MEASURING AND MODELING BUBBLES IN SHIP WAKES, AND THEIR EFFECT ON ACOUSTIC PROPAGATION

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Abstract: This paper presents measurements and model predictions of the density and size distribution of bubbles in the wakes of full scale ships, and the effects of the bubbles on acoustic propagation. The ship wake is primarily a hydrodynamic process, characterized by temporally and spatially varying flow conditions (i.e., local fluid velocity, pressure, turbulent kinetic energy and dissipation, etc.) coupled with the transport of large numbers of bubbles. We present results of ship wake modelling utilizing hydrodynamic and bubble transport equations in which we assume that the bubble density is such that, to a very good approximation, the bubbles have negligible effect on the momentum of the local (water) flow. This allows transport of the bubble field to be performed after the flow calculation, i.e., there is no feedback. Bubble transport includes the following processes: buoyancy, drag, turbulent dispersion, and response to pressure fluctuations. We present measurements of acoustical backscattering from full-scale ship wakes, which reveal the wake’s time-varying shape and dependence upon operating conditions (e.g., speed). The bubbles dominate the optical and acoustic characteristics of the wake, which means that remote probing of the wake provides direct information about bubble distribution and only by inference other properties such as fluid velocity. Finally, the effect of the bubbles on acoustic propagation is investigated using a full-field acoustic propagation model that responds to the range-dependent refractive and attenuative effects of the bubbles. Both sound speed and attenuation depend upon the bubble radius, pressure and acoustic frequency, and the radius depends upon pressure (i.e., depth). The nature of the wake bubble field leads to depths and frequencies at which sound is attracted toward or away from the wake.

Keywords: ship wake model, ship wake bubbles, ship wake measurements, ship wake acoustics
MODELING THE SHIP WAKE

A Reynolds-Averaged Navier-Stokes (RANS) methodology has been used to compute bubble density in ship wakes based on the two-fluid formulation by Drew [1] for treating the liquid-air bubble mixture. The equations are solved in a coordinate system that moves with ship such that the ship is stationary and the fluid is in motion. We assume that the liquid and bubble phases are one-way coupled in the domain of interest due to the low bubble void fraction. This reduces the mass conservation equation for the liquid phase to

\[ \nabla \cdot \hat{\mathbf{v}}_l = 0 \]  

and to the Navier-Stokes equation for conservation of momentum:

\[ \rho_l \left( \frac{\partial \hat{\mathbf{v}}_l}{\partial t} + \mathbf{v}_l \cdot \nabla \hat{\mathbf{v}}_l \right) = -\nabla P + \nabla \left[ 2\tau_i + \mathbf{T}_l^{\text{re}} \right] + \rho_l \hat{\mathbf{g}} \]  

Here \( \hat{\mathbf{v}}_l \) is the density-weighted average liquid velocity and the laminar and turbulent (Reynolds) stresses are respectively given by \( \tilde{\tau}_i \) and \( \mathbf{T}_l^{\text{re}} \). The closure for the Reynolds stresses is obtained from the eddy-viscosity hypothesis and is calculated in conjunction with the blended \( k/\omega \) model (details in [2]). Currently, density stratification, temperature, and salinity variations are not implemented, but the numerical structure for these calculations is already incorporated in the Computational Fluid Dynamics (CFD) software CFDSHIP [2]. For the bubble phase, the governing equation for mass conservation is

\[ \frac{\partial \alpha \tilde{\rho}_g}{\partial t} + \nabla \cdot \left( \alpha \tilde{\rho}_g \hat{\mathbf{v}}_g \right) = 0 \]  

and for momentum conservation is

\[ 0 = -\alpha \nabla P + \alpha \left( \tilde{\rho}_g - \tilde{\rho}_l \right) \left( -|\mathbf{g}|e_z \right) + \frac{3}{8} \frac{\alpha \rho_l C_D}{r} \left| \mathbf{v}_l - \mathbf{v}_g \right| \left( \mathbf{v}_l - \mathbf{v}_g \right) \]  

\[ -\frac{3}{8} \frac{\rho_l C_D}{r} \frac{\left| \mathbf{v}_l - \mathbf{v}_g \right|}{S_{cb}} \mathbf{v}_l \cdot \nabla \alpha. \]

Velocity and pressure in the liquid phase are calculated using CFDSHIP, which is especially suited to naval applications. CFDSHIP solves the incompressible Navier-Stokes equations (2) with free surface deformation, which allows for generation of a surface wave field that changes dramatically with increasing speed, particularly in the transom region. This is a consequence of increasing Froude number \( \left( \frac{\hat{\mathbf{v}}_l}{\sqrt{gL}} \right) \), where \( g \) is acceleration due to gravity and \( L \) is ship length. The wake computations include a model for the thrust and torque action of the propellers on the flow. We have added code to solve equations (3) and (4) and transport the bubble phase code.
The liquid velocity field is solved in the entire domain surrounding the ship and throughout the wake using a boundary-fitted grid. For the bubble phase, a bubble size distribution (BSD) (discussed further below) is used as an initial condition near the stern because air entrainment and bubble generation around the ship hull are not yet fully understood and modelled. The dynamics governing the evolution of the bubbly phase include a large, but relatively weak vortex that transports bubbles away from the core and into the deeper region of the wake.

SHIP WAKE MEASUREMENTS

The acoustic measurements of bubble density in full-scale ship wakes utilize an autonomous underwater vehicle (AUV) fitted with a single beam upward-looking sonar. The single element transducer has a 9° two-way beam width and transmits a one msec 215 kHz pulse every 55 msec using a source level of 211 dB re 1 μPa at 1yd. The AUV crossed under the wake (depth ≈ 1.5*ship draft) a number of times while recording acoustic backscatter from the ship wake. The received signal backscattered by the wake is band pass filtered and recorded continuously.

Dumbrell [3] and Trevorrow et. al. [4] describe how the scattered signal from each ping is processed to estimate wake bubble density, scattering strength and attenuation. Briefly:

a. Beginning with the 1st point after a blanking time, total backscattered energy is estimated using the sonar equation and the transducer two-way 6 dB beam width.
b. From the backscattered energy, the number of bubbles is estimated for the first cell using the backscattering cross section for single bubbles and the bubble size spectrum. The procedure is repeated for the second data point (or cell), except that attenuation due to bubbles in the first cell is taken into account when computing the backscattered energy and bubble density. This procedure is repeated for each point in the uplook sonar data.

The size distribution of the bubbles is required. Dumbrell [3] has compared backscatter and attenuation measurements in the ship wake and found good agreement between direct measurements of attenuation and attenuation inferred from uplook sonar backscattering.

MODEL – MEASUREMENT COMPARISON

Figure 1 compares modelled and measured bubble density in a cross-wake plane approximately 2 kyd astern of the ship. The RANS model wake is for a naval ship moving at 18 kt speed while the in-water measurements are for the same hull moving at 16 kts. (The speed difference is unavoidable; all scales are the same). Note that the RANS model predicts mean bubble density, which varies smoothly and is symmetric about the wake center, whereas the measurements are snapshots of a random field. The measured wakes are strongly inhomogeneous and contain clumps and “fingers” of bubbles that reach downward from the surface and leave bubble-free holes between them, apparently the result of turbulent velocity structures. These bubble structures cause the measured wakes to look very different from the RANS wake model predictions and limit the model-measurement comparison to a qualitative comparison of wake size and density.

Gray patches inside the measured wakes indicate areas where bubble density cannot be estimated for one of two reasons. The first is that the received level for the first point after the blanking distance in a single ping of uplook sonar data corresponds to an attenuation of 1 dB or more, indicating that the sonar was in the wake. The second possible reason is
that the cumulative attenuation applied to sound propagating upward through the wake reached a level indicating that sound cannot penetrate farther.

The model wake is 60% as wide as the measured wake for all bubble radii, is significantly denser than the measured wake for the 25 \( \mu m \) and 75 \( \mu m \) bubbles, and is somewhat denser for the 250 \( \mu m \) bubbles. Modelled bubble density generally matches the measured values within the dense fingers that extend down from the surface, but is much higher elsewhere. It appears that the modelled wake is not as deep as the measured wake; the latter is truncated because the sonar crossed through the bottom of the wake.

![Log10 bubble density for 18 kt modelled wake (a) and 16 kt measured wake (b) @ ~2 kyds.](image1)

**Fig. 1:** \( \log_{10} \) bubble density for 18 kt modelled wake (a) and 16 kt measured wake (b) @ ~2 kyds.

Figure 2 shows the density of 25, 75 and 250 \( \mu m \) bubbles in cross-wake planes approximately 2 kyd astern. The modelled wake is for the naval ship moving at 28 kt while the measurements are for the same hull moving at 26 kts. The modelled wake is considerably narrower than the measured wake from the surface down to 50’ depths, especially for the 250 \( \mu m \) bubbles. Modelled wake intensity matches the measured values in the lobes extending down from the surface but not elsewhere. Wake depth cannot be compared because the measured wake is cut off due to shallow AUV run depth. In general, the modelled wakes are smoother and more compact than the measured wakes.

![Log10 bubble density for 28 kt modelled wake (a) and 26 kt measured wake (b) @ ~2 kyds.](image2)

**Fig. 2:** \( \log_{10} \) bubble density for 28 kt modelled wake (a) and 26 kt measured wake (b) @ ~2 kyds.
DETACHED EDDY SIMULATION (DES)

A characteristic of RANS wake models is that they calculate the mean velocity field and use it to transport bubbles through the wake. In a real wake, however, bubbles are transported by the instantaneous, turbulent velocity field. The absence of turbulent scales in the RANS velocity field causes the resulting bubble field to vary smoothly over location. A significant improvement in model fidelity is provided by the Detached-Eddy Simulation (DES) method [5] which combines RANS solutions in the boundary layer where the turbulence scales are smaller than the grid spacing, with Large Eddy Simulation (LES) solutions in regions where the flow is separated from the boundary and contains large scale turbulence. DES senses the grid spacing and switched the calculation between RANS and LES, resulting in a computational load that can be substantially less than that of LES. In separated regions like the ship wake, DES retains large scale turbulent structure that would be averaged out in the RANS calculation.

Figure 3 compares normalized fluid velocity (we cannot predict bubble density yet) predicted using RANS and DES along the center line of a ship wake. Note that the coordinate system moves with the ship in the positive x direction, and in this reference frame the water is flowing to the left (negative x direction). Both calculations show a recirculation zone near the stern (indicated by the orange color). However, the mean fluid velocity field computed using the RANS model (upper) changes smoothly while the DES field (lower) contains large scale structure in the velocity field. The DES calculation is unsteady, meaning that the velocity field is not constant in time.

![Normalized water velocity predicted using RANS (upper) and DES (lower) models. The coordinate system moves with the ship. Red or green dots mark flow stream lines.](image)

ACOUSTIC PROPAGATION THROUGH THE WAKE

The presence of bubbles in water causes the sound speed to be dispersive (vary with frequency) and acoustic signals to be attenuated (cf. [6]). These two effects can be
incorporated into a complex effective sound speed. For a mixture of bubbles of different sizes, the effective sound speed at range \( r \), depth \( z \) and acoustic frequency \( f \) is

\[
c(r, z, f) = \frac{c_0(z)}{\sqrt{1 + S(r, z, f)}}
\]

(5)

Here \( c_0(z) \) is the sound speed without the bubbles present, which varies with \( z \) but not with frequency or range. The effect of the bubbles is incorporated in the term

\[
S(r, z, f) = \frac{c_0^2(z)}{\pi f^2} \sum_{j=1}^{M} \frac{a_j N(r, z, a_j)}{a_j N(r, z, a_j)}
\]

(6)

The summation in (6) is over the \( M \) bubble radii \( a_j \). The quantity \( N(r, z, a_j) \) is the density of bubbles with radius between \( a_j \) and \( a_j + da \) at position \( r, z \), where \( da = 1 \mu m \). The resonant frequency of a bubble of radius \( a_j \) at depth \( z \) is \( f_0(z, a_j) \), and \( \delta(a_j) \) is the damping due to re-radiation, shear viscosity of sea water, and thermal conductivity.

Fig. 4 shows sound speed and attenuation vs frequency for an exponential BSD [3]. The speed of sound in bubble-free water is 1500 m/s. Fig. 4 shows that at 30 kHz for example, the sound speed is higher than 1500 m/s near the surface but is lower than 1500 m/s at greater depth. Because sound is refracted away from higher speed regions and toward lower speed regions, near the surface 30 kHz sound will be refracted away from
the wake, while at greater depth it will be refracted toward the wake. The wake attenuation peak (bottom panel) moves to higher frequency as depth increases.

Sound speed and attenuation in the ship wake can be used with an acoustic propagation model to calculate transmission loss for sources and receivers located in or near the ship wake [7]. The Parabolic approximation to the wave Equation (PE) model is well suited for predicting propagation through dense, random distributions of bubbles such as those that can be found in surface ship wakes. Beginning with a 3D distribution of bubbles, a vertical slice containing the source and receiver is extracted, and the 2D (range vs. depth) sound speed and attenuation fields are calculated (e.g., Fig. 5, upper and middle plots, respectively). A Green's Function parabolic equation (GF-PE) code takes the source-receiver geometry and sound speed and attenuation fields as input and computes the complex pressure field (e.g., the bottom panel in Fig. 5).

Fig. 5 utilizes the bubble density data shown in the left panel of Fig. 2. The resolution used in Fig. 5 is 5 m in range and 1 m in depth. The upper panel shows sound speed (m/s) at 30 kHz, which we see is higher than ambient (1500 m/s) near the surface but lower than ambient at deeper depths. The middle panel shows attenuation (dB/m) at 30 kHz, which is higher in the shallow part of the wake and lower in the deeper wake. This is explained in Fig. 4. The bottom panel is transmission loss (dB) for a source at 1.0 times the ship draft. There is a shadow zone behind the wake into which acoustic energy enters by refraction and diffraction under the wake.

![Figure 5: Sound speed, m/s (a) and attenuation (dB/m) (b) at 30 kHz calculated from the ship wake bubble field in Fig. 2a. Panel (c) is transmission loss (dB) calculated using PE and a source 1.0 times the ship draft. Note the acoustic shadow cast by the wake.](image-url)
SUMMARY

We have discussed measurements and modelling of the density and size distribution of bubbles in the wakes of full scale ships, and the effect of the bubbles on acoustic propagation. The velocity field in the wake was modelled using the Reynolds-Averaged Navier Stokes (RANS) and Detached Eddy Simulation (DES) approaches and used to transport bubbles, and the resulting bubble fields compared with measured wakes. Both RANS and DES calculate the velocity field all around the ship hull and throughout the wake. However, RANS calculates the average field, which is smooth and devoid of turbulent velocity. The measured wakes are strongly inhomogeneous, apparently due to bubbly fingers advected downward by turbulent velocity. The DES model retains some turbulent scales and thus appears to possess the variability seen in measured wakes. RANS is a mature and widely used tool; DES is a newer technique that is the subject of current computational fluid dynamics (CFD) research.

Our ultimate interest is in the acoustic properties of the bubbly wake and the wake’s effect on acoustic propagation. The acoustics of the ship wake depend strongly on the bubble size distribution (BSD) and depth. For acoustic frequencies in the 10’s of kHz, we are in the Rayleigh scattering region where resonant scattering is often dominant but off-resonant scattering can be important. The change in bubble radius caused by depth (pressure) changes has a strong effect on wake acoustics. As an example, we used an exponential bubble distribution and showed that increasing pressure increases the frequency of the attenuation peak and at 30 kHz, lowered sound speed and attenuation. Although this result depends upon the BSD and the acoustic frequency, a general, conclusion is that low frequency sound is refracted toward the wake while high frequencies are refracted away from the wake.

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REFERENCES