Inequalities, Roots

Return by Monday 4th February

1. Let $a, b, \alpha, \beta$ be real numbers with $0 < b, 0 < \beta$ and

$$\frac{a}{b} < \frac{\alpha}{\beta}.$$ (i) Prove that $a\beta < ab$.

(ii) Prove that

$$\frac{a}{b} < \frac{a + \alpha}{b + \beta} < \frac{\alpha}{\beta}.$$ (ii) Prove that

2. Let $x$ and $y$ be real numbers. Prove that $x^4 - 4x^2y^2 + 6y^4 \geq 0$.

3. Prove that if $0 < x < 1$ if and only if $0 < x^3 < 1$.

4. Find all real values of $x$ such that

$$\frac{x}{x + 1} < \frac{x - 1}{x + 2}.$$ (ii) Prove that

5. Simplify the following.

(i) $64^{2/3}$,

(ii) $3125^{1/5}$,

(iii) $27^{-4/3}$.