The object of this homework is to use Lehman’s method to find factors. The first two problems could be done by hand, but it would be more convenient to use a good calculator, or Pari/gp. I do not claim that for the numbers below Lehman’s method is necessarily faster than trial division, although it is a lot less tedious. More to the point, you can see the possibilities. Lehman’s method runs as follows.

1. Try trial division $d|n$ for $d \leq n^{1/3}$ for $d$ in the range. It suffices to take prime values of $d$ only and for the first two numbers $n$ below these are well known! If a factor is found, stop. Pari/gp has a list of primes built in.

2. For each $t \in \mathbb{N}$ with $1 \leq t \leq n^{1/3}$ and $x \in \mathbb{N}$ with $(4tn)^{1/2} \leq x \leq (4tn + n^{2/3})^{1/2}$ (for the first two numbers below the intervals that arise contain at most one integer) compute $x^2 - 4tn$ and check to see if this is a perfect square $y^2$. If it is compute $(x + y, n)$ and stop.

For full marks, list the remainders on division by primes $3, 7, 11, 13, 17, \ldots$ up to $n^{1/3}$. Then for each $t$ with $1 \leq t \leq n^{1/3}$ list the possible values for $x$, and if $x^2 - 4tn$ is a perfect square $y^2$, then compute $(x + y, n)$ and stop. Note that Pari/gp has the gcd function built in. If you use a different programming language, then you will probably need to construct the gcd algorithm from scratch.

1. Find a non–trivial factor of 19109.
2. Find a non–trivial factor of 39757.
3. Find a non–trivial factor of 2048129.