1. Suppose that \( \{u_n\} \) is uniformly distributed (mod 1), and let \( c \) be a real number. Put \( v_n = u_n + c \). Show that \( \{v_n\} \) is uniformly distributed.

2. Let \( \alpha_n = \log n - \lfloor \log n \rfloor \)
   (a) Show that
   \[
   \limsup_{N \to \infty} \frac{1}{N} \text{card}\{n : 1 \leq n \leq N, \alpha_n \in [0, 1/2]\} = \frac{e - e^{1/2}}{e - 1}.
   \]

   (b) Show that
   \[
   \liminf_{N \to \infty} \frac{1}{N} \text{card}\{n : 1 \leq n \leq N, \alpha_n \in [0, 1/2]\} = \frac{e^{1/2} - 1}{e - 1}.
   \]

   (c) Show that
   \[
   \frac{1}{N} \sum_{n=1}^{N} e^{(k \log n)} = \frac{N^{2 \pi i k}}{2 \pi i k + 1} + O\left(\frac{|k|}{N}\right).
   \]
   Hint: Try comparing the sum on the left with the corresponding integral.

   (d) Show that the sequence \( \{\alpha_n\} \) is not uniformly distributed (mod 1).

3. Suppose that the sequence \( \alpha_n \) satisfies \( \lim_{n \to \infty} (\alpha_{n+1} - \alpha_n) = \beta \). Prove that if \( \beta \in \mathbb{R} \setminus \mathbb{Q} \) then \( \alpha_n \) is uniformly distributed modulo 1. Hint: Consider \( \sum_{m=1}^{n} e^{(h \alpha_{m+1})} \).