1. (25 marks) Prove that if \( m \in \mathbb{N} \) and \( n \in \mathbb{N} \), then there are integers \( a, b \) such that \( \gcd(a, b) = m \) and \( ab = mn \) if and only if \( m \mid n \).

2. (25 marks) Find all pairs of integers \( x \) and \( y \) such that \( 922x + 2163y = 7 \).

3. (25 marks) Prove that \( 1365 \mid n^{13} - n \).

4. (25 marks) Prove that \( 23n - 1 \) is never a perfect square.