MATH 401 INTRODUCTION TO ANALYSIS-I, SPRING TERM 2019, PRACTICE EXAM 1

Note that the first exam is on Wednesday 30th January, at 1:25 in Room 105 Wartik.

1. (25 points) Differentiate with respect to \(x\).
   (i) \((2x^3 - 1) \sin x\)
   (ii) \(7x^{-4} \ln x\)

2. (25 points) Let \(A, B, C\) be sets. Prove that \((A \cap B) \cup C = (A \cup C) \cap (B \cup C)\).

Summary of order axioms from class (slightly different from the textbook): There is a relation “<” which satisfies the following axioms. \(a, b, c\) denote real numbers.
   O1. Exactly one of \(a < b\), \(a = b\), \(b < a\) holds.
   O2. If \(a < b\) and \(b < c\), then \(a < c\).
   O3. If \(a < b\), then \(a + c < b + c\) for all \(c\).
   O4. If \(a < b\) and \(0 < c\), then \(ac < bc\).

The expression \(a > b\) means \(b < a\). We also use \(a \leq b\) to mean “either \(a < b\) or \(a = b\)”.

3. (25 points) Suppose that \(x\) and \(y\) are real numbers with \(x < y\). Prove that \(2x < x + y\) and \(x + y < 2y\). Here \(2x\) means \(x + x\), of course.

4. (25 points) Suppose that \(a\) and \(b\) are real numbers with \(0 < ab\). Prove that either (i) \(0 < a\) and \(0 < b\), or (ii) \(a < 0\) and \(b < 0\).