These exercises are essentially the same as in the text, so I have included a cross reference.

1. §1.1. Exercise 2. Prove that if \( f_1, f_2 \ldots \) are real continuous functions of \( \mathbb{R} \) and if for each \( x \in \mathbb{R} \) we have \( \lim_{n \to \infty} f_n(x) \) exists, then \( A = \{ x : 0 \leq f(x) < 1 \} \) is measurable. Hint: Prove that \( A = \bigcup_{k \geq 1} \bigcup_{m \geq 1} \bigcap_{n \geq m} \{ x : f_n(x) \leq 1 - 1/k \} \).

2. §1.2. Exercise 2. Check that for fixed \( \beta \), the inner product \( (\alpha, \beta) \) is a continuous function of \( \alpha \).

3. §1.3. Exercise 9. For \( x \in [0, 1] \) define the Haar function \( e^k_n \) by \( e^k_n(x) = 1 \) and, when \( n \geq 0, 1 \leq k \leq 2^n \), by

\[
    e^k_n(x) = \begin{cases} 
        2^{n/2} & \text{when } k - 1 \leq 2^n x < k - 0.5, \\
        -2^{n/2} & \text{when } k - 0.5 \leq 2^n x < k, \\
        0 & \text{otherwise}.
    \end{cases}
\]

Prove that they form a unit–perpendicular basis for \( L^2[0, 1] \). Hint: One route to showing that they span is first to show that if \( f \) is perpendicular to them all, then \( \int_0^x f = 0 \) for all \( x \) of the form \( k2^{-n} \), and deduce that \( \int_B f = 0 \) for every measurable \( B \subset [0, 1] \).

4. §1.3. Exercise 14. Show that the family \( \{ f_n \} \) spans \( L^2(Q) \) iff \( (f, f_n) = 0 \) for every \( n \) implies \( f \equiv 0 \). Hint: What is the annihilator of the family?