1. Write a program to find \(x\) and \(y\) such that \(mx + ny = \gcd(m, n)\) where
   (i) \(m = 8148657527, n = 8148653735,\)
   (ii) \(m = 8418785375, n = 7849911069.\)
   A copy of your program should be submitted with your solutions to gain credit.

2. Let \(a, b, c \in \mathbb{Z}\) with \(a\) and \(b\) not both zero. Prove each of the following.
   (i) If \((a, b) = 1\) and \(a | bc\), then \(a | c\).
   (ii) \(\left(\frac{a}{(a,b)}, \frac{b}{(a,b)}\right) = 1\).
   (iii) \((a, b) = (a + cb, b)\).

3. Show that if \((a, b) = 1\), then \((a - b, a + b) = 1\) or 2. Exactly when is the value 2?

4. (i) Show that if \(m\) and \(n\) are integers of the form \(4k + 1\), then so is \(mn\).
   (ii) Show that if \(m, n \in \mathbb{N}\), and \(mn\) is of the form \(4k - 1\), then so is one of \(m\) and \(n\).
   (iii) Show that every number of the form \(4k - 1\) has a prime factor of this form.
   (iv) Show that there are infinitely many primes of the form \(4k - 1\).

5. Show that if \(ad - bc = \pm 1\), then \((a + b, c + d) = 1\).