1. Evaluate the series

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \]

This is \( L(3, \chi) \) where \( \chi \) is the non-trivial Dirichlet character modulo 4. Hint: Problems 3 can be useful.

2. Find a Fourier series proof that if \( m \) and \( n \) are non-negative integers, then

\[ \sum_{k=0}^{n} \binom{n}{k} \binom{k}{m} = \binom{n}{m} 2^{n-m}. \]

3. Suppose that \( g : [0, 1] \to \mathbb{C}, g \in L^2[0,1], c \in \mathbb{C}, f(x) = \int_{0}^{x} g(y)dy + c \quad (0 \leq x \leq 1), \)
\[ c_0(f) = \int_{0}^{1} f(x)dx, \quad c_0(g) = \int_{0}^{1} g(x)dx. \]
Then prove that

\[ \int_{0}^{1} \left| f(x) - (f(1) - f(0))(x - \frac{1}{2}) - c_0(f) \right|^2 dx \leq \frac{1}{4\pi^2} \int_{0}^{1} |g(x) - c_0(g)|^2 dx \]

with equality if and only if \( f \) is of the form \( ax + b + c^+ e^{2\pi ix} + c^- e^{-2\pi ix} \).