Easier problems
1. Show that if \( p \) is a prime number and \( 1 \leq j \leq p - 1 \), then \( p \) divides the binomial coefficient \( \binom{p}{j} \).
2. Show that \( n|(n - 1)! \) for all composite \( n > 4 \).
3. Exhibit a complete residue system modulo 17 composed entirely of multiples of 3.
4. Solve \( 11x \equiv 21 \pmod{105} \).
5. Prove that \( 3n^2 - 1 \) can never be a perfect square.

Harder problems
6. Prove that no polynomial \( f(x) \) of degree at least 1 with integral coefficients can be prime for every positive integer \( x \).
7. If \( 2^n + 1 \) is an odd prime for some integer \( n \), prove that \( n \) is a power of 2.
8. Show that if \( p \) is an odd prime, then the number of solutions (i.e., the number of ordered pairs of residues modulo \( p \)) of the congruence \( x^2 - y^2 \equiv a \pmod{p} \) is \( p - 1 \) when \( a \not\equiv 0 \pmod{p} \) and \( 2p - 1 \) when \( a \equiv 0 \pmod{p} \).