells them the value of upgrading (both) of those lines. As a planning matter, the cost of the upgrade would then have to be weighed against its benefits. What the OPF does not tell planners or operators is that removing or de-energizing line S_{23} (the Wheatstone bridge) will relieve congestion, perhaps even at a lower cost. In this sense, the active constraint is on line S_{23} – it is line S_{23} that is really "causing" the congestion on lines S_{12} and S_{34}.

Conclusions and Future Work

In restructuring the electric power industry, policymakers have tried to move away from centralized planning and decision-making, towards a reliance on decentralized market participants acting on information contained in market prices. Our analysis of the Braess Paradox in electric power systems suggests that relying on prices as they are currently calculated may lead to system-suboptimal decisions. Lines may be upgraded in cases where removing lines might relieve constraints at a lower cost. Further, location matters in electric power systems, and the incentives of merchant generators or transmission firms may lead them to locate in parts of the grid where they may cause or worsen congestion.

Future work in this area revolves around the question of whether the behavior described here for small systems is also found in larger electric networks. Milchtaich (2005) shows that even very complex networks can be categorized as series-parallel (including radial networks) or as having an embedded Wheatstone network. This suggests that electric power networks could be decomposed into small sub-systems consisting of series-parallel and Wheatstone networks. The larger network could then be analyzed in a piecewise fashion.

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