1. Evaluate \((\frac{313}{367})_J, (\frac{367}{401})_J, (\frac{401}{313})_J\). \(313 \equiv 401 \equiv 1 \pmod{8}\), \(367 \equiv 7 \pmod{8}\). Thus \((313|367)_J = (367|313)_J = (6|313)_J = (2|313)_J(3|313)_J = (313|3)_J = 1\), \((367|401)_J = (401|367)_J = (2|367)_J(17|367)_J = (367|17)_J = (10|17)_J = (5|17)_J = (25)_J = -1\), \((401|313)_J = (88|313)_J = (11|313)_J = (5|11)_J = (1|5)_J = 1\).

2. Show that the congruence \(x^6 - 11x^4 + 36x^2 - 36 \equiv 0 \pmod{p}\) is soluble for every prime \(p\). \(x^6 - 11x^4 + 36x^2 - 36 = (x^2 - 2)(x^2 - 3)(x^2 - 6)\). If 2 or 3 is a 0 or QR we are done. If not, then 6 is a QR.

3. Suppose that \(a \in \mathbb{Z}\backslash\{0\}\), and there is a \(b \in \mathbb{Z}\) such that \(a = -b^2\). Show that there is an odd positive integer \(m\) such that \((\frac{a}{m})_J = -1\). Deduce that there is an odd prime \(p\) such that \((\frac{a}{p})_J = -1\). Choose \(m \equiv -1 \pmod{4b}\). Then \((b^2|m)_J = (b|m)_J^2 = -1\). Moreover not all the primes \(p\) with \(p|m\) can satisfy \((-b^2|p)_l = 1\).

4. Suppose that \(a \in \mathbb{Z}\backslash\{0\}\) and \(a = \pm 2^n b\) where \(u \in \mathbb{N}\) and \(b \in \mathbb{N}\) with both \(u\) and \(b\) odd. Show that there is an odd positive integer \(m\) such that \((\frac{a}{m})_J = -1\). Deduce that there is an odd prime \(p\) such that \((\frac{a}{p})_J = -1\). Choose \(m\) so that \(m \equiv 5 \pmod{8}\) and \(m \equiv 1 \pmod{b}\). Then \((\pm 2^u b|m)_J = (2|m)_J(b|m)_J = -(m|b)_J = -1\). Moreover not all the primes \(p\) with \(p|m\) can satisfy \((\pm 2^u b|m|p)_L = 1\).

5. Suppose that \(a \in \mathbb{Z}\backslash\{0\}\) and \(a = \pm 2^n b q^t\) where \(u\) is a non-negative integer, \(b \in \mathbb{N}\) and \(t \in \mathbb{N}\) with both \(b\) and \(t\) odd, and \(q\) is an odd prime with \(q \nmid b\). Show that there is an odd positive integer \(m\) such that \((\frac{a}{m})_J = -1\). Deduce that there is an odd prime \(p\) such that \((\frac{a}{p})_J = -1\). Choose \(m\) so that \(m \equiv 1 \pmod{4b}\) and \(m\) is a QR \(n\) (mod \(q\)). Then \((\pm 2^n b q^t|m)_J = (m|b)_J(m|q)_L = -1\). Moreover not all the primes \(p\) with \(p|m\) can satisfy \((\pm 2^n b q^t|p)_L = 1\).

6. Show that an integer \(a\) is a perfect square if and only if it is a quadratic residue for every prime \(p\) not dividing \(a\). If \(a\) is a perfect square then at once it is a QR modulo \(p\) for every such \(p\). If \(a\) is not a perfect square, then it is of one of the forms \(-b^2, \pm 2^n b\) where \(u \in \mathbb{N}\) and \(b \in \mathbb{N}\) with both \(u\) and \(b\) odd, or \(\pm 2^n b q^t\) where \(u\) is a non-negative integer, \(b \in \mathbb{N}\) and \(t \in \mathbb{N}\) with both \(b\) and \(t\) odd, and \(q\) is an odd prime with \(q \nmid b\). Hence by questions 3, 4 or 5, there is always an odd prime \(p\) such that \(a\) is a QNR modulo \(p\).