

Communication and Allocation of Decision Rights in Multi-Agent Environments

by

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Abstract

In many economic environments such as multidivisional organizations, principals have to resolve the conflict between coordinating the decisions of self interested agents and successfully adapting to local conditions. We model the interdependencies that require coordination as complementarities (or substitutabilities) and (positive or negative) externalities between actions taken by these agents. Also, each agent has private knowledge of local conditions that the organization should adapt to. We consider centralization and decentralization as two alternative regimes of communication and decision making. Under centralization, agents send simultaneous signals about their private information to the principal who then decides on actions, while under decentralization, agents communicate with each other prior to taking actions on their own. In this paper we provide an analysis of strategic communication in these two regimes. We also investigate which regime is optimal (efficient) in different organizations. In both regimes, the quality of communication varies with the nature of the interaction between complementarities and externalities. In the decentralized regime, the extent of informative communication is very limited, while the centralized one allows for more informative communication. In particular, complementarities and positive externalities result in no information being transmitted under decentralization. We also show how the informativeness of communication influences the efficiency of different regimes.

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To my loving family...

Chapter 1

Introduction

Economists and management scientists have long been interested in the analysis of various communication protocols among agents who are asymmetrically informed about the real state of the world, and who have possibly diverging preferences over the courses of actions to be taken. More specifically, researchers have been trying to understand how different decision schemes and communication protocols contribute to the value of multi-unit organizations.

The seminal work on *cheap talk* by Crawford and Sobel (1982) has provided a new understanding of issues surrounding strategic communication schemes. In the canonical cheap talk model, there is an informed agent who can send costless messages to an uninformed decision maker (principal). Agent's information matters for the best action from the perspectives of the agent and the principal. However, the agent is biased in the sense that his best action is different from the principal's best action. Furthermore, agent's information is not contractible. Crawford and Sobel (1982) show that if the agent's bias is not large, then communication is informative to a certain extent. Our communication structure is based on this cheap talk model.

The first communication protocol that we are interested in is between an uninformed principal and two informed agents, which we will refer to as *vertical communication*. Each agent is locally and privately informed about an independent aspect of the

state of the world. Upon observing the messages transmitted by the informed agents, the principal takes a multidimensional decision that affects the welfare of all of the agents. Although a similar information and communication structure has been proposed in the organizational economics literature [[Alonso et al. \(2008b\)](#), [Rantakari \(2008a\)](#)], the cheap talk literature with multiple agents (experts) has been interested in the case where each expert observes the same signal of the true state [[Krishna and Morgan \(2001a\)](#), [Battaglini \(2002, 2004\)](#), [Ambrus and Takahashi \(2008\)](#)]. Other related papers [[McGee and Yang \(2009\)](#), [Hori \(2009\)](#)] consider the case in which experts have independent private signals each of which constitutes one aspect of the unidimensional state variable.

The second communication structure we analyze is the information sharing between two agents who are privately informed about only one dimension of the state. After sending messages about their private information, each agent takes an action which determines the welfare of both agents. Throughout this paper, this mechanism will be referred to as *horizontal communication*. Similar to the vertical communication case, the literature on communication in multi-unit organizations has also focused on this type of communication and decision making structures [[Alonso et al. \(2008b\)](#), [Rantakari \(2008a\)](#)]. There has been attempts to understand when credible communication about intended actions can occur [[Aumann \(1990\)](#), [Farrell \(1988\)](#), [Farrell and Rabin \(1996\)](#)], but these have been limited to environments with complete information. There are few papers that analyze the properties of communication [[Baliga and Morris \(2002\)](#), [Baliga and Sjöström \(2004, 2010\)](#), and [Fey et al. \(2007\)](#)]. However, results in this literature are limited to specific games and environments. In contrast, we try to understand the main forces that determine the informativeness of communication in general environments with incomplete information.

In particular, our focus will be on how complementarities between the decision variables and externalities that these decision variables have on the payoffs of the other agents jointly affect the communication outcome. Actions of the agents are said to be *strategic complements* (*substitutes*) if the payoff to increasing own action

is increasing (decreasing) in the level of the other agent's action. On the other hand, *externalities* refer to the positive or negative direct impacts of an agent's action on the payoff of the other agent. We refer to the total impact of an agent's action on other's payoff, which result from complementarities (substitutabilities) and externalities, as *spillovers*. We also assume that own action and own private information satisfy the increasing differences property, which means that the payoff of an agent to increasing her action is higher when her local private information signals that she is a higher type. This assumption, though not strictly necessary, functions as a sorting condition and allows us to obtain clear-cut results. The principal's payoff function, which plays a role in vertical communication, is the sum of the payoff functions of the agents. Hence, the principal cares only about efficiency.

A natural application of the framework offered by vertical and horizontal communication is the analysis of optimal communication and decision making structures in multi-unit organizations. As John Roberts suggests in his excellent book on the study of multidivisional firms, to achieve efficiency in a business, the managers of that business have to find a fit between the strategy, the organization, and the environment.¹ In his view, two of the key elements of the organization are the architecture and the routines, which, along with the other elements, require the managers to devise well defined decision making and communication structures that fit the other elements of the business.² Similarly, Alfred Sloan, the former CEO and the president of General Motors, defines the main responsibility of the central management as deciding on the centralization and decentralization of various divisions.³

Our model is particularly suited to analyze issues of communication and allocation of decision rights in multi-unit organizations due to the following salient features of such organizations. First, information is dispersed within the organization, so our approximation that each unit observes only one aspect of the state of nature is

¹Roberts (2004, p12)

²Roberts (2004, p17). The other elements are the people and the culture, which altogether, form the synonym PARC.

³Sloan et al. (1990, p431)

congruent with the structure of these organizations. For instance, Lew Platt, the former CEO and the president of HP, once remarked that "I wish we knew what we knew at HP", pointing out to the prominence and dispersion of private information within HP.⁴ Second, most of this information is soft in the sense that information held by agents in the hierarchy is not verifiable. This justifies modeling speeches or reports of the agents as cheap talk messages. Moreover, there are numerous studies that point out the pervasiveness of complementarities among decision variables of different units and inter-divisional externalities resulting from the activities of other units.⁵ In addition, following [Hart and Moore \(1990\)](#) and [Grossman and Hart \(1986\)](#), we assume that the decisions that the organization takes are complex, and therefore they are not contractible. Hence, the organization lacks commitment and the only formal authority is the allocation of decision rights. In particular, we consider only two forms of allocating decision rights: decisions are made by the central management (centralization), or they are delegated to the informed parties (decentralization).

We can illustrate how our model applies to multi-unit organizations via a simple example. Consider a firm that has two divisions, each producing a distinct good for a distinct market. It is conceivable that, due to economies of scale (diseconomies of scale), increasing the production of one product may decrease (increase) the costs of producing more of the other product, which implies complementarity (substitutability) between the quantities produced of these products. Moreover, these two products may be complements or substitutes for the consumers, which is another source of complementarity or substitutability. It is also possible that these two products use the same production or storage facilities. Thus a higher quantity produced by one division may impose costs on the other. This may affect the profits of the other department in the margin, thus leading to substitutabilities, or there may be costs that are proportional to the quantity produced by the other division which implies negative externalities. Also, the headquarters may be allocating capital for

⁴[O'Dell and Grayson \(1998\)](#)

⁵See for instance [Roberts \(2004\)](#), [Milgrom and Roberts \(1990b, 1995\)](#), and [Vives \(1990, 2005\)](#)

the next periods based on the production volumes of these divisions which is another source of negative externality. On the other hand, the compensation schemes may imply positive externalities if headquarters use a wage scheme such that one division's payoff includes a component that is proportional to the volume of production by another division. Another sort of spillover is the culture within the organization. This may be through sharing of know-how, or through the divisions' perception of the goals of the organization as common goals (or through envy towards other divisions). Besides, each division might hold unverifiable private information regarding the markets they operate. For example, they may observe the tastes of the consumers better than the headquarters or the other division. There may also be private information about the cost structures or productivity of a division. Hence, one of the tasks that the headquarters face is to determine whether to employ decentralized communication and decision making, or to centralize production decisions and ask the division managers to report their private information.

Our model can be applied to various other problems. For instance, [Baliga and Morris \(2002\)](#) analyze an example in which two distinct firms engage in cheap talk communication before choosing their technology adoption levels. Each firm is privately informed about its cost of adoption and there are strategic complementarities and positive externalities between their choices. Another example is suggested by [Gal-Or \(1986\)](#) who analyze the incentives of two duopolists to share information about their private costs. Among other problems that can be analyzed in our setting are bank-runs, political alliances, trade associations, merger behavior, joint ventures, and trade agreements.

In this paper, we assume a specific formulation of complementarities and externalities. More precisely, we assume that agent i 's payoff function is $\theta_i a_i - \alpha a_i^2 + \beta a_i a_j + \gamma a_j$, where a_i and a_j are the actions of agent i and j , and θ_i is the private information of agent i . We assume that the principal's payoff function is the sum of the payoff functions of agents 1 and 2. In horizontal communication protocol, agents simultaneously send messages to each other and then independently choose their

actions. In vertical communication protocol, agents simultaneously send messages to the principal and then the principal chooses both actions. In both models, we use the Perfect Bayesian Equilibrium (Fudenberg and Tirole (1991)) as our solution concept and focus on the most informative one when there are multiple equilibrium outcomes. Our main results can be summarized as follows:

VERTICAL COMMUNICATION

Under this protocol, communication from the agents takes the form of a partition equilibrium, i.e., the state space is partitioned into intervals and agents report which interval their private information belongs to.⁶ The structure of the partition depends on how the spillovers are aligned. For instance, when there are spillovers in the form of complementarities and positive externalities ($\beta > 0$, $\gamma > 0$) the principal would like to internalize them. This induces him to choose actions that are higher than the individual best responses of the agents. Since there are increasing differences between the types and actions of each agent, the principal would like to choose higher actions for higher types. Therefore, agent 1 for instance, faces the following trade-off: her payoff is increasing in the action taken on behalf of agent 2, and the principal would like to choose higher actions for agent 2 only if he finds it optimal to choose higher actions for agent 1. This is only possible by making the principal believe that agent 1 has a higher type. On the other hand, if the principal believes that agent 1 has a higher type, then he takes an action on behalf of agent 1 that is higher than the one that is optimal from agent 1's perspective. This turns out to reduce the welfare of agent 1 more than the gain brought about by increasing agent 2's action. As a result, agents have an incentive to pretend that their types are lower. We further show that information revelation is possible only non-generically.

HORIZONTAL COMMUNICATION

⁶This is similar in nature to the results obtained in previous literature. See, for instance, Crawford and Sobel (1982), Rantakari (2008a), Alonso et al. (2008b), and Harris and Raviv (2005).

Under this protocol, full revelation of the private information occur only if there is no strategic interaction between the agents, i.e., the complementarity parameter, β , is equal to zero. However, this is the case only because the best response of an agent is independent of the action of the other agent. Hence, agents do not care about the private information of other agents and communication has no consequences on the outcome of the game.

When the complementarity parameter is not zero, agents can credibly convey limited information to each other. Basically, agents can only signal whether their type is lower or higher than a threshold value that is determined by the spillover parameters of the model. The reason behind partitioning into two is that depending on the parameter configuration, each type of each agent faces either negative or positive spillovers through the actions of the other agents. Therefore, the agents who face negative spillovers pool together to make one of the partition elements, whereas the others form the other partition element. In particular, when there are strategic complementarities and positive externalities, informative communication is impossible because each type of each agent faces positive spillovers and they would like the other agent to choose the highest possible action. As a result, in equilibrium, the agents cannot receive any information from the messages of the other agents.

EFFICIENCY

We provide a mapping between the spillover parameters of the model and the optimal organizational design. First, when there are spillovers in the form of strategic complementarities and positive externalities, as spillovers decrease, the relative performance of decentralization to centralization increases. Moreover, if centralization is informative enough, then decentralization is optimal. The intuition behind these results is that lower spillovers decrease the need for coordination. On the other hand, the need for adaptation is still important. Meanwhile, the adaptation gain in decentralization due to increased communication (as a result of lower spillovers) is not sufficient for the optimality of centralization. With a similar line of reasoning,

when there are strategic substitutabilities and negative externalities, i.e., $\beta < 0$ and $\gamma < 0$, as spillovers decrease in absolute value, decentralization performs better.

For the rest of the parameters, i.e., $\beta < 0$ and $\gamma > 0$, or $\beta > 0$ and $\gamma < 0$, we provide numerical simulations due to analytical complexity. In these regions, the importance of both horizontal and vertical communication in the determination of the optimal regime is more substantial compared to the first two cases.

This paper is organized as follows. Chapter 2 summarizes the related literature in detail. In Chapter 3, we present the model. In Chapters 4 and 5, we analyze vertical communication and horizontal communication respectively. Chapter 6 provides an analysis of the optimal regime in a multi-unit organization, and lastly in Chapter 7 we conclude the paper with possible extensions of the current work.

Chapter 2

Literature Review

Our study is related to various strands of the literature, which we classify and review in this chapter.

2.1 Literature on Experts

The vertical communication structure can be thought of as a situation in which the principal consults two different experts about different dimensions of uncertainty before taking a decision. Therefore, our study is related to a number of papers in the literature on experts.

As we discussed in the introduction, [Crawford and Sobel \(1982\)](#), in a general framework, analyze the quality of communication when an uninformed principal consults to an informed but biased agent. They show that equilibrium always involves noisy information transmission in which the principal can only infer to which equilibrium partition element the agent's private information belongs to. [Spector \(2000\)](#) complements this paper by showing that as the bias of the agent tends to vanish, full revelation of information is possible. [Ottaviani \(2000\)](#), using a uniform-quadratic specification, considers possible extensions of the [Crawford and Sobel \(1982\)](#) model,

such as allowing for naive receivers or senders, or assuming that the expert has a noisy signal of the true state. He also considers cases where the bias of the agent is random, and that monetary transfers are possible conditional on the reported messages. [Morgan and Stocken \(2003\)](#) elaborates on the possibility of the expert's bias to be random in the context of stock recommendation reports by financial analysts, whereas [Krishna and Morgan \(2008\)](#) study the case of monetary transfers in detail. Besides, [Ottaviani and Squintani \(2006\)](#), [Kartik et al. \(2007\)](#), and [Chen \(2010\)](#) analyze the case of less strategically sophisticated receivers or senders in more detail.

The papers cited above are only interested in the case of single senders, whereas we study information transmission by multiple experts. There are other papers that study multiple experts. [Krishna and Morgan \(2001a\)](#) present an expertise model where two experts with different biases who observe the same unidimensional state of nature are sequentially consulted. They show that consulting two experts is beneficial only when experts are biased in opposite directions. They also show that full revelation is not possible in their setting. [Battaglini \(2002\)](#) extends the multiple expert model to a multidimensional setting. In his model, each expert observes the multidimensional state of nature perfectly and the principal takes a multidimensional action following simultaneous reports by the agents. He shows that full revelation of information is possible under quite general conditions. [Ambrus and Takahashi \(2008\)](#) modify this model by allowing the state space to be closed subsets of the Euclidean space and provide conditions under which full revelation is possible. [Battaglini \(2004\)](#) considers a similar model where experts receive noisy signals of the true state. He finds that, unlike the previous models, full revelation of information is not possible. [Battaglini \(2004\)](#) also considers delegation as an alternative to communication and finds that delegating decision making authority is never optimal. Our paper differs from these papers in two respects. First, these papers assume that the experts either perfectly or imperfectly observe all of the dimensions of the uncertainty, whereas we assume that each expert observes only one dimension of the uncertainty. Second, these papers assume that the bias of

the experts is exogenously determined while we endogenize the bias so that it is determined by the underlying spillover parameters of the model and actions of the other agents.

2.2 Pre-play Communication in Incomplete Information Games

The horizontal communication structure can be thought of as a game of two-sided incomplete information with pre-play communication.

The role of pre-play communication in complete information games has been previously studied in the literature. [Farrell and Rabin \(1996\)](#) analyze the role of cheap talk in normal form games using examples. They illustrate when adding cheap talk will (or will not) alter the equilibria, and when it might allow the players to coordinate on efficient outcomes.¹

Closer to our model is [Baliga and Morris \(2002\)](#). In 2-person, finite type, finite action games where one player has incomplete information, they characterize conditions for truthful revelation by the informed player. In particular, they show that when there are strategic complementarities and positive externalities, there is no information transmission at the cheap talk stage. A similar result can be obtained when our model is modified such that only one player has incomplete information in the game.² They suggest an example such that if both sides have private information and the game has strategic complementarities and positive spillovers, there are equilibria that are non-monotonic in types, where lowest and highest types send the same message whereas medium types send a different message. In contrast, our research shows that strategic complementarities and positive externalities rule out informative communication. As will be discussed in more detail in chapter 5, the difference arises because in [Baliga and Morris \(2002\)](#), action spaces are finite.

¹See also [Aumann \(1990\)](#).

²A detailed proof of this, in a general environment, may be obtained from the author.

Baliga and Sjöström (2004) generalize the two-sided incomplete information example given in Baliga and Morris (2002) to a continuum of types. They analyze communication in an arms-races game where each country can either build or not build arms. We can choose the parameters and restrict the action spaces in our model such that the two models are equivalent, and this reveals that there are strategic complementarities and negative externalities in this game. They show that, absent communication, an arms-race takes place with probability 1, whereas adding a communication stage results in an equilibrium similar to the one in Baliga and Morris (2002), with a significantly smaller probability of an arms race. On the other hand, in our model there is at most two-interval equilibrium in the presence of strategic complementarities and negative externalities. Again, this difference is due to the restrictions placed on the action space in Baliga and Sjöström (2004). In a related work, Baliga and Sjöström (2010) introduce another player, the extremist, into the above game. This player is able to observe one player's private information and send public messages about that private information. They show that, even though the extremist is a dominant strategy type, his messages may be able to transmit some information regarding the player that he can observe.

Fey et al. (2007) present a model similar to ours in that agents' actions exhibit strategic complementarities or substitutabilities. In contrast to ours, there is an equilibrium with full revelation of information in their model. The difference results from their specification of the payoff functions of the agents. In their model, each player has an incentive to truthfully reveal their private information because they obtain the best possible payoff if they respond to the local information jointly. In contrast, in our model, there are almost always incentives to manipulate information. Also, they show that imposing bounds on the available actions may reduce the informativeness of communication. In a similar vein, we show that when actions are constrained to be positive, then strategic substitutabilities and negative externalities prevent information transmission, whereas when actions are unconstrained, informative communication is possible.

2.3 Communication and Decision Making in Organizations

One of the major objectives of our study is to apply our main results to the analysis of optimal organization designs. There is, in fact, a growing literature in organizational economics that is concerned with the quality of information exchange and decision making in organizations.

[Dessein \(2002\)](#) considers a setup similar to the one in [Crawford and Sobel \(1982\)](#) and analyzes the efficiency consequences of choosing delegation versus centralization of decision rights. He shows that delegation is preferred to centralization if and only if the divergence of the preferences between the principal and the agent is small. We obtain a similar result in the cases of complementarities and positive externalities, and substitutabilities and negative externalities. On the other hand, in our model, the basic tradeoff is between adaptation and coordination whereas in Dessein's it is between adaptation and control over the actions. [Ivanov \(2010\)](#), in a complementary paper to [Dessein \(2002\)](#), shows that the ability of the principal to manipulate the information structure of the agent limits the value of delegation. Also, [Alonso and Matouschek \(2008\)](#) compare communication in [Dessein \(2002\)](#) to an alternative governance structure in which the principal delegates authority but she also has the ability to constrain the action set from which the agent chooses. In a related work, [Krishna and Morgan \(2001b\)](#) analyze efficiency of different legislative rules.

There are some papers that analyze communication and governance structures in organizations where the agents are partially informed about the state of nature. [Harris and Raviv \(2005\)](#) consider a model in which the unbiased CEO and the biased division manager has both private information. The underlying state corresponds to the sum of the private information that these agents hold. They find that the CEO would like to delegate authority and communicate his information to the division manager if and only if the importance of division manager's information relative to the CEO's private information is sufficiently high. [McGee and Yang](#)

(2009) consider a model with one uninformed principal and two partially informed and biased agents. The real state is a mapping from the agents' information to a univariate ideal action of the principle. They analyze how this mapping affects the level of information transmitted under centralization and decentralization, and which regime is optimal for the principal. Hori (2009) considers a more general mapping from the private information of agents' information to the real state of the world and analyze various communication and decision making structures. The crucial difference between our study and these papers is how information structure affects the payoffs. Unlike these papers which use a mapping to a univariate state of nature, we are interested in the case of a multidimensional state of nature. Moreover, there is only one action to be taken in these models. This restricts the set of decision making structures. For instance, the decentralization framework that we offer requires at least two distinct actions to be taken.

Alonso et al. (2008a) investigate the efficiency consequences of centralization versus decentralization for a firm that sells a single product in different markets whose local characteristics known only to the division manager in that market. When the firm is constrained to choose a single price, decentralization is optimal when the departments face sufficiently different environments. On the other hand, Alonso (2008) characterizes the optimal allocation of decision rights in a firm which needs to take a two-dimensional decision in an environment with unidimensional uncertainty. The basic result of this paper is that when actions are substitutes, full delegation is optimal, and when there are complementarities, the principal acquires control of one of the activities.

Two closely related papers are Alonso et al. (2008b) and Rantakari (2008a). In these papers, a multi-divisional firm needs to resolve the conflict between the need for coordination among the divisions and the need for adaptation to the local conditions by each division. Alonso et al. (2008b) show that increased need for coordination may favor decentralization since communication between division managers is more informative in such a situation. They also show that vertical communication is

more informative than horizontal communication. [Rantakari \(2008a\)](#) extends the analysis to the case where divisions can be asymmetric in size and investigates other regimes such as allocating both decision rights to a particular division. Our paper departs from the analysis of these papers in various aspects. First, these papers regard coordination as an exogenous requirement, but in our paper, the need for coordination arises endogenously from the interactions within the firm. Second, we also consider substitutabilities between the choice variables whereas these papers only look at a specific form of complementarities. [Rantakari \(2008b\)](#) extends the above framework by analyzing optimal governance structures within a firm while treating the importance of coordination and incentive contracts as choice variables. In his setting, decision makers can acquire information at a cost and if the local information of managers is sufficiently important, then decentralization is optimal.

Recently, [Alonso et al. \(2009\)](#) independently analyzed a model that is similar to ours in some respects. They are mainly concerned with how competition faced by a multi-divisional firm affects its choice to decentralize. In their model, they use a slightly different payoff formulation. For instance, when division i 's profits are P^i , then that division's payoff function is $P^i + \lambda P^j$ where $\lambda \in [0, 1/2]$. In this setting, since $\lambda \in [0, 1/2]$, it is not possible to consider the role of negative externalities. Besides, in this setup, for $i = 1, 2$, P^i has either strategic complementarities or substitutabilities. They find that decentralized communication results in an equilibrium that is similar to [Crawford and Sobel \(1982\)](#), whereas we find that at most two partition equilibrium exists. Moreover, their communication results suggest that vertical communication is always more informative than horizontal communication, but our results show that it is possible to have more informative horizontal communication than vertical communication.

2.4 Other Literature

The second stage (after communication) game in our horizontal communication setting is a supermodular game.³ [Van Zandt and Vives \(2007\)](#) investigate equilibria of Bayesian games of strategic complementarities and show that, in equilibrium, higher actions are chosen by both players if there is an increase in beliefs regarding the other players' types (with respect to first order stochastic dominance). As we will show in chapter 5, there is a correspondence between this result and our finding that positive spillovers and strategic complementarities rule out informative horizontal communication.

Another related line of research is interested in identifying complementarities among organizational choice variables. For instance [Roberts \(2004\)](#) provides a comprehensive analysis of how a firm must adjust its strategy and organization to the complementarities within. [Milgrom and Roberts \(1990b, 1995\)](#), apply the theory of supermodularity to show how the design parameters in an organization may respond together to changes in the environment. We contribute to this strand of literature by introducing issues of information and communication, and the implications for optimal governance.

Another related literature is on communication networks. [Calvo-Armengol and de Martí \(2007\)](#) and [Calvo-Armengol and Beltran \(2009\)](#) are concerned about the role of communication in facilitating coordination where each agent observes a noisy signal of the underlying state. They show that the organization of the communication network is the key to achieve efficiency. While these papers assume non-conflicting objectives among agents and emphasize the role of the network structure in enhancing coordination, [Hagenbach and Koessler \(2010\)](#) provide a strategic communication approach to the formation of networks. Agents observe part of a binary state⁴ and decide whether to communicate with other agents endogenously.

³See [Topkis \(1979\)](#), [Vives \(1990\)](#), and [Milgrom and Roberts \(1990a\)](#) for more on supermodular games.

⁴They use a model similar to that of [McGee and Yang \(2009\)](#).

Also, [Calvó-Armengol et al. \(2009\)](#) consider a model of multi-agent communication and decision making structure where each agent, as in our case, observes a different dimension of the underlying state. They characterize the communication structure when it is costly to send and receive signals, hence, unlike in our model, the communication structure does not take the form of cheap talk.

Chapter 3

The Model

In our environment, there are three players: agent 1 and 2, and a principal.¹ The payoff function of agent $i \in \{1, 2\}$ is given by

$$U^i(\theta_i, a_i, a_j) = \theta_i a_i - \alpha a_i^2 + \beta a_i a_j + \gamma a_j, \quad (3.1)$$

where $j \neq i$, a_i is the action of agent i and θ_i is his private information. The principal's payoff function is simply the sum of the payoffs of both agents, i.e.,

$$U^P(\theta_1, \theta_2, a_1, a_2) = U^1(\theta_1, a_1, a_2) + U^2(\theta_2, a_1, a_2). \quad (3.2)$$

We assume that $a_i \in \mathbb{R}$, $i = 1, 2$, and θ_1 and θ_2 are drawn independently from two uniform distributions supported on $[0, 1]$ interval. We will investigate the problem under the restriction that $\alpha > 0$. Also, we assume that $|\alpha| > |\beta|$, so that the principal's payoff function is strictly concave in a_1 and a_2 .

At this point, it should be remarked that the payoff functions have a few nice properties. First, the payoff function of agent i is strictly concave in a_i , so that given θ_i, a_j , it has a unique maximum in a_i . Second, U^i has increasing differences in

¹Each agent will be referred to as she, and the principal will be referred to as he.

(θ_i, a_i) , i.e., $U_{12}^i > 0$, which implies that given a_j , the maximizer a_i is an increasing function of θ_i . Similarly, when $\beta > 0$ (< 0), U^i has strategic complementarities (substitutabilities) in (a_i, a_j) , and the maximizer a_i is an increasing (decreasing) function of a_j . In the payoff functions, γ represent the externalities. If $\gamma > 0$, there are positive externalities, whereas when $\gamma < 0$, there are negative externalities.²

In this environment, we study two different games. First, we will analyze the vertical communication game in which the agents simultaneously send costless messages to the principal, as in the standard cheap talk framework, and the principal chooses both a_1 and a_2 . Second, we study the horizontal communication game in which agents simultaneously send messages to each other, after which agent i chooses a_i . We also investigate the problem under autarchy, where agents cannot communicate and each agent chooses on her own action.

²Our definition of externalities is a particular specification for externalities and there may be other formulations of externalities. For instance, [Alonso et al. \(2009\)](#) use an externality parameter λ such that agent i 's payoff function is the sum of her profit and λ times the profit of agent j .

Chapter 4

Vertical Communication

The vertical communication game is composed of three stages. First, nature independently chooses θ_1 and θ_2 . Then, agent i observes her private information θ_i and chooses m_i from a set of feasible signals $\mathcal{M}_i = [0, 1]$. Lastly, the principal observes (m_1, m_2) and chooses $(a_1, a_2) \in \mathbb{R}^2$. We denote the strategy of agent i as $\mu_i : \Theta_i \rightarrow \mathcal{M}_i$ and the strategy of the principal as $y : \mathcal{M}_1 \times \mathcal{M}_2 \rightarrow \mathbb{R}^2$. An assessment is given by (μ_1, μ_2, y, P) , where $P(\cdot | m_1, m_2)$ is the density of the principal's beliefs conditional on (m_1, m_2) .

We use Perfect Bayesian Equilibrium ([Fudenberg and Tirole \(1991\)](#)) as the solution concept. The following conditions characterize an equilibrium of the game:

- The principal's beliefs over (θ_1, θ_2) conditional on observing (m_1, m_2) are formed using Bayes' rule whenever possible.
- Given $P(\cdot | m_1, m_2)$, the principal chooses (a_1, a_2) to maximize $E[U^P(\theta_1, \theta_2, a_1, a_2) | m_1, m_2]$, i.e.,

$$y(m_1, m_2) = (a_1, a_2) = \arg \max_{\bar{a}_1, \bar{a}_2} \int_{(\theta_1, \theta_2) \in (\Theta_1 \times \Theta_2)} U^P(\theta_1, \theta_2, \bar{a}_1, \bar{a}_2) dP(\theta_1, \theta_2 | m_1, m_2). \quad (4.1)$$

- Given y and μ_j , agent i chooses m_i to maximize $E[U^i(\theta_i, a_1, a_2)]$, i.e.,

$$\mu_i(\theta_i) = m_i = \arg \max_{\bar{m}_i \in \mathcal{M}_i} \int_0^1 U^i(\theta_i, y(\bar{m}_i, \mu_j(\theta_j))) d\theta_j. \quad (4.2)$$

Notice that the optimal strategy of the principal depends only on the expectations of θ_1, θ_2 conditional on the received messages. More precisely, when the message profile is (m_1, m_2) the principal faces the following problem:

$$\max_{(a_1, a_2)} \int_{(\theta_1, \theta_2) \in (\Theta_1 \times \Theta_2)} [\theta_1 a_1 + \theta_2 a_2 + 2\beta a_1 a_2 - \alpha(a_1^2 + a_2^2) + \gamma(a_1 + a_2)] dP(\theta_1, \theta_2 | m_1, m_2), \quad (4.3)$$

or more compactly,

$$\max_{(a_1, a_2)} E[\theta_1 a_1 + \theta_2 a_2 + 2\beta a_1 a_2 - \alpha(a_1^2 + a_2^2) + \gamma(a_1 + a_2) | m_1, m_2]. \quad (4.4)$$

Since the messages are chosen independently, the problem reduces to:

$$\max_{(a_1, a_2)} a_1 E[\theta_1 | m_1] + a_2 E[\theta_2 | m_2] + 2\beta a_1 a_2 - \alpha(a_1^2 + a_2^2) + \gamma(a_1 + a_2) \quad (4.5)$$

which is solved as:

$$a_i(E[\theta_1 | m_1], E[\theta_2 | m_2]) = \frac{2\alpha E[\theta_i | m_i] + 2\beta E[\theta_j | m_j] + 2\gamma(\alpha + \beta)}{4\alpha^2 - 4\beta^2} \quad (4.6)$$

Therefore, from now on we will use

$$y(m_1, m_2) = (a_1(E[\theta_1 | m_1], E[\theta_2 | m_2]), a_2(E[\theta_1 | m_1], E[\theta_2 | m_2])) \quad (4.7)$$

interchangeably whenever the meaning is clear. Note that we are abusing the notation as we are implicitly assuming that $E[\theta_i | m_i, m_j] = E[\theta_i | m_i]$, but since the messages are chosen independently, this assumption is justified.

Lemma 4.1 shows that, in equilibrium, the set of the messages chosen with positive probability by some agent type θ_i is finite. Before proceeding further, let us

introduce some terminology.

Definition 4.1. Let (μ_1, μ_2, y, P) be a Perfect Bayesian Equilibrium. An action profile (a_1, a_2) is on-the-equilibrium path if there exists a type profile (θ_1, θ_2) who chooses (m_1, m_2) and $y(m_1, m_2) = (a_1, a_2)$. More formally, (a_1, a_2) is on-the-equilibrium path if there exists θ_1, θ_2 such that $(a_1, a_2) = y(\mu_1(\theta_1), \mu_2(\theta_2))$. Similarly, we say that $t_i \in [0, 1]$, $i = 1, 2$, is an on-the-equilibrium path expectation if there exists a type θ'_i such that $t_i = E[\theta_i | \mu_i(\theta'_i)]$.

Lemma 4.1. The number of on-the-equilibrium path action profiles is finite if

$$\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} \neq 0.$$

Proof. We will show that the number of on-the-equilibrium path expectations is finite, which implies the claim from equation 4.6. Without loss of generality, suppose that there exists an interval $[\underline{\theta}_1, \bar{\theta}_1]$ such that for any $\theta \in [\underline{\theta}_1, \bar{\theta}_1]$, there exists a corresponding message m_1 which is played by some type of agent 1 and $E[\theta_1 | m_1] = \theta$. Let $\theta \in (\underline{\theta}_1, \bar{\theta}_1)$ and $\varepsilon \in [\underline{\theta}_1 - \theta, \bar{\theta}_1 - \theta]$. The expected payoff of agent 1 of type θ to sending a message that would induce the conditional expectation equal to θ is given by

$$\begin{aligned} & \int_0^1 [\theta a_1(\theta, E[\theta_2 | \mu_2(\theta'_2)]) - \alpha a_1(\theta, E[\theta_2 | \mu_2(\theta'_2)])]^2 \\ & + \beta a_1(\theta, E[\theta_2 | \mu_2(\theta'_2)]) a_2(\theta, E[\theta_2 | \mu_2(\theta'_2)]) + \gamma a_2(\theta, E[\theta_2 | \mu_2(\theta'_2)]) d\theta'_2. \end{aligned} \quad (4.8)$$

The expected payoff of this agent to sending a message that would induce an expectation $\theta + \varepsilon$ is

$$\begin{aligned} & \int_0^1 [\theta a_1(\theta + \varepsilon, E[\theta_2 | \mu_2(\theta'_2)]) - \alpha a_1(\theta + \varepsilon, E[\theta_2 | \mu_2(\theta'_2)])]^2 \\ & + \beta a_1(\theta + \varepsilon, E[\theta_2 | \mu_2(\theta'_2)]) a_2(\theta + \varepsilon, E[\theta_2 | \mu_2(\theta'_2)]) + \gamma a_2(\theta + \varepsilon, E[\theta_2 | \mu_2(\theta'_2)]) d\theta'_2. \end{aligned} \quad (4.9)$$

After simplification, we obtain the following difference between the above payoffs:

$$4.9 - 4.8 = -\frac{\alpha}{4\alpha^2 - 4\beta^2} \varepsilon \left[\beta \int_0^1 E[\theta_2 | \mu_2(\theta'_2)] d\theta'_2 + \alpha\varepsilon + \gamma(2\alpha - \beta) \right]. \quad (4.10)$$

Observe that

$$\int_0^1 E[\theta_2 | \mu_2(\theta'_2)] d\theta'_2 = E[\theta_2] = 1/2 \quad (4.11)$$

by the law of total expectations. Thus, combining 4.10 and 4.11, we get:

$$4.9 - 4.8 = -\frac{\alpha^2}{4\alpha^2 - 4\beta^2} \varepsilon \left[\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} + \varepsilon \right]. \quad (4.12)$$

Notice that the first part of the expression in 4.12 is negative. If $\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} > 0$, let $\varepsilon = \max \left[-\frac{\beta + 2\gamma(2\alpha - \beta)}{4\alpha}, \theta_1 - \theta \right]$. If $\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} < 0$, let $\varepsilon = \min \left[-\frac{\beta + 2\gamma(2\alpha - \beta)}{4\alpha}, \bar{\theta}_1 - \theta \right]$. Under this specification, we observe that agent 1 of type θ would strictly prefer to send the message that would induce an expectation of $\theta + \varepsilon$. Therefore, agent 1 of type θ would never send a signal that will result in a conditional expectation of θ .

Second, we will make use of the earlier remark that $U_{12}^1(\theta_1, a_1, a_2) > 0$. There are two cases to consider. First, let $\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} < 0$ with its corresponding deviation $\varepsilon = \min \left[-\frac{\beta + 2\gamma(2\alpha - \beta)}{4\alpha}, \bar{\theta}_1 - \theta \right]$. Let $\theta' > \theta$, m_1 be the signal inducing the expectation θ , m'_1 be the signal inducing the expectation $\theta + \varepsilon$ and m_2 be an arbitrary signal sent by an agent 2 type. For notational simplicity, let $a_1 = a_1(m_1, m_2)$, $a'_1 = a_1(m'_1, m_2)$, $a_2 = a_2(m_1, m_2)$, and $a'_2 = a_2(m'_1, m_2)$. For any m_2 , $a'_1 > a_1$ because the principal's optimal a_1 is increasing in his expectation regarding agent 1's type. Now, observe that:

$$U^1(\theta', a'_1, a'_2) - U^1(\theta', a_1, a_2) = U^1(\theta, a'_1, a'_2) - U^1(\theta, a_1, a_2) + \underbrace{(a'_1 - a_1)(\theta' - \theta)}_{>0} \quad (4.13)$$

Integrating the right hand side of the equation with respect to θ_2 and using equation 4.12, we see that sending a signal that would yield the conditional expectation $\theta + \varepsilon$ will yield a strictly higher payoff than choosing a signal that results in the conditional

expectation θ . Therefore, we can conclude that a signal that yields a conditional expectation θ is never played with positive probability by agent 1 of types $\theta' \geq \theta$ in equilibrium. Therefore we obtain a contradiction that the signal that yields an expectation θ should actually yield an expectation less than θ . This analysis extends to the case where $\frac{\beta+2\gamma(2\alpha-\beta)}{2\alpha} > 0$ and also to the second agent.

Now assume for contradiction that the number of on-the-equilibrium path expectations is infinitely countable. Thus, without loss of generality, for every ε , there exists agent 1 types $\theta'_1 > \theta_1$ such that $\theta'_1 - \theta_1 = \delta \leq \varepsilon$. Consider the case where $\frac{\beta+2\gamma(2\alpha-\beta)}{2\alpha} < 0$ and let $\varepsilon = -\frac{\beta+2\gamma(2\alpha-\beta)}{4\alpha}$. Then for an agent 1 type θ_1 , the difference in payoffs to sending a signal that would yield a conditional expectation θ'_1 and a signal that would yield the conditional expectation θ_1 is:

$$-\frac{\alpha^2}{4\alpha^2 - 4\beta^2} \delta \left[\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} + \delta \right]. \quad (4.14)$$

Since $0 < \delta \leq \varepsilon$, the expression in 4.14 is strictly positive. Therefore, by a similar analysis as above, no agent 1 type $\theta \geq \theta_1$ will prefer to induce θ_1 in equilibrium, resulting in a contradiction. Again, the analysis extends to the case where $\frac{\beta+2\gamma(2\alpha-\beta)}{2\alpha} > 0$ and also to the second agent.

As a result, since the set of on-the-equilibrium path expectations is finite for both types, the actions chosen by the principal in equilibrium, given by 4.6, is also finite.

□

Corollary. If $\frac{\beta+2\gamma(2\alpha-\beta)}{2\alpha} = 0$, then there is an equilibrium in which each agent fully reveals his private information.

Proof. The proof follows directly from Lemma 4.1. Let $\mu_i(\theta_i) = \theta_i$. Then $E[\theta_i | \mu_i(\theta_i)] = \theta_i$ for $i = 1, 2$. According to the calculations in Lemma 4.1, if a type θ_i reports other than θ_i , she would get a strictly worse payoff than reporting θ_i because in expression 4.12 we see that a deviation would result a utility change of $-\frac{\alpha^2 \varepsilon^2}{4\alpha^2 - 4\beta^2} < 0$. □

The following lemma will establish the fact that different types of each agent will optimally form an interval in terms of their strategies in equilibrium:

Lemma 4.2. If $\underline{\theta}_i$ and $\bar{\theta}_i$ prefer to send m_i to the principal, then any $\theta_i^0 \in [\underline{\theta}_i, \bar{\theta}_i]$ also prefers to send m_i .

Proof. Let $E[\theta_i|m_i]$ be the conditional expectation of the principal to type θ_i upon receiving the message m_i . Also, let m_i be another signal such that $E[\theta_i|m'_i] > E[\theta_i|m_i]$. For simplicity, denote $a_1(m_i, m_j) = a_1$, $a_2(m_i, m_j) = a_2$ and $a_1(m'_i, m_j) = a'_1$, $a_2(m'_i, m_j) = a'_2$. Clearly, $a'_1 > a_1$ for every m_j . Notice that for any m_j ,

$$[U_i(\theta_i^0, a_1, a_2) - U_i(\theta_i^0, a'_1, a'_2)] = [U_i(\bar{\theta}_i, a_1, a_2) - U_i(\bar{\theta}_i, a'_1, a'_2)] + \underbrace{(\theta_i^0 - \bar{\theta}_i)(a_1 - a'_1)}_{>0}. \quad (4.15)$$

Integrating the right hand side of equation 4.15 with respect to θ_2 , we see that the expected payoff of type θ_i^0 to choosing m_i is strictly higher than the payoff to choosing m'_i . With a similar argument with roles of $\underline{\theta}_i$ and $\bar{\theta}_i$ switched, we conclude that θ_i^0 will never find it optimal to choose a message that would yield a conditional expectation lower than $E[\theta_i|m_i]$.

□

Since both agents are identical from the perspective of the principal, we will focus on symmetric reporting strategies. Namely, a reporting strategy profile is symmetric if $\theta_1 = \theta_2$ implies $\mu_1(\theta_1) = \mu_2(\theta_2)$. In order to make the notation less cumbersome, we will also focus on the pure reporting strategies. This is without loss of generality because all the other equilibria are economically equivalent to the pure reporting strategy equilibria that we define in proposition 4.1. Moreover, we will not provide a restriction on beliefs for messages that are out-of-equilibrium path because we can support the equilibrium we find in 4.1 with different off-the-equilibrium beliefs.

Proposition 4.1. If $\frac{2\gamma(\beta-2\alpha)-\beta}{\alpha} \neq 0$, then under symmetric reporting strategies, there exists $\bar{N} = N(\frac{2\gamma(\beta-2\alpha)-\beta}{\alpha}) \in \mathbb{N}^+$ such that for every $N \in \{1, 2, \dots, \bar{N}\}$, there exists an equilibrium where $0 = k_0 < k_1 < \dots < k_N = 1$, and

i.

$$\mu_i(\theta) = \begin{cases} \frac{k_n + k_{n+1}}{2}, & \text{if } \theta \in [k_n, k_{n+1}); \\ \frac{k_{N-1} + k_N}{2}, & \text{if } \theta = 1. \end{cases}$$

ii.

$$P\left(\theta_1, \theta_2 \mid \frac{k_n + k_{n+1}}{2}, \frac{k_m + k_{m+1}}{2}\right) = \begin{cases} \frac{1}{(k_{n+1} - k_n)(k_{m+1} - k_m)}, & \text{if } (\theta_1, \theta_2) \in [k_n, k_{n+1}) \times [k_m, k_{m+1}); \\ 0, & \text{otherwise.} \end{cases}$$

iii.

$$\begin{aligned} a_1\left(\frac{k_n + k_{n+1}}{2}, \frac{k_m + k_{m+1}}{2}\right) &= \frac{\alpha(k_n + k_{n+1}) + \beta(k_m + k_{m+1}) + 2\gamma(\alpha + \beta)}{4\alpha^2 - 4\beta^2}, \\ a_2\left(\frac{k_n + k_{n+1}}{2}, \frac{k_m + k_{m+1}}{2}\right) &= \frac{\alpha(k_m + k_{m+1}) + \beta(k_n + k_{n+1}) + 2\gamma(\alpha + \beta)}{4\alpha^2 - 4\beta^2}. \end{aligned}$$

iv.

$$k_{n+1} = 2k_n - k_{n-1} + \frac{2\gamma(\beta - 2\alpha) - \beta}{\alpha} \text{ for every } n = 1, 2, \dots, N - 1.$$

Moreover, \bar{N} is the greatest integer smaller than

$$\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{8}{\left| \frac{2\gamma(\beta-2\alpha)-\beta}{\alpha} \right|}}.$$

Proof. Given lemma 4.1 and lemma 4.2, we necessarily have a partition equilibria, i.e., type space is partitioned into intervals such that types in the same interval report the same message. More precisely, there exists $\{k_n\}_{n=0}^N$ such that $0 = k_1 < k_2 < \dots < k_N = 1$ and if $\theta, \theta' \in (k_n, k_{n+1})$, then $\mu_i(\theta) = \mu_i(\theta')$.

If the reporting strategies of agents are given as in (i), then (ii) and (iii) follow immediately. Therefore, we should determine the partition $\{k_n\}_{n=0}^N$ so that each type of each agent is best responding. Since U^i is continuous in θ_i , for each $k_n = k_1, k_2, \dots, k_{N-1}$, the expected payoff of sending the message $\frac{k_{n-1}+k_n}{2}$ and $\frac{k_{n+1}+k_n}{2}$ should be equal, which implies the following arbitrage condition:

$$E[U^i(k_n, y(\frac{k_{n+1}+k_n}{2}, \mu_j(\theta_j))) - U^i(k_n, y(\frac{k_{n-1}+k_n}{2}, \mu_j(\theta_j)))] = 0. \quad (4.16)$$

Let $a_i(\frac{k_n+k_{n-1}}{2}, \frac{k_m+k_{m-1}}{2}) = a_i(k_n, k_m)$, and observe that for each $n = 1, \dots, N$:

$$\begin{aligned} & \sum_{j=1}^N [k_j - k_{j-1}] [k_n(a_i(k_{n+1}, k_j) - a_i(k_n, k_j)) - \alpha(a_i(k_{n+1}, k_j)^2 - a_i(k_n, k_j)^2) \\ & + \beta(a_i(k_{n+1}, k_j)a_j(k_{n+1}, k_j) - a_i(k_n, k_j)a_j(k_n, k_j)) + \gamma(a_j(k_{n+1}, k_j) - a_j(k_n, k_j))] = 0. \end{aligned} \quad (4.17)$$

Further simplification leads to

$$k_{n+1} = 2k_n - k_{n-1} + \frac{2\gamma(\beta - 2\alpha) - \beta}{\alpha}, \quad \text{for every } n = 1, 2, \dots, N-1. \quad (4.18)$$

Next we will search for the explicit solution for the k_n 's. Assume first that $\frac{2\gamma(\beta-2\alpha)-\beta}{\alpha} > 0$. Given $k_0 = 0$, the linear non-homogenous difference equation in 4.18 has the explicit solution parameterized by k_1 :

$$k_n = k_1 n + \left[\frac{2\gamma(\beta - 2\alpha) - \beta}{2\alpha} \right] (n^2 - n). \quad (4.19)$$

\bar{N} is the greatest integer such that $\left[\frac{2\gamma(\beta-2\alpha)-\beta}{2\alpha} \right] (N^2 - N) < 1$, which is the greatest integer that is smaller than

$$\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{8}{\frac{2\gamma(\beta-2\alpha)-\beta}{\alpha}}}. \quad (4.20)$$

Notice that $\left[\frac{2\gamma(\beta-2\alpha)-\beta}{2\alpha} \right] (N^2 - N)$ is increasing in N , so we have an equilibrium for every $N = 1, 2, \dots, \bar{N}$.

Now consider the case where $\frac{2\gamma(\beta-2\alpha)-\beta}{\alpha} < 0$. Given $k_N = 1$, we can get the explicit formula for k_n parameterized by k_{N-1} as:

$$k_n = \left[1 - N + Nk_{N-1} + \frac{2\gamma(\beta-2\alpha)-\beta}{2\alpha}(N^2 - N) \right] + \left[\frac{2\gamma(\beta-2\alpha)-\beta}{2\alpha} \right] n^2 + \left[1 - k_{N-1} - \frac{2\gamma(\beta-2\alpha)-\beta}{2\alpha}(2N-1) \right] n \quad (4.21)$$

For this case, \bar{N} is the greatest integer such that $\left[1 + \frac{2\gamma(\beta-2\alpha)-\beta}{2\alpha}(N^2 - N) \right] > 0$, which is the greatest integer that is smaller than

$$\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{8}{\frac{2\gamma(\beta-2\alpha)-\beta}{\alpha}}}. \quad (4.22)$$

□

Equilibrium reporting behavior of the agents crucially depends on the marginal benefits to manipulating information. To see this, let us first assume that any expectation, x , can be induced in equilibrium. Let $\lambda \equiv \beta/\alpha$ and $\lambda_D \equiv \beta/(2\alpha)$. Also, let $T_V \equiv 2[\lambda_D + 2\gamma(1 - \lambda_D)]$ ¹ be defined as the communication bias in vertical communication. Then, for agent 1

$$\frac{\partial}{\partial x} E[U^1(x, y(x, \mu_2(\theta_2)))] = C[\theta_1 - x + (\gamma(\lambda_D - 1) - \lambda_D \mu)] \quad (4.23)$$

$$= C\left[\theta_1 - x - \frac{T_V}{4}\right] \quad (4.24)$$

where C is some positive constant that depends on the parameters of the model. As the second derivative of the expression in 4.23 with respect to x is negative, an agent of type θ_1 would like the principal to believe that her type is $\theta_1 - \frac{T_V}{4}$. This confirms our previous result that we would have a fully revealing equilibrium only if $T_V = 0$. Now, if T_V is positive and very high, then each type of each agent would

¹Note that

$$T_V = 2 \left[\frac{\beta + 2\gamma(2\alpha - \beta)}{2\alpha} \right]$$

like the principal to believe that she is the lowest type. As a result, informative vertical communication will not be feasible. On the other hand, when T_V is small, then agents in an interval would not like to exaggerate their information too much, and different types of agents may find it in their interests to distinguish themselves from a type outside that interval.

There are various tradeoffs that an agent faces in communicating her information. For example, when there are strategic complementarities and positive externalities, and hence $T_V > 0$, the agents' incentives are to induce lower expectations regarding their types. In this case equilibrium actions are always positive and hence there are positive spillovers, i.e., the payoff of an agent is increasing in the level of the action chosen for the other player. This gives the agent incentives to masquerade as a higher type because this is the only way she can induce the principal to take higher actions for the other player. However, the principal, in order to internalize the spillovers, chooses an action for this agent that is higher than the action that is optimal from the perspective of the agent. Therefore, the agent would like to manipulate the principal into believing that she is a lower type, because this is the only way that she can induce the principal to take a lower action on her behalf. In equilibrium, the second effect dominates the first one and agents' incentives are towards making the principal believe that they are lower types.

In summary, our results regarding vertical communication show that, in general, it is not possible to have an equilibrium in which agents fully reveal their private information to the principal. The communication bias is independent of the types, and the parameters of the model create this bias endogenously. As a result, the equilibrium takes a similar form to that of [Crawford and Sobel \(1982\)](#), and we obtain partition equilibria in which each type of each agent reports to which partition element that her private information belongs to.

Chapter 5

Horizontal Communication

As a benchmark case, we will first analyze the autarchy game in which agents cannot communicate their private information. Later, we will look at what happens when communication is possible.

5.1 Autarchy

In the autarchy game, the game is played between the agents. In this setting, agent i observes θ_i , but not θ_j . After observing their types, agent i and agent j simultaneously choose a_i and a_j respectively to maximize their utilities. We will adopt the Bayesian equilibrium ([Harsanyi \(1968\)](#)) as the equilibrium concept. Since the game is ex-ante symmetric, we will focus on symmetric strategies.

Proposition 5.1. The symmetric equilibrium of the autarchy game is given by $f : \Theta_i \rightarrow \mathbb{R}$ such that

$$f(\theta_i) = \frac{\theta_i}{2\alpha} + \frac{\beta}{4\alpha(2\alpha - \beta)}. \quad (5.1)$$

where player i of type θ_i plays according to f defined above.

Proof. Let us suppose that agent j plays according to f . Given the strategy of agent j , the objective of agent i of type θ_i is:

$$\max_{a_i} \int_0^1 [\theta_i a_i - \alpha a_i^2 + \beta a_i f(\theta_j) + \gamma f(\theta_j)] d\theta_j. \quad (5.2)$$

Since the objective function is strictly concave in a_i , the first order condition

$$a_i = \frac{\theta_i + \beta \int_0^1 f(\theta_j) d\theta_j}{2\alpha} \quad (5.3)$$

is necessary and sufficient for a maximum. Therefore, in equilibrium

$$f(\theta_i) = \frac{\theta_i + \beta \int_0^1 f(\theta_j) d\theta_j}{2\alpha}. \quad (5.4)$$

This equation is a Fredholm integral equation of the second kind with the simplest degenerate kernel¹ whose solution is given by:

$$f(\theta_i) = \frac{\theta_i}{2\alpha} + \frac{\beta}{4\alpha(2\alpha - \beta)}. \quad (5.5)$$

□

5.2 Horizontal Communication

In the horizontal communication game, after observing their private information, agent i independently and simultaneously sends a message m_i to the other agents, and after observing the messages, independently chooses a_i . We will, as in the vertical communication game, confine our attention to the Perfect Bayesian Equilibria of the game.

¹See [Polyanin and Manzhirov \(1998\)](#).

Remark. Let m_1, m_2 be sent in equilibrium. If θ_i sends m_i , the optimal action, a_i , solves:

$$\max_{\bar{a}_i} E[\theta_1 \bar{a}_i - \alpha \bar{a}_i^2 + \beta \bar{a}_i a_j + \gamma a_j | m_j] \quad (5.6)$$

which is solved at:

$$a_i = \frac{\theta_i}{2\alpha} + \frac{\beta}{2\alpha} E[a_j | m_j]. \quad (5.7)$$

Now let us plug a_j back into 5.7 to get:

$$a_i = \frac{\theta_i}{2\alpha} + \frac{\beta}{2\alpha} E\left[\frac{\theta_j}{2\alpha} + \frac{\beta}{2\alpha} E[a_i | m_i] | m_j\right]. \quad (5.8)$$

Since m_i and m_j are chosen independently, the equation in 5.8 reduces to:

$$a_i = \frac{\theta_i}{2\alpha} + \frac{\beta}{4\alpha^2} E[\theta_j | m_j] + \frac{\beta^2}{4\alpha^2} E[a_i | m_i]. \quad (5.9)$$

Taking expectations of 5.9 conditional on m_i yields:

$$E[a_i | m_i] = \frac{2\alpha}{4\alpha^2 - \beta^2} E[\theta_i | m_i] + \frac{\beta}{4\alpha^2 - \beta^2} E[\theta_j | m_j]. \quad (5.10)$$

Therefore, combining 5.8 and 5.9, we get:

$$a_i = \frac{\theta_i}{2\alpha} + \frac{\beta^2}{2\alpha(4\alpha^2 - \beta^2)} E[\theta_i | m_i] + \frac{\beta}{4\alpha^2 - \beta^2} E[\theta_j | m_j]. \quad (5.11)$$

Since a_i depends only on θ_i, m_1 and m_2 , we will denote the optimal actions by $a_i(\theta_i, m_1, m_2)$ where appropriate.

Having characterized the equilibrium decision rules, we are now ready to determine the equilibrium reporting rules.

Proposition 5.2. If $\beta \neq 0$, there is at most two on-the-equilibrium path conditional expectations for each agent.

Proof. Let θ and $\theta' = \theta + \varepsilon$ be two conditional expectations associated with m_1 and m'_1 on agent 1's signaling strategy. The difference in utility of agent 1 of type θ_1 between sending m'_1 and m_1 is:

$$\underbrace{\frac{\beta^2}{2\alpha(4\alpha^2 - \beta^2)^2}}_{K_1} \cdot \varepsilon \cdot [\theta_1 \underbrace{(4\alpha^2 - \beta^2)}_{K_2} + \frac{\beta^2 \varepsilon}{2} + \beta^2 \theta + \underbrace{\frac{\alpha\beta^2 + 2\alpha\gamma(4\alpha^2 - \beta^2)}{\beta}}_{K_3}] \quad (5.12)$$

or more compactly:

$$K_1 \cdot \varepsilon \cdot [\theta_1 K_2 + \frac{\beta^2 \varepsilon}{2} + \beta^2 \theta + K_3] \quad (5.13)$$

Assume for a contradiction that three on-the-equilibrium path expectations θ , $\theta' = \theta + \varepsilon_1$, and $\theta'' = \theta' + \varepsilon_2$ exist with $\varepsilon_1, \varepsilon_2 > 0$. Since θ' is on-the-equilibrium path, there exists an agent 1 type θ_1^* who prefers θ' to θ , i.e.

$$K_1 \cdot \varepsilon_1 \cdot [\theta_1^* K_2 + \frac{\beta^2 \varepsilon_1}{2} + \beta^2 \theta + K_3] \geq 0. \quad (5.14)$$

Now, for this agent let us find the difference in utility between sending a signal that would yield the conditional expectation $\theta'' = \theta' + \varepsilon_2$ and the signal that yields θ' :

$$K_1 \cdot \varepsilon_2 \cdot [\theta_1^* K_2 + \frac{\beta^2 \varepsilon_2}{2} + \beta^2(\theta + \varepsilon_1) + K_3] \quad (5.15)$$

which can be written as:

$$K_1 \cdot \varepsilon_2 \cdot [\theta_1^* K_2 + \frac{\beta^2 \varepsilon_1}{2} + \beta^2 \theta + K_3] + K_1 \cdot \varepsilon_2 \cdot [\frac{\beta^2(\varepsilon_1 + \varepsilon_2)}{2}] \quad (5.16)$$

The first part of the expression in 5.16 is greater than zero due to the expression in 5.14. Moreover, the second part is strictly greater than zero since $\beta \neq 0$. Therefore the expression in 5.14 is strictly greater than zero. This means that type θ_1^* would strictly prefer to induce the expectation θ'' to θ' . This contradicts with the assumption that θ'' is induced. Therefore, at most two on-the-equilibrium conditional expectations can be induced.

□

Corollary. If $\beta = 0$, there exists a fully revealing equilibrium.

Proof. According to the expression in 5.12, when $\beta = 0$, each type of each agent is indifferent between sending any of the signals. Therefore, the signaling strategy defined by $\mu_i(\theta_i) = \theta_i$ is part of an equilibrium. □

Remark. From 5.12, it is evident that when $\beta \neq 0$, all equilibria will take the form of partition equilibria. Also, it is straightforward to identify the conditions when a two-partition equilibrium exists. Confining our attention to symmetric strategies, this occurs when there exists a type $k \in (0, 1)$ for each agent which is indifferent between sending a signal yielding a conditional expectation $\frac{k}{2}$ and $\frac{1+k}{2}$. So, equating the equation in 5.12 to zero with the restrictions that $\theta_1^* = k$, $\theta = \frac{k}{2}$, $\epsilon = \frac{1}{2}$, we get the following condition:

$$k = -\frac{4\alpha\beta^2 + \beta^3 + 8\alpha\gamma(4\alpha^2 - \beta^2)}{16\alpha^2\beta - 2\beta^3} \in (0, 1). \quad (5.17)$$

Note that since $\lambda_D = \beta/(2\alpha)$, we can rewrite condition 5.17 as

$$k = -\frac{\lambda_D^2(1 + \frac{\lambda_D}{2}) + 2\gamma(1 - \lambda_D^2)}{2\lambda_D(1 - \frac{\lambda_D^2}{2})} \in (0, 1). \quad (5.18)$$

the quality of communication under horizontal communication is given by, $V^D = E[(\theta_i - E[\theta_i|m_i])^2]$, or more explicitly

$$V^D = \frac{1}{12} - \frac{k}{4} + \frac{k^2}{4}. \quad (5.19)$$

When agent 1 can induce any expectation x on the beliefs of other agent, her marginal payoff to lying will be:

$$\begin{aligned} \frac{\partial}{\partial x} E[U^1(x, a_1(x, \mu_2(\theta_2)), a_2(x, \mu_2(\theta_2)))] &= \underbrace{\frac{\lambda_D^4}{(1 - \lambda_D^2)^2}}_{K_1 \geq 0} x + \underbrace{\frac{\lambda_D^2}{2\alpha(1 - \lambda_D^2)}}_{K_2 \geq 0} \theta_1 \\ &+ \underbrace{\frac{\lambda_D^5}{4(1 - \lambda_D^2)^2} + \frac{\lambda_D^3}{4\alpha(1 - \lambda_D^2)} + \frac{\gamma\lambda_D}{2\alpha(1 - \lambda_D^2)}}_{K_3} \end{aligned} \quad (5.20)$$

Notice that, unlike vertical communication, the payoff function is convex in x . This is the main difference between horizontal and vertical communication. In horizontal communication, each type would like to send the message that would yield either the highest, or the lowest expectation. On the other hand, in vertical communication, there is a unique expectation that maximizes the expected payoff of an agent, and the agent does not find it optimal to exaggerate her private information too much. Figure 1 depicts three different possible outcomes associated with different parameter configurations. Depending on the values of K_1 , K_2 , and K_3 , the lines labeled by l_i represent configuration of the parameters where the identity in 5.20 is equal to zero, i.e. $x = -\frac{K_2}{K_1}\theta_1 - \frac{K_3}{K_1}$. Combined with the convexity of $E[U^1(\cdot)]$ in x , this line depicts the expectations that yield the minimum utility for agent 1. In figure 5.1.a l_1 is drawn when K_3 is positive. In this case, for every θ_1 , the marginal utility at every x is positive. This means that every type of agent 1 would like agent 2 to believe that she is the highest type. Therefore, informative communication will not be possible. In figure 5.1.b, K_3/K_1 is negative but high in absolute value. Similarly, each type of agent 1 would like agent 2 to believe that she is the lowest type, which again makes informative communication impossible. On the other hand, in figure 5.1.c, K_3/K_1 is sufficiently small (in absolute value) that the (θ_1, x) space can be partitioned into two. Now, we can see graphically why our assertion in the proof of proposition 5.2 holds. If we have three expectations, say x_1, x_2, x_3 , induced in equilibrium, then none of the types would like to choose the message yielding the expectation x_2 because hitting the highest or lowest expectation is strictly better

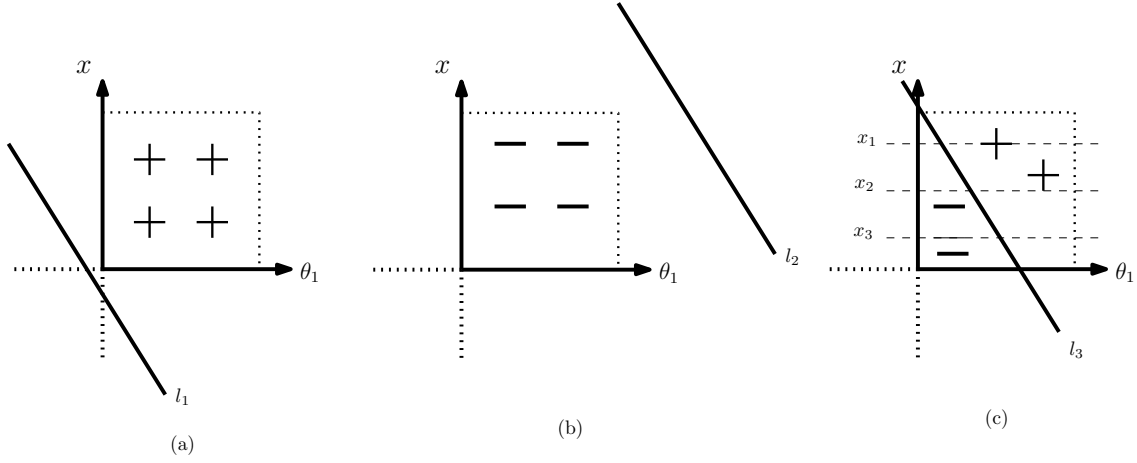


FIGURE 5.1: Informativeness of horizontal communication

compared to choosing x_2 . Therefore, in this case, there will be a cutoff type and below this cutoff type, types will prefer lower expectations (as marginal utility is negative) and above this cutoff type, types will prefer higher expectations. Thus, this gives rise to a two-partition equilibrium.

When there are strategic complementarities and positive externalities, horizontal communication is completely uninformative. One way to see this is that in 5.20, $\lambda_D > 0$ and $\gamma > 0$ implies that K_3 is positive, and thus each type of each agent would like to exaggerate her private information infinitely. Another way is to consider

$$\frac{\partial}{\partial a_j} U_i(\cdot) = \beta a_i + \gamma. \quad (5.21)$$

According to equation 5.11, each a_i is positive when there are strategic complementarities and positive externalities. Thus, agent i would like the other agent to take a high action since her payoff is increasing in the other's action by equation 5.21. The only way that i can increase j 's action is to make her believe that she is high type (see equation 5.11). Therefore, as indicated above, every type has incentive to exaggerate her private information.

This result can also be established using proposition 16 of Van Zandt and Vives (2007). Since in the second stage games that we analyze there is a unique equilibrium, this proposition implies that if the beliefs of players change to a first-order

stochastically dominating belief, then the equilibrium actions of both players will increase. Consider now player 1. Assume that two beliefs about player 1's types can be induced in equilibrium, say $p_H > p_L$.² Let a_L^1 and a_L^2 (a_H^1 and a_H^2) be the equilibrium actions of player 1 and player 2 when beliefs are given by p_L (p_H). Then $U_1(a_H^1, a_H^2) \geq U_1(a_L^1, a_H^2) > U_1(a_L^1, a_L^2)$ where the first inequality follows from the fact that a_H^1 is a best response, whereas the second inequality follows from the existence of positive spillovers. Therefore, each type of agent 1 would like to choose a message that would yield p_H , and hence in equilibrium only one belief can be induced.

By analogy, it is tempting to conclude that horizontal communication is completely uninformative when there are strategic substitutes and negative externalities. However, this is not correct. There is a region where horizontal communication can be informative. To see why, consider equation 5.21. If actions were always positive, then each type would have an incentive to make the other agent believe that she is a lower type. However, actions are not restricted to positive values, and a quick inspection of the decision rules in equation 5.11 shows that negative actions are induced for some parameter configurations. The reason for the informativeness of horizontal communication can also be attributed to K_3 in equation 5.20 being negative for some (λ_D, γ) pairs. Notice that as λ_D gets smaller, we also can make K_3 negative for smaller γ . Therefore, as λ_D gets smaller, the region for the γ levels yielding informative horizontal communication will be wider.

Remark. Fey et al. (2007) show that imposing bounds on the action spaces may prevent transmission of information when actions are strategic substitutes. The above observation, in which we remarked that if actions were non-negative then information transmission would not be possible, is in line with the result of Fey et al. (2007).

²In this case only expectations matter, so first order stochastic dominance is equivalent to saying that expectation of player 1's types when beliefs are p_H is higher than when the beliefs are p_L

Remark. In the literature review, we discussed an example by [Baliga and Morris \(2002\)](#) where there are two agents each of whom has three types and two actions available to them. They find an equilibrium in which high types and low types send the same message whereas the medium type sends a different message. Their equilibrium construction relies on selection from multiple equilibria in the second stage game. Our model can be tailored to match to their example by using only two different actions $a_L = 1, a_H = 2$ and choosing the following parameters: $\alpha = 10, \beta = 5, \gamma = 1$. The distribution over types to obtain such an equilibrium is: $p_M = 0.4, p_L = 0.5, p_H = 0.1$. Similarly, the model presented by [Baliga and Sjöström \(2004\)](#), which has a continuum of types and non-monotonic equilibria, can be represented as a transformation of our model with binary actions as a game of strategic complementarities and negative externalities. We do not have such equilibria in our model because we assume that the action spaces of the agents are unrestricted. Therefore, with strictly concave utilities, it is not possible to have multiple equilibria after the communication stage, and this implies monotonic equilibria.

To sum up, we see that horizontal communication is fully informative only if there is no link in the form of strategic complementarities and substitutabilities between actions of agents. If this link exists, the equilibrium is such that each type of each agent can at most signal that their type is lower or higher than a threshold value. In particular, when there are strategic complementarities and positive externalities, informative horizontal communication is not possible.

Chapter 6

The Optimal Regime

In this chapter, we will investigate the role of complementarities (substitutabilities) and externalities on the optimality of vertical or horizontal communication regimes from the viewpoint of the principal. Since the principal's payoff function is the sum of the agents' payoff functions, our analysis will also allow us to compare these two regimes in terms of efficiency. To achieve this objective, we will first analyze the first best decisions of the principal, i.e., we will look at the problem when the principal is fully informed about the agents' types. Consecutively, we will compare each regime with the first best in terms of principal's payoff. Since there is the possibility of multiple equilibrium, we will only consider the most informative equilibrium.¹

¹We are using the most informative equilibrium in order to observe the full influence of the communication stage on the payoffs. There are various papers that justify the use of the most informative equilibrium. For instance, [Chen et al. \(2008\)](#) propose a refinement of equilibria in games like the one in [Crawford and Sobel \(1982\)](#). They show that only the most informative equilibrium satisfies the *no incentive to seporate* condition, which is a regularity condition about these games.

6.1 Rewriting the Decisions

6.1.1 First Best Decisions

The principal's problem under complete information is given by:

$$\max_{a_1, a_2} \theta_1 a_1 + \theta_2 a_2 + 2\beta a_1 a_2 - \alpha a_1^2 - \alpha a_2^2 + \gamma(a_1 + a_2). \quad (6.1)$$

Let a_i^{FB} denote the action choices of the principal under complete information and $a_i^{FB} = \bar{a}_i^{FB} + \Delta_i^{FB}$ where $\bar{a}_i^{FB} = E_{\theta_1, \theta_2}[a_i^{FB}]$ and Δ_i^{FB} denote average decisions and deviations from the average decisions respectively. Also, let $\mu = 1/2$ denote the expected value of θ_i , $\sigma^2 = 1/12$ denote the variance of θ_i , and $\lambda = \beta/\alpha$ for illustration purposes. As a result, the first order conditions of the above problem imply:

$$a_i^{FB} = \frac{\theta_i + \gamma}{2\alpha} + \lambda a_j^{FB} = \frac{\theta_i + \lambda\theta_j + \gamma(1 + \lambda)}{2\alpha(1 - \lambda^2)} \quad (6.2)$$

$$\bar{a}_i^{FB} = \frac{\mu + \gamma}{2\alpha} + \lambda \bar{a}_j^{FB} = \frac{\mu + \gamma}{2\alpha(1 - \lambda)} \quad (6.3)$$

$$\Delta_i^{FB} = \frac{\theta_i - \mu}{2\alpha} + \lambda \Delta_j^{FB} = \frac{\theta_i - \mu}{2\alpha(1 - \lambda^2)} + \lambda \frac{\theta_j - \mu}{2\alpha(1 - \lambda^2)}. \quad (6.4)$$

Accordingly, the ex-ante payoff of agent i is given by:

$$\begin{aligned} \Pi_i(a_i^{FB}, a_j^{FB}) &= E_{\theta_i, \theta_j}[\theta_i a_i^{FB} - \alpha(a_i^{FB})^2 + \beta a_i^{FB} a_j^{FB} + \gamma a_j^{FB}] \\ &= \underbrace{\mu \bar{a}_i^{FB} - \alpha(\bar{a}_i^{FB})^2 + \beta \bar{a}_i^{FB} \bar{a}_j^{FB} + \gamma \bar{a}_j^{FB}}_{\Pi_i(\bar{a}_i^{FB}, \bar{a}_j^{FB}) = \frac{(\mu + \gamma)^2}{4\alpha(1 - \lambda)}} + \underbrace{E[\theta_i \Delta_i^{FB} - \alpha(\Delta_i^{FB})^2 + \beta \Delta_i^{FB} \Delta_j^{FB}]}_{F_i(\Delta_i^{FB}, \Delta_j^{FB}) = \frac{\sigma^2}{4\alpha(1 - \lambda^2)}} \end{aligned} \quad (6.5)$$

Here $\Pi_i(\bar{a}_i^{FB}, \bar{a}_j^{FB}) = \frac{(\mu + \gamma)^2}{4\alpha(1 - \lambda)}$ represents agent i 's ex-ante rigid payoff due to the average production decisions, whereas $F_i(\Delta_i^{FB}, \Delta_j^{FB}) = \frac{\sigma^2}{4\alpha(1 - \lambda^2)}$ is agent i 's ex-ante

payoff resulting from flexibility. Therefore, the principal's first best expected payoff is

$$\Pi(a_i^{FB}, a_j^{FB}) = \Pi_i(a_i^{FB}, a_j^{FB}) + \Pi_j(a_i^{FB}, a_j^{FB}) \quad (6.6)$$

$$= \frac{(\mu + \gamma)^2}{2\alpha(1 - \lambda)} + \frac{\sigma^2}{2\alpha(1 - \lambda^2)} \quad (6.7)$$

6.1.2 Centralization

We have already analyzed the decisions under centralization and we will reformulate our results to simplify the analysis. Recall that the principal's problem in this setting is:

$$\max_{(a_1, a_2)} E[\theta_1 a_1 + \theta_2 a_2 + 2\beta a_1 a_2 - \alpha(a_1^2 + a_2^2) + \gamma(a_1 + a_2) | m_1, m_2]. \quad (6.8)$$

The actions of the principal under centralization is given by:

$$a_i^C = \frac{E[\theta_i | m_i] + \gamma}{2\alpha} + \lambda a_j^C = \frac{E[\theta_i | m_i] + \lambda E[\theta_j | m_j] + \gamma(1 + \lambda)}{2\alpha(1 - \lambda^2)} \quad (6.9)$$

where we implicitly assume the dependence of a_i^C on the expectations of the private information conditional on the transmitted signals. The corresponding average decisions and deviations from these averages are:

$$\bar{a}_i^C = \frac{\mu + \gamma}{2\alpha} + \lambda \bar{a}_j^C = \frac{\mu + \gamma}{2\alpha(1 - \lambda)}, \quad (6.10)$$

$$\Delta_i^C = \frac{E[\theta_i - \mu | m_i]}{2\alpha} + \lambda \Delta_j^C = \frac{E[\theta_i - \mu | m_i]}{2\alpha(1 - \lambda^2)} + \lambda \frac{E[\theta_j - \mu | m_j]}{2\alpha(1 - \lambda^2)}. \quad (6.11)$$

Let $V^C = E[(\theta_i - E[\theta_i | m_i])^2]$ be the measure of the quality of information flows in centralization. More explicitly, given an equilibrium of vertical communication, the

quality of communication under centralization depends on the number of partitions N , and the conflict of interest T_V via the formula:

$$V^C = \frac{1}{12N^2} + \frac{T_V^2(N^2 - 1)}{48}. \quad (6.12)$$

Then, the ex-ante payoff of agent i is given by:

$$\Pi_i(a_i^C, a_j^C) = \underbrace{\mu \bar{a}_i^C - \alpha(\bar{a}_i^C)^2 + \beta \bar{a}_i^C \bar{a}_j^C + \gamma \bar{a}_j^C}_{\Pi_i(\bar{a}_i^C, \bar{a}_j^C) = \frac{(\mu + \gamma)^2}{4\alpha(1 - \lambda^2)}} + \underbrace{E[E[\theta_i \Delta_i^C - \alpha(\Delta_i^C)^2 + \beta \Delta_i^C \Delta_j^C] | m_i, m_j]}_{F_i(\Delta_i^C, \Delta_j^C) = \frac{\sigma^2}{4\alpha(1 - \lambda^2)} - \frac{V^C}{4\alpha(1 - \lambda^2)}}$$

Therefore, the expected payoff of the principal under centralization is:

$$\Pi(a_i^C, a_j^C) = \Pi_i(a_i^C, a_j^C) + \Pi_j(a_i^C, a_j^C) \quad (6.13)$$

$$= \frac{(\mu + \gamma)^2}{2\alpha(1 - \lambda)} + \frac{\sigma^2}{2\alpha(1 - \lambda^2)} - \frac{V^C}{2\alpha(1 - \lambda^2)} \quad (6.14)$$

The objective functions of the principal under centralization and the first best (6.1 and 6.8) differ only in terms of the information regarding the environments while he can choose any covariance between the actions in both situations. Hence, the principal's strength in aligning a_i^C and a_j^C is identical to the first best while the difference between the decisions under the first best and under centralization lies in the amount of information as observed in the FOCs (6.2 and 6.9) of these two settings. As the information transmission in vertical communication gets better, the decision rules summarized in these FOCs get closer to each other. This confirms the result in 6.14. When V^C gets smaller, the noise in the communication gets smaller and centralization yields payoffs closer to the first best. The loss of the centralized organization with respect to the first best will be referred to as adaptation loss.

6.1.3 Decentralization

Recall that $\lambda_D = \lambda/2 = \beta/2\alpha$ and $V^D = E[(\theta_i - E[\theta_i|m_i])^2]$ is the measure of the quality of information transmission in decentralized communication. Then

$$a_i^D = \frac{\theta_i}{2\alpha} + \lambda_D E[a_j^D|m_i, m_j] = \frac{\theta_i}{2\alpha} + \frac{\lambda_D^2}{2\alpha(1 - \lambda_D^2)} E[\theta_i|m_i] + \frac{\lambda_D}{2\alpha(1 - \lambda_D^2)} E[\theta_j|m_i] \quad (6.15)$$

and the corresponding average decisions and deviations under decentralization is given by:

$$\bar{a}_i^D = \frac{\mu}{2\alpha(1 - \lambda_D)} \quad (6.16)$$

$$\Delta_i^D = \frac{\theta_i - \mu}{2\alpha} + \frac{\lambda_D^2}{2\alpha(1 - \lambda_D^2)} E[\theta_i - \mu|m_i] + \frac{\lambda_D}{2\alpha(1 - \lambda_D^2)} E[\theta_j - \mu|m_i]. \quad (6.17)$$

Then, the principal's payoff due to average decisions is:

$$\Pi(\bar{a}_i^D, \bar{a}_i^D) = \Pi(\bar{a}_i^{FB}, \bar{a}_i^{FB}) - \alpha(1 - 2\lambda_D)[\bar{a}_i^{FB} - \bar{a}_i^D] \quad (6.18)$$

$$= \frac{(\mu + \gamma)^2}{2\alpha(1 - \lambda)} - \frac{(\mu\lambda_D + \gamma(1 - \lambda_D))^2}{2\alpha(1 - 2\lambda_D)(1 - \lambda_D)^2} \quad (6.19)$$

$$= \frac{(\mu + \gamma)^2}{2\alpha(1 - \lambda)} - \frac{T_V^2}{32\alpha(1 - 2\lambda_D)(1 - \lambda_D)^2} \quad (6.20)$$

where, as we remarked earlier, $T_V = 2[\lambda_D + 2\gamma(1 - \lambda_D)]$ is the incentive conflict between the principal and each agent under vertical communication. On the other hand, the principal's expected payoff due to flexibility is:

$$\begin{aligned} F(\Delta_i^D, \Delta_j^D) &= F(\Delta_i^{FB}, \Delta_j^{FB}) - \left[\frac{2\lambda_D^2 V^D}{\alpha(1 - 4\lambda_D^2)} + \lambda_D^2 \frac{(1 + 5\lambda_D^2)(\sigma^2 - V^D)}{2\alpha(1 - \lambda_D^2)^2(1 - 4\lambda_D^2)} \right] \\ &= \frac{\sigma^2}{2\alpha(1 - \lambda^2)} - \left[\frac{2\lambda_D^2 V^D}{\alpha(1 - 4\lambda_D^2)} + \lambda_D^2 \frac{(1 + 5\lambda_D^2)(\sigma^2 - V^D)}{2\alpha(1 - \lambda_D^2)^2(1 - 4\lambda_D^2)} \right] \end{aligned} \quad (6.21)$$

The FOCs 6.2 and 6.15 of first best and decentralization differ in 3 aspects. First, γ does not enter the decisions of agents, meaning that the agents does not take into account the direct externalities that their decisions have on the other agent's payoff function. Second, the interdependency between actions in the first best is

$\lambda = 2\lambda_D$ whereas it is only λ_D in decentralization. This, on the other hand, is due to agents not taking into account the externalities that their actions have on the marginal payoffs of the other. Third, in decentralization, each agent has uncertainty regarding the other agent's type and hence will rely only on an expectation of the other agent's decision. All of these factors stem from the coordination failures of the agents in decentralized organization. Therefore, we will refer to the losses in 6.20 and 6.21 as the coordination loss of decentralization.

On the other hand, there is a gain in responsiveness of the decisions to the local information with respect to centralization. When we compare the FOCs 6.2 and 6.15 again, we observe that the local information enters the decision rules in the same way, as opposed to the noise in centralized FOCs. In other words, under the first best and decentralization, local information is fully employed, whereas under centralization, local information is utilized only to the extent of information transmitted by agents in centralized communication.

6.2 The Optimal Regime

In this section, we will analyze whether centralization or decentralization yields a higher utility for the principal depending on the parameter values. The above analysis allows us to write the payoff difference between decentralization and centralization as:

$$U^D - U^C = \frac{V^C}{2\alpha(1 - \lambda^2)} - \frac{(\mu\lambda_D + \gamma(1 - \lambda_D))^2}{2\alpha(1 - 2\lambda_D)(1 - \lambda_D)^2} - \left[\frac{2\lambda_D^2 V^D}{\alpha(1 - 4\lambda_D^2)} + \lambda_D^2 \frac{(1 + 5\lambda_D^2)(\sigma^2 - V^D)}{2\alpha(1 - \lambda_D^2)^2(1 - 4\lambda_D^2)} \right] \quad (6.22)$$

where U^D and U^C denote the payoffs under decentralization and centralization respectively.

6.2.1 Strategic Complementarities and Positive Externalities

When there are strategic complementarities and positive externalities, i.e., $\lambda_D > 0$, $\gamma > 0$, the horizontal communication is completely uninformative. Therefore $V^D = E[(\theta_i - \mu)^2] = \sigma^2 = 1/12$.

Now, we will investigate the parameter space in order to determine the optimal regime. Initially, let us consider the case where vertical communication is also uninformative. This happens when $T_V \geq 1$ and this corresponds to the region $\gamma \geq \frac{1-2\lambda_D}{4(1-\lambda_D)}$ in the parameter space (γ, λ_D) . Letting U_t^R denote the utility of the principal under regime R with t partitions, in this case the utility difference simplifies to:

$$U_1^D - U_1^C = \frac{1}{32\alpha} \left[\frac{4}{3} - \frac{T_V^2}{(1-2\lambda_D)(1-\lambda_D)^2} \right] \quad (6.23)$$

$$= \frac{1}{8\alpha} \left[\frac{1}{3} - \frac{(\lambda_D + 2\gamma(1-\lambda_D))^2}{(1-2\lambda_D)(1-\lambda_D)^2} \right]. \quad (6.24)$$

Thus, when the maximum number of partitions in vertical communication is 1, decentralization performs better when

$$\gamma < \sqrt{\frac{1-2\lambda_D}{12}} - \frac{\lambda_D}{2(1-\lambda_D)} \quad (6.25)$$

and centralization is desirable when the above inequality reverses.

Second, consider the parameter values that correspond to a maximum partition size 2 in vertical communication. This region is given by $T_V \in [\frac{1}{3}, 1)$, i.e.,

$$\frac{1-6\lambda_D}{12(1-\lambda_D)} \leq \gamma < \frac{1-2\lambda_D}{4(1-\lambda_D)}. \quad (6.26)$$

The principal's expected utility difference is:

$$U_1^D - U_2^C = \frac{1}{96\alpha(1 - 4\lambda_D^2)(1 - \lambda_D)^2} \left[(1 - 16\lambda_D^2)(1 - \lambda_D)^2 + 3T_V^2\lambda_D(\lambda_D - 4) \right]. \quad (6.27)$$

Since $\lambda_D \in (0, 0.5)$, the second term in the square brackets in equation 6.27 is always negative. On the other hand, when $\lambda_D \geq 0.25$, the first term is also non-positive, and thus the whole expression is negative. Therefore, if $\lambda_D \geq 0.25$, centralization performs better when the parameters are restricted according to the expression in 6.26. If $\lambda_D < 0.25$, then decentralization yields a higher utility for the principal when

$$\gamma < \sqrt{\frac{(1 - 16\lambda_D^2)}{48\lambda_D(4 - \lambda_D)}} - \frac{\lambda_D}{2(1 - \lambda_D)}. \quad (6.28)$$

We have the following general result when the maximum equilibrium partition number is greater than two:

Lemma 6.1. Let $\beta > 0$ and $\gamma > 0$. If we have an equilibrium consisting of more than two partitions in centralized communication, then decentralization is always optimal for the principal.

Proof. Let the maximum number of equilibrium partition size in centralization, \bar{N} , be greater than 2. Then since \bar{N} is the greatest integer smaller than $\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8}{|T_V|}}$, we have:

$$T_V \in \left[\frac{2}{\bar{N}(\bar{N} + 1)}, \frac{2}{\bar{N}(\bar{N} - 1)} \right), \quad (6.29)$$

and since $T_V = 2[\lambda_D + 2\gamma(1 - \lambda_D)]$ and $\beta > 0, \gamma > 0$,

$$\lambda_D \in \left[\frac{1}{\bar{N}(\bar{N} + 1)}, \frac{1}{\bar{N}(\bar{N} - 1)} \right). \quad (6.30)$$

The principal's expected utility difference when the number of maximum equilibrium partitions is \bar{N} can be written as:

$$U_1^D - U_{\bar{N}}^C = \frac{(1 - (2\bar{N}\lambda_D)^2)}{24\alpha(1 - 4\lambda_D^2)\bar{N}^2} + \frac{T_V^2}{96\alpha(1 - 4\lambda_D^2)(1 - \lambda_D)^2} [(\bar{N}^2 - 1)(1 - \lambda_D^2) - 3(1 + 2\lambda_D)] \quad (6.31)$$

First, let us consider the first term in equation 6.31. By equation 6.30,

$$2\bar{N}\lambda_D \in \left[\frac{2}{(\bar{N} + 1)}, \frac{2}{(\bar{N} - 1)} \right) \quad (6.32)$$

and since $\bar{N} \geq 3$, $2\bar{N}\lambda_D < 1$. Consequently, $(1 - (2\bar{N}\lambda_D)^2) > 0$, and the first term in equation 6.31 is positive. Now, let us look at the last term of equation 6.31. As $\bar{N} \geq 3$, the lowest value that $\bar{N}^2 - 1$ can get is 8. Also, from 6.30, λ_D is always smaller than $\frac{1}{6}$ since $\bar{N} \geq 3$. Thus, $(1 - \lambda_D^2) > \frac{35}{36}$. Similarly, $3(1 + 2\lambda_D) < \frac{4}{3}$. Hence,

$$(\bar{N}^2 - 1)(1 - \lambda_D^2) - 3(1 + 2\lambda_D) > 8\left(\frac{35}{36}\right) - 4 = \frac{54}{9} > 0, \quad (6.33)$$

so the last term of 6.31 is also positive. This proves the claim that $U_1^D - U_{\bar{N}}^C > 0$ when $\bar{N} \geq 3$. \square

We can organize our findings regarding the optimal regime in the following proposition:

Proposition 6.1. Let $\lambda_D > 0$ and $\gamma > 0$. Then, decentralization is optimal if

$$\gamma < \begin{cases} \sqrt{\frac{1-2\lambda_D}{12}} - \frac{\lambda_D}{2(1-\lambda_D)}, & \text{if } \lambda_D \leq 0.069; \\ \sqrt{\frac{(1-16\lambda_D^2)}{48\lambda_D(4-\lambda_D)}} - \frac{\lambda_D}{2(1-\lambda_D)}, & \text{otherwise.} \end{cases} \quad (6.34)$$

Figure 6.1 summarizes our findings regarding the first quadrant of (β, γ) space. The dash-dotted line corresponds to the parameter configuration where $T_V = 1$. Thus, above this line, the number of maximal set of partitions in vertical communication is 1. Below this line and above the $T_V = 1/3$ line (dashed line), the vertical communication has two partitions at maximum. As we go further close to

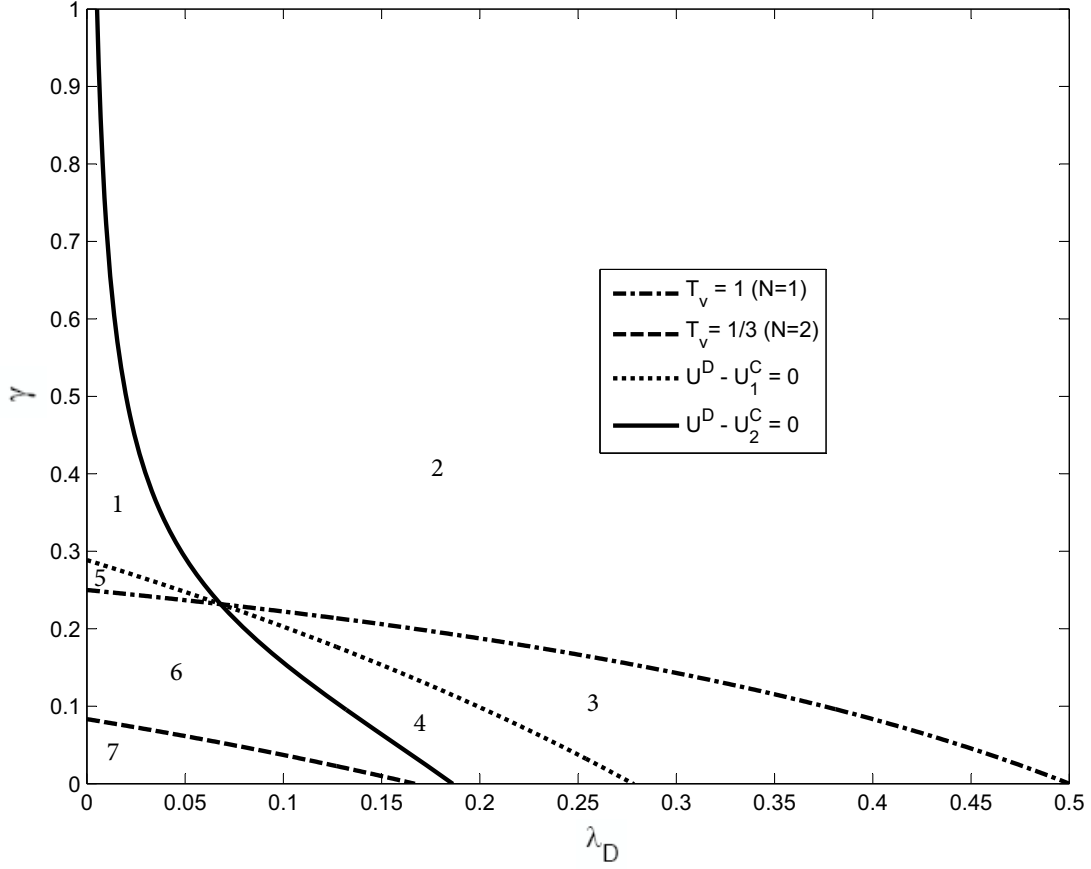


FIGURE 6.1: Analysis of optimality when $\lambda_D > 0$ and $\gamma > 0$
 Centralization is optimal in regions 1, 2, 3, and 4,
 whereas decentralization is optimal in 5, 6, and 7.

the origin, the number of maximal set of partitions gradually increase and in the limit, where $(\beta, \gamma) \rightarrow 0$, vertical communication gets completely informative. These observations are in accordance with our previous remarks. When T_V is high, the incentives to misrepresent information increases, and the equilibrium involves more noisy signaling. As the agents' interests and the principal's interests get closer, more information is transmitted in equilibrium.

In figure 6.1, the dotted line shows the following: If the equilibrium consists of one partition, then the area above this line would constitute the parameter configuration where centralization is better and below decentralization is optimal. Note that this line is meaningful for the whole parameter space because for every parameter configuration, we have an uninformative equilibrium.

Initially, let us concentrate our attention to the space where $\bar{N} = 1$, which corresponds to the union of the regions 1, 2, and 5 in figure 6.1. In regions 1 and 2, centralization performs better while in 5 decentralization is more efficient. To understand this picture, it is helpful to analyze the first order conditions:

$$\begin{aligned} a_i^{FB} &= \frac{\theta_i + \gamma}{2\alpha} + \lambda a_j^{FB} \\ a_i^C &= \frac{E[\theta_i|m_i] + \gamma}{2\alpha} + \lambda a_j^C \\ a_i^D &= \frac{\theta_i}{2\alpha} + \lambda_D E[a_j^D|m_i, m_j]. \end{aligned} \tag{6.35}$$

Notice that when λ and γ are relatively high, as seen in first best decision rules, the need for coordinating decisions are stronger relative to the need for adapting the decisions to the private information of the agents. Therefore, keeping the level of transmitted information constant in both vertical and horizontal communication, as λ and γ get higher centralized decisions yield a better approximation for the actions that the principal would ideally take. This also explains why an increase in either of the parameters can only result in a shift from the optimality of decentralization to the optimality of centralization.

This observation can explain why decentralization is optimal only in the small region 5. In this region, the externalities due to λ and γ are sufficiently low that the need for coordination is not very high. Thus, the importance of the coordination of the actions to the corresponding local information gets relatively low. The adaptation loss of the centralized regime is high, thus decentralization performs better.

Next, we will look at the region where $\bar{N} = 2$ in vertical communication. This region arises as the union of 3, 4, and 6. As we remarked earlier, the dotted line ($U_1^D - U_1^C = 0$) is still meaningful. If we were instead interested in a comparison of decentralization and centralization in terms of efficiency for the uninformative equilibrium in centralization even though $\bar{N} = 2$, then we would have concluded that in 3 centralization is optimal, whereas in 4 and 6, decentralization is optimal. Now, the solid line ($U_1^D - U_2^C = 0$) is the boundary where to the right of this line,

centralization is optimal and to the left decentralization performs better. Therefore, informativeness of vertical communication is the only reason why centralization is optimal in region 4.

The last region is 7. Now, we are in the subset of the parameter space that is of interest to lemma 6.1. Here, as we proved above, decentralization is always optimal. The reasoning is as follows: When we have more partitions in vertical communication, this necessarily means that β and γ are smaller. The rate of the decrease in the adaptation loss of the centralized organization is lower than the rate of the decrease in the coordination loss of the decentralized organization. This divergence arises from the fact that this time not only β and γ decrease, but also equilibrium actions get smaller. Therefore, the need for coordination reduces faster and this gives another edge to decentralization.

6.2.2 Strategic Substitutabilities and Negative Externalities

We will now restrict our analysis to the parameter values where $\beta < 0$, and $\gamma < 0$. A priori, the calculations seem to be more cumbersome due to the possibility of informative horizontal communication. As we will see, this complexity does not have a bite in this region.

The vertical communication structure is quite similar to the analysis of $\lambda_D > 0$ $\gamma > 0$ quadrant. Again as we get closer to the origin, T_V , which is negative now, gets smaller in absolute value. Therefore, the agents' interests gets closer to that of the principal's. Since $T_V < 0$, now each type has an incentive to induce an expectation higher than her type. Thus, the partitions get wider, since higher types send less credible messages.

In order to compare the two regimes, first, let us consider the region where vertical communication is uninformative, i.e., $V^C = \sigma^2$. This region is defined by:

$$\gamma \leq -\frac{1 + 2\lambda_D}{4(1 - \lambda_D)}. \quad (6.36)$$

Moreover, initially assume that horizontal communication is also uninformative. Then, the utility difference between decentralization and centralization is given by:

$$U_1^D - U_1^C = \frac{1}{32\alpha} \left[\frac{4}{3} - \frac{T_V^2}{(1 - 2\lambda_D)(1 - \lambda_D)^2} \right] \quad (6.37)$$

$$= \frac{1}{8\alpha} \left[\frac{1}{3} - \frac{(\lambda_D + 2\gamma(1 - \lambda_D))^2}{(1 - 2\lambda_D)(1 - \lambda_D)^2} \right]. \quad (6.38)$$

Therefore, when both vertical and horizontal communication are uninformative, decentralization is optimal if

$$\gamma > -\sqrt{\frac{1 - 2\lambda_D}{12}} - \frac{\lambda_D}{2(1 - \lambda_D)} \quad (6.39)$$

However, since we are interested in the most informative equilibria, we need to determine whether possible informativeness of horizontal communication affects the result above. From equation 5.18, horizontal communication is possible if

$$\gamma > -\frac{\lambda_D^2(1 + \frac{\lambda_D}{2})}{2(1 - \lambda_D^2)}. \quad (6.40)$$

We claim that the regions where centralization is optimal (assuming that we are in the region where centralization is uninformative and ignoring communication in decentralization) and where horizontal communication is possible do not intersect. Therefore, as informativeness of horizontal communication works in favor of decentralization, horizontal communication will only exacerbate the necessity of decentralization where decentralization is already optimal. In other words, if there

was an area where centralization is optimal and horizontal communication is feasible, we might have had a chance to support decentralization against centralization with regard to the baseline parameter restrictions. To prove this claim, we will next show that if 6.40 holds, then 6.39 also holds. We will determine the sign of the first expression in the following array:

$$-\frac{\lambda_D^2(1 + \frac{\lambda_D}{2})}{2(1 - \lambda_D^2)} + \sqrt{\frac{1 - 2\lambda_D}{12}} + \frac{\lambda_D}{2(1 - \lambda_D)} \quad (6.41)$$

$$= \frac{1}{2(1 - \lambda_D^2)} \left[\underbrace{\sqrt{\frac{1 - 2\lambda_D}{3}}}_{> \frac{1 - 2\lambda_D}{3}} (1 - \lambda_D^2) + \lambda_D - \frac{\lambda_D^3}{2} \right] \quad (6.42)$$

$$> \frac{1}{12(1 - \lambda_D^2)} [2 - 2\lambda_D^2 + 2\lambda_D + \lambda_D^3]. \quad (6.43)$$

If the expression in square brackets in equation 6.43 is positive, then we are done. This is so. The derivative of this part with respect to λ_D is:

$$\frac{d(2 - 2\lambda_D^2 + 2\lambda_D + \lambda_D^3)}{d\lambda_D} = -4\lambda_D + 2 + 3\lambda_D^2 \quad (6.44)$$

which is positive in the interval $[-0.5, 0]$. Therefore, $2 - 2\lambda_D^2 + 2\lambda_D + \lambda_D^3$ is bounded from below by $2 - 2(-0.5)^2 + 2(-0.5) + (-0.5)^3 = 0.3750$. Thus the expression in 6.41 is positive, which proves the claim.

Lemma 6.2. Let $\lambda_D < 0$ and $\gamma < 0$. If the number of maximum equilibrium partitions in vertical communication (\bar{N}) is more than 1, then decentralization is optimal for the principal.

Proof. The proof is quite similar to the case where $\beta > 0$ and $\gamma > 0$. We will again look at the case where horizontal communication is uninformative and show that even when benefits of information transmission in decentralization is ignored, still decentralization does better than centralization for the principal.

First, let $\bar{N} = 2$, then the sign of $U_1^D - U_2^C$ will be solely determined by the sign of $[(1 - 16\lambda_D^2)(1 - \lambda_D)^2 + 3T_V^2\lambda_D(\lambda_D - 4)]$ as in equation 6.27. Consider the following set of inequalities:

$$\begin{aligned}
(1 - 16\lambda_D^2)(1 - \lambda_D)^2 &+ \underbrace{3}_{>4\lambda_D^2} \underbrace{T_V^2\lambda_D(\lambda_D - 4)}_{>0} \\
&> (1 - 16\lambda_D^2)(1 - \lambda_D)^2 + 12\lambda_D^3(\lambda_D - 4) \\
&= -\underbrace{(1 + 2\lambda_D)}_{>0} \underbrace{(1 + \lambda_D)}_{>0} \underbrace{(\lambda_D - (-\frac{5}{4} + \frac{\sqrt{33}}{4}))}_{<0} \underbrace{(\lambda_D - (-\frac{5}{4} - \frac{\sqrt{33}}{4}))}_{>0} \\
&> 0.
\end{aligned}$$

Thus, decentralization does better when $\bar{N} = 2$ in vertical communication even if we ignore communication in decentralization.

The rest of the proof, i.e. showing that decentralization is optimal without consideration of information transmission in horizontal communication, is analogous to the proof of lemma 6.1 with slight differences. Now, instead of equations 6.29 and 6.30, we have:

$$T_V \in \left(-\frac{2}{\bar{N}(\bar{N} - 1)}, -\frac{2}{\bar{N}(\bar{N} + 1)} \right], \quad (6.45)$$

and since $T_V = 2[\lambda_D + 2\gamma(1 - \lambda_D)]$ and $\beta < 0, \gamma < 0$,

$$\lambda_D \in \left(-\frac{1}{\bar{N}(\bar{N} - 1)}, -\frac{1}{\bar{N}(\bar{N} + 1)} \right]. \quad (6.46)$$

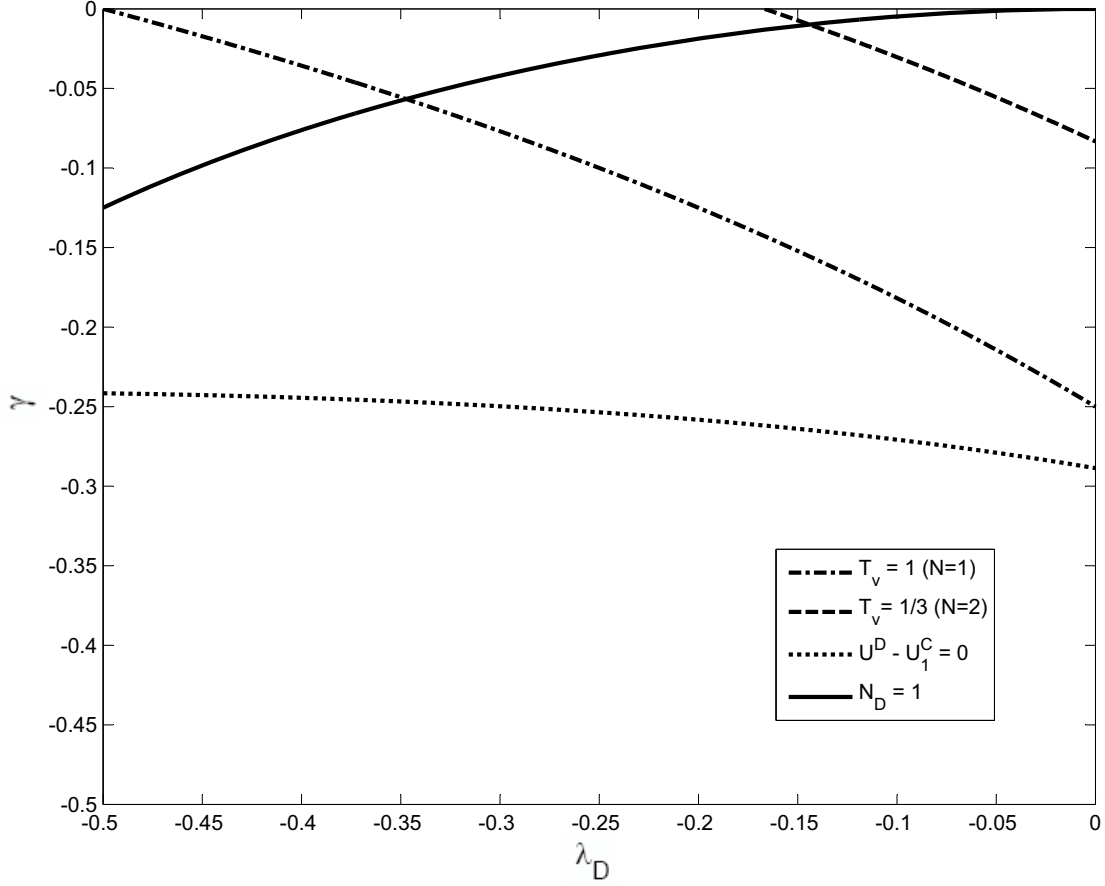
As $\bar{N} \geq 3$, $\lambda_D > -\frac{1}{6}$, and the calculations in 6.33 continue to hold. Therefore, decentralization performs better when vertical communication is informative.

□

Again, let us organize our findings into a proposition:

Proposition 6.2. Let $\lambda_D < 0$ and $\gamma < 0$. Then, decentralization is optimal if

$$\gamma > -\sqrt{\frac{1 - 2\lambda_D}{12}} - \frac{\lambda_D}{2(1 - \lambda_D)}. \quad (6.47)$$

FIGURE 6.2: Analysis of optimality when $\lambda_D < 0$ and $\gamma < 0$

The qualitative implications of the above proposition is similar to the first case where $\lambda_D > 0$ and $\gamma > 0$. In Figure 6.2, we summarize the results for strategic substitutabilities and negative externalities. Above the dash-dotted line, vertical communication is uninformative and above the dashed line, a two partition equilibrium is possible. As we get closer to the origin, the number of partitions in vertical communication increases and eventually becomes completely informative in the limit. Horizontal communication is informative above the solid line.

The dotted line is the frontier where centralization and decentralization are equally efficient for the principal. Above this line, decentralization is optimal and below centralization does better. Now when γ is very low, centralization is the optimal organizational structure. This is intuitive because the agents do not take into account large negative externalities that their decisions have on the other agent's

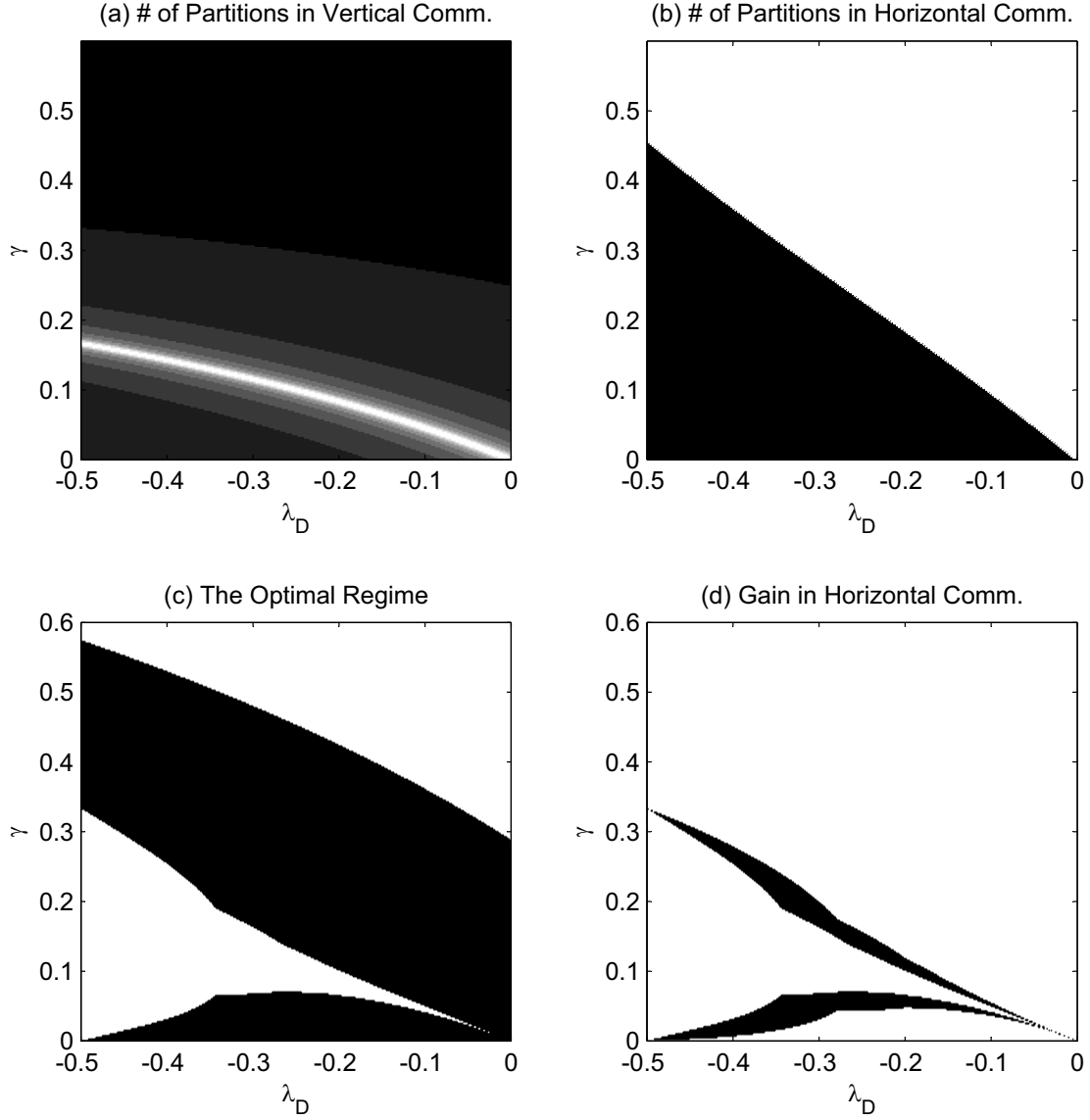
payoff in this case. On the other hand, unlike the $\lambda_D > 0$ and $\gamma > 0$ case, for strong complementarities (substitutabilities in this region), we can always find a corresponding γ such that for every $\gamma' > \gamma$ decentralization is optimal. This happens because when $\lambda_D > 0$, the last term in equation 6.20 becomes arbitrarily large as $\lambda_D \rightarrow 0.5$. Therefore, coordination becomes very important. On the other hand, when $\lambda_D < 0$, this term is always bounded. Therefore, the principal's need to coordinate decisions are lower which allows decentralization to become optimal for a wider range of λ_D values.

By a similar argument, in the region where vertical communication becomes informative, decentralization is always optimal. Now, we have a smaller adaptation loss due to less noisy communication in vertical communication. Also, the coordination loss is smaller. However, the need for coordination decreases faster, making coordination loss small relative to the gain in adaptation in centralization.

6.2.3 $\lambda_D < 0$ and $\gamma > 0$, or $\lambda_D > 0$ and $\gamma < 0$

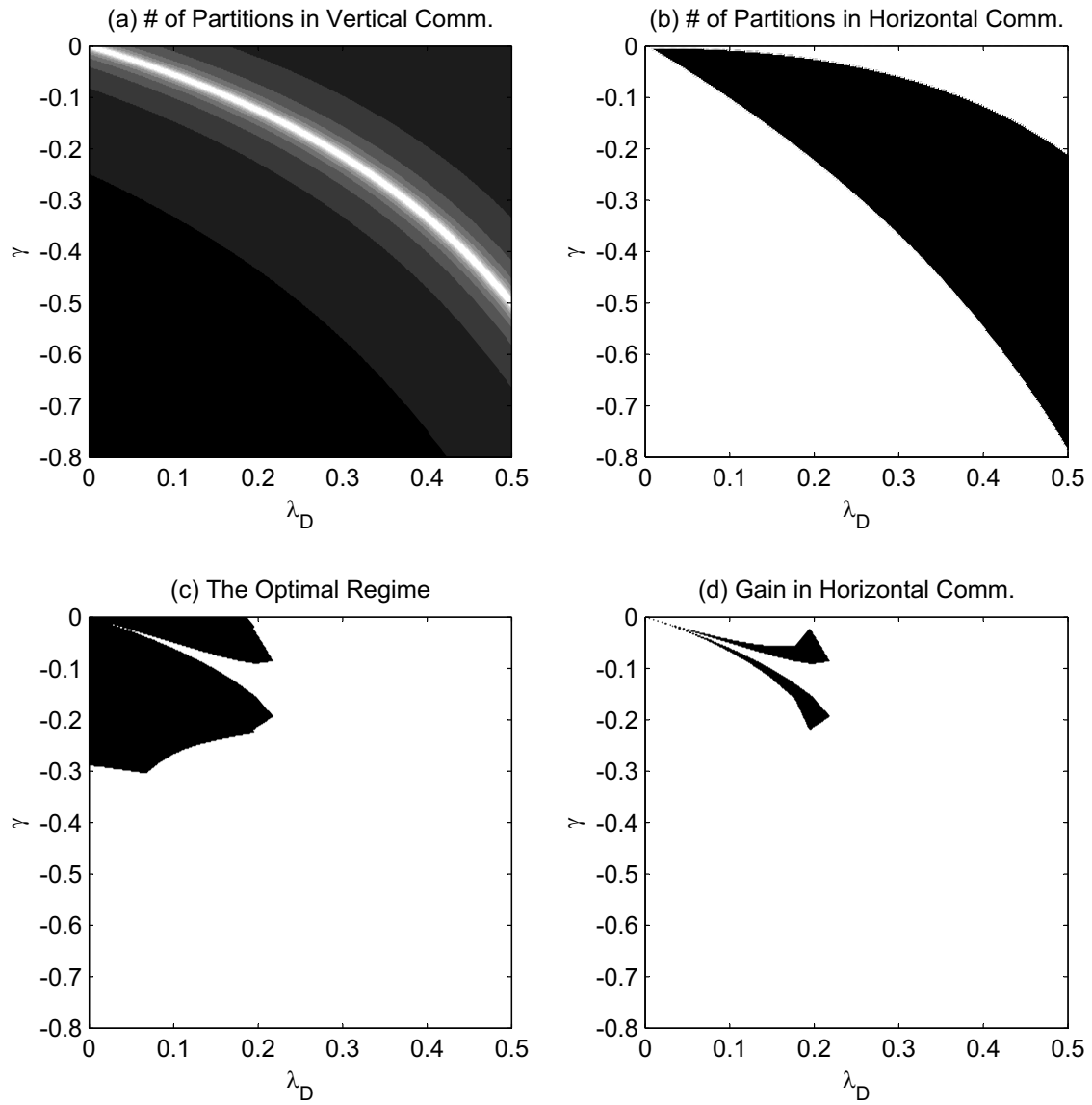
For the rest of the parameter configurations, the analytical complexity, especially due to possible informative vertical communication, does not allow us to obtain clear-cut results. Instead, we will rely on numerical analysis to assess the performance and quality of communication in each regime.

Figures 6.3 and 6.4 are plotted in Matlab for $\alpha = 10$. In these figures, panel a depicts the number of partitions in the most informative equilibrium in vertical communication. As we approach to the lighter region, the equilibrium involves more partitions. In panel b, on the other hand, we highlight the region (colored in black) where horizontal communication is informative. The black region in panel c is the set of parameter configurations where decentralization is more efficient, given that we are working with the most informative equilibria. Lastly, panel d shows the area where decentralization is optimal only when communication in horizontal

FIGURE 6.3: Analysis of optimality and communication when $\lambda_D < 0$ and $\gamma > 0$

communication is taken into account, i.e. if we ignored horizontal communication, then we would be subtracting the region in panel (d) from panel (c).

Let's first consider figure 6.3. In the plots in figure 6.3, we have strategic substitutabilities and positive externalities. As shown in panel a, to achieve the same level of informativeness, we need to increase γ if we decrease λ_D . In panel b, we observe

FIGURE 6.4: Analysis of optimality and communication when $\lambda_D > 0$ and $\gamma < 0$

that lower levels of λ_D are associated with a wider range of informative horizontal communication. This is again due to the form of K_3 in 5.20.

Before studying panel c, it might be helpful to answer the following question: What would have happened if we assumed uninformative vertical communication everywhere? In this case, the answer is that decentralization is optimal in the union of the shaded region in panel c and the white region that is buried inside the black region. Therefore, informativeness of vertical communication is now very crucial because it allows centralization to perform better in a wide range of parameter values.

To simplify the analysis, let's ignore horizontal communication and rewrite equation 1.26 as:

$$U^D - U^C = \frac{V^C}{2\alpha(1-\lambda^2)} - \frac{(\mu\lambda_D + \gamma(1-\lambda_D))^2}{2\alpha(1-2\lambda_D)(1-\lambda_D)^2} - \frac{2\lambda_D^2 V^D}{\alpha(1-4\lambda_D^2)} \quad (6.48)$$

$$= \frac{\sigma^2}{2\alpha} - \frac{(\mu\lambda_D + \gamma(1-\lambda_D))^2}{2\alpha(1-2\lambda_D)(1-\lambda_D)^2} - \frac{\text{gain in horizontal comm.}}{1-4\lambda_D^2} \quad (6.49)$$

Therefore, this implies that the adaptation gain from vertical communication becomes important as $\lambda_D \rightarrow -0.5$. Notice that if λ_D is very close to -0.5 , centralization becomes optimal as long as vertical communication is possible. As we approach to the origin, the importance of this gain gets smaller even though it may get very precise. This happens because even though centralization facilitates coordination given the information, the need for coordination is small.

Considering figure 6.4, we see that the results are equally dramatic for the case with $\lambda_D > 0$ and $\gamma < 0$ as well. When we look at panel c of this figure and compare this with figure 6.5, in which vertical communication is ignored, we see a wide area where centralization now performs better. Again, when we go through southeast direction, coordination becomes important and as more information is transmitted, the adaptation gain in vertical communication increases very quickly.

In short, our results regarding efficiency show the tradeoff between the need for adaptation and the need for coordination. Given information, centralized regime can

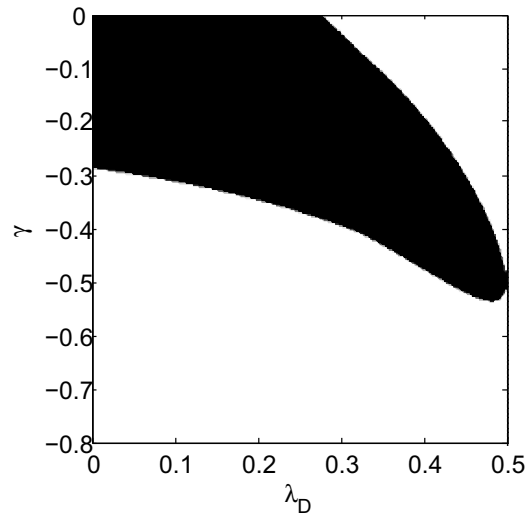


FIGURE 6.5: Comparison of uninformative centralization and decentralization

better coordinate decisions whereas decentralized regime allows for more adaptation to the local conditions. Particularly, when there are strategic complementarities and positive externalities, or strategic substitutabilities and negative externalities, if centralized communication is informative enough, then decentralization is optimal.

Chapter 7

Concluding Remarks

In this paper, we attempted to characterize various communication and decision making structures and their relationship to the design of authority and communications in organizations. We were able to show that the strategic considerations of an informed agent due to complementarities (substitutabilities) and externalities (positive or negative) lead her to distort her information in communication. A particular example is that, when there are strategic complementarities and positive externalities, the messages of the agents in horizontal communication are not credible since each type of each agent would like to distort her private information too much. On the other hand, an agent also has an incentive not to distort her information too much in communication, because otherwise, the decision makers might take actions that are suboptimal for the informed agent. These considerations lead agents to form intervals in which each type in an interval reports the same message.

The framework we offered can be beneficial to understand structural changes and their impacts in terms of efficiency within organizations. For instance, the Du Pont Company, initially a centralized firm, changed its organizational structure to decentralization in 1921. The changes in 1921 were the keys to the success of the company in the subsequent years. On the other hand, William Durant, the founder of the General Motors Company, had a vision of decentralized business from the

very beginning. However, this view did not enable Mr. Durant to compete with Henry Ford's centralized Ford Company in the early 1900s. These examples point out to the importance of understanding various complementarities and externalities in an organization and finding a fit between the organizational structure and the nature of the interaction within an organization.

In this study, we were able to consider only two particular forms of communication and decision making structures. Various other communication protocols can also be analyzed. For instance, more rounds of communication may be added to the vertical and horizontal communication games. One alternative governance structure is centralizing decisions for one of the departments whereas delegating part of the authority to the other department. Another alternative is proposed by [Rantakari \(2008a\)](#), where the decision rights for both dimensions of the actions are delegated to one of the agents and the other agent communicates her information to the agent who has the decision rights. Also, we only considered simultaneous communication structures. This problem can also be investigated in the presence of sequential communication protocols.

We considered only the case of two symmetric divisions. It would be interesting to see how communication and organizational choice is affected when there are asymmetric divisions or there are more than two divisions. Also, existence of other agents whose objective function is between the principal's and the agents' may yield different results in terms of communication and in terms of efficiency. Moreover, the parameters of the model can be endogenized so that the principal chooses both the strategic complementarity levels and externality levels as well as the allocation of decision rights.

Other than these lines of future research, since this study is a first order approximation into the study of communication and decision making under complementarities and externalities, our framework can be extended to a more general framework. For instance, we analyzed communication and decision making only under the assumption of uniform distribution of types. We might also be able to see whether

our results are robust to different distributions in a more general framework. Also, instead of using particular spillover formulations, one might rely on general assumptions regarding the payoff functions.

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