This paper describes the possible effects of feedback on learning (associations) using a connectionist tool, the delta rule. Feedback in instruction can be described in terms of the interaction of stimulus inputs and response outputs, an associationist perspective. Here the delta rule is applied to each instance that an input and an output likely interact; the delta rule is used to describe the direction and magnitude of increase or inhibition of each instance. The total effect for a type of feedback is then described by the combination of the instances that make up that type. The findings of this investigation may provide seminal foundations for understanding feedback effects. The paper focuses on the following topics: an overview of the connectionist approach; implications of lesson item difficulty; delta rule calculations for immediate and delayed feedback; and implications. Several graphs and mathematical calculations illustrate the discussion. (Author/AEP)
Differential memory effects for immediate and delayed feedback:  
A delta rule explanation of feedback timing effects

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Abstract
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connectionist tool, the delta rule. Feedback in instruction can be described in terms of the  
interaction of stimulus inputs and response outputs, an associationist perspective. Here the delta  
rule is applied to each instance that an input and an output likely interact; the delta rule is used to  
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for a 'type' of feedback is then described by the combination of the instances that make up that  
type. The findings of this investigation may provide seminal foundations for understanding  
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Focus of this Paper
This presentation will focus on:  
- Overview of the Connectionist approach  
- Implications of lesson item difficulty  
- Delta rule calculations for immediate and delayed feedback  
- Implications

Connectionist Approach
Recently, connectionist investigations have revived associationist views of memory.  
Connectionist apply various mathematical rules within neural network computer simulations in a  
search to mimic and describe memory associations and learning. For example, trained neural nets
have been shown to be capable of pattern matching, recognition, and classification (Haberlandt, 1997) among other things. This approach provides additional ways to understand cognitive activity (McClelland & Rumelhart, 1985).

Among a number of connectionist rules, the delta rule (Shanks, 1995; Widrow & Hoff, 1960) is one of the simplest that include the effects of feedback on learning. The delta rule describes the change in association weight, termed Δw, between an input unit and an output unit at each learning trial, \( a_\Delta \Delta w_{i0} = \alpha a_{iin} (t_o - a_{oout}) \), where \( \alpha \) is the learning rate parameter, \( a_{iin} \) is the activation level of input units, \( t_o \) is the desired response (the \( t \) refers to 'teacher', in this case \( t_o \) is item feedback), and \( a_{oout} \) is the activation level of the output units (from Shanks, 1995).

In instructional terms, learning is an increase in the association (e.g., an increase in \( \Delta w_{i0} \)) between the stimulus \( (a_{iin}) \) and the correct response \( (a_{oout}) \), with a relative decrease in association (e.g., a decrease in \( \Delta w_{i0} \)) for incorrect responses. For example, suppose you move to a new city; for the question "What is your postal zip code?", your old zip code at that instance will have a probability of being recalled of say .53, while your new zip code would have a probability of .24 (refer to Figure 1). Over time, the old will decrease (if it is not recollected) while the new increases. Note that the two are separate but probably correlated units. Interference (RI and PI) and also retrieval competition between the two is likely, and so context of recall is important.

![Diagram](image)

Figure 1. Two visualizations of output activation levels (possible responses) of about 100 output units \( a_{oout} \) given an input activation \( a_{iin} \) (such as a lesson question). Activation levels, regardless of absolute activity, are shown as proportions which sum to 1.
In the delta rule equation, feedback impacts learning in the term \( t_o - a_{out} \). Customarily, the values for \( t_o \) and \( a_{out} \) are constrained between zero to one (McLeod, Plunkett, & Rolls, 1998). The value for \( t_o \) equals one if the activation level of the input unit matches the desired response (i.e., with correct responses) and \( t_o \) equals zero if the activation level of the input unit does not match the desired response (i.e., with incorrect responses). So with correct responses the association weight increases since \( 1 - a_{out} \) is positive, while with incorrect responses the association weight decreases since \( 0 - a_{out} \) is negative.

**Item Difficulty and Feedback Effects**

Though not yet a part of the feedback canon, feedback probably has its greatest effect with difficult lesson items. For example, Bangert-Drowns, Kulik, Kulik, and Morgan (1991) state:

> If feedback's primary importance is the correction of errors, then one would expect to see larger effects for instruction with higher error rates (more difficult lessons). This is exactly what happens. The correlation between error rate and effect size is .48, a significant relation. (p.230)

So feedback is most effective with difficult lessons. Lesson item difficulty and number of lesson errors reflect the same construct, so it is a small step to state that feedback is most effective with difficult lessons items. (Note: Have previous studies of feedback been confounded by lesson difficulty?)

To apply the delta rule, lesson average item difficulty values are assumed to be reasonable estimates of initial lesson \( a_{out} \). Item difficulty is a simple item statistic obtained by averaging correct responses to an item, for example an item difficulty of .20 indicates that 20% of the learners responded correctly to that item. Item difficulty values range from 0 to 1 with low values indicating difficult items and high values indicating easy items. Using lesson average item difficulty values as estimates of lesson \( a_{out} \) seems justified in that lesson average item difficulty is the actual averaged probability of selecting the correct answer for that item during the lesson for that population of learners.
Using the Delta Rule to Predict Feedback Effects (posttest performance)

Assumptions

In comparing immediate and delayed feedback, first the delta rule would predict that for correct lesson responses, memory of both initial lesson responses (ILRs) and of correct responses (CRs) would be strengthened in general for both immediate and delayed feedback, since \( t_o = 1 \) and so \( (t_o - a_{out}) \) is positive. Second, for incorrect lesson ILRs (i.e., errors), the ILR association with the item stem would be weakened for immediate feedback since \( t_o = 0 \) and \( (t_o - a_{out}) \) is negative, but not for delayed feedback. Note further that the delta rule can predict both magnitude and direction of the post-feedback memory association levels of ILRs for immediate and delayed feedback as well as for correct responses. The results of applying the delta rule across a range of possible lesson item difficulty values are shown below in detail.

The delta rule (Shanks, 1995) estimates change in association weight \( (\Delta w_{in}) \) as a result of feedback training as \( \Delta w_{in} = \alpha \ a_{in} \ (t_o - a_{out}) \). The retention test association weight of \( a'_{out} \) can be estimated as \( a'_{out} = a_{out} + \Delta w_{in} \) (the prime symbol in this paper indicates retention posttest values of the variable, while variables without a prime symbol are lesson values). In order to perform delta rule calculations, it is necessary to estimate two item related variables in the equation, \( \alpha \) the learning rate parameter, and \( a_{in} \), the activity level of the input unit (in this case the question stem). The term \( \alpha \) ranges from 0 to 1, and most likely relates to the familiarity \( \alpha \) the content domain and the form of the question. In this case, \( \alpha \) will be set equal to 0.5, the mid-point, as the best-guess estimate. The term \( a_{in} \) also ranges from 0 to 1. Hopefully, the vocabulary of item stems as well as their relationship to the lesson text make items meaningful and understandable, with a resulting average \( a_{in} \) nearly equal to 1. In this case \( a_{in} \) will be set equal to 0.8. (Note: Future studies may attempt to directly measure \( a_{in} \) by asking learners to self-report the meaningfulness of each item.) Both \( \alpha \) and \( a_{in} \) for an item combine to impact the change in association weight in the delta rule. In this study \( \alpha \ * \ a_{in} = 0.5 \ * \ 0.8 \), so in the calculations to follow, \( \alpha \ * \ a_{in} = 0.4 \).

For any item labeled \( g \), lesson item difficulty value \( p_g \) provides a measure of the lesson activation level of item \( g \). To solve for retention test \( a'_{out} \) for any lesson \( p_g \), substitute \( p_g \) for lesson \( a_{out} \) in the delta rule and substitute 0.4 as an estimate of \( \alpha \ a_{in} \):

\[
a'_{out} = p_g + 0.4 \ (t_o - p_g)
\]
This approach shown in Equation 1 can be applied across a range of possible lesson item difficulty values providing retention test activation levels of both ILRs and of correct responses for both delayed and immediate feedback.

Predicting ILR Posttest Values for Immediate Feedback

When correct, \( t_0 = 1 \) and so the activation weight of the initial lesson response is increased per the delta rule:

\[
p_{ILR-C}^{'} = p_b + 0.4 (1 - p_b) \tag{2}
\]

When the lesson response is incorrect estimate the activation level of the ILR using Equation 3, then since \( t_0 = 0 \) decrease the activation level of the ILR per the delta rule:

\[
p_{ILR-I} = (1 - p_b) \tag{3}
\]

\[
p_{ILR-I}^{'} = p_{ILR-I} + 0.4 (0 - p_{ILR-I}) \tag{4}
\]

Now combine Equations 2 and 4 (see Figure 2):

\[
p_{ILR}^{'} = (p_b \cdot p_{ILR-C}^{'} ) + [(1 - p_b) \cdot p_{ILR-I}^{'}] \tag{5}
\]

\[\text{Figure 2. Graph of the Predicted ILR Posttest Values for Immediate Feedback.}\]
Note that the correct and incorrect items contribute to final predicted retention test $p'_ILR$ proportionally across different values of $p_b$. For example, when $p_b = 0.3$ (a difficult item with 30% of the learners getting it right and 70% missing it during the lesson), $p'_{ILR-C}$ in Equation 2 contributes 0.3 and $p'_{ILR-I}$ in Equation 4 contributes 0.7 to the total $p'_ILR$ score. Equation 5 shows this proportional combination of $p'_{ILR-C}$ and $p'_{ILR-I}$ relative to the item difficulty $p_b$.

Predicting ILR Posttest Values for Delayed Feedback

When correct, the ILR association increases with the initial commitment per the delta in Equation 6, then since $t_0 = 1$ the ILR increases during the presentation of feedback per the delta rule in Equation 7:

$$p_{ILR-C} = p_b + 0.4 (1 - p_b)$$  \hspace{1cm} 6

$$p'_ILR-C = p_{ILR-C} + 0.4 (1 - p_{ILR-C})$$  \hspace{1cm} 7

When the lesson response is incorrect, first estimate the activation level of the ILR using Equation 8, then since $t_0 = 1$ increase the activation level of the ILR per the delta rule:

$$p_{ILR-I} = (1 - p_b)$$  \hspace{1cm} 8

$$p'_ILR-I = p_{ILR-I} + 0.4 (1 - p_{ILR-I})$$  \hspace{1cm} 9

Now combine Equations 7 and 9 (see Figure 3):

$$p'_ILR = (p_b \cdot p'_ILR-C) + [(1- p_b) \cdot p'_ILR-I]$$  \hspace{1cm} 10

In the steps shown in Equations 6 and 7, delayed feedback receives two positive increments based on the assumption that both the initial response and then the feedback response constitute two separate learning exposures. Note especially in Equation 9 that $t_0 = 1$ rather than 0 since the learner presumes that their initial response is correct. Comparing the curves shown by Equation 5 in Figure 2 and Equation 10 in Figure 3 indicates that posttest memory of ILRs will be considerably larger under delayed feedback compared to immediate feedback.
Predicting CR Posttest Values for Immediate Feedback

When correct, \( t_0 = 1 \) and so the activation weight of the correct response is increased per the delta rule:

\[
p'_{CR-C} = p_g + 0.4 \left(1 - p_g\right) \quad (11)
\]

When incorrect, estimate the initial error activation level \( p_{ILR-1} \) in terms of \( p_g \) using Equation 12, next use this value of \( p_{ILR-1} \) to determine the final activation level of the error \( p'_{ILR-1} \), since \( t_0 = 0 \), the error association decreases (Equation 13), then determine the adjusted value of \( p_{CR-1} \) in terms of \( p'_{ILR-1} \) using Equation 14, then determine \( p'_{CR-1} \) from \( p_{CR-1} \) using Equation 15 with \( t_0 = 1 \):

\[
p_{ILR-1} = (1 - p_g) \quad (12)
\]

\[
p'_{ILR-1} = p_{ILR-1} + 0.4 \left(0 - p_{ILR-1}\right) \quad (13)
\]

\[
p_{CR-1} = (1 - p'_{ILR-1}) \quad (14)
\]

\[
p'_{CR-1} = p_{CR-1} + 0.4 \left(1 - p_{CR-1}\right) \quad (15)
\]

Now combine Equations 11 and 15 (see Figure 4):
\[ p'_{\text{CR}} = (p_8 \times p'_{\text{CR-C}}) + [(1 - p_8) \times p'_{\text{CR-1}}] \]

With immediate feedback, the ILR error interacts with attaining the correct response. Specifically, the delta rule change at Equation 13 results in a decrease in the ILR error association, which increases the activation (relative) of the correct response in Equation 14, and then the delta rule change at Equation 15 results in a second absolute (not just relative) increase in the correct response association. This results in the odd situation that for any given item with difficulty \( p_8 \), posttest gain will be greater when it is missed during the lesson (per equation 15) than when it is correct during the lesson (per Equation 11) mainly due to the error decrease in Equation 13.

![Graph of the Predicted CR Posttest Values for Immediate Feedback.](image)

**Figure 4.** Graph of the Predicted CR Posttest Values for Immediate Feedback.

**Predicting CR Posttest Values for Delayed Feedback**

When correct, the correct response association increases with the initial commitment per the delta in Equation 17, then since \( t_0 = 1 \) the correct response increases again during the presentation of feedback per the delta rule in Equation 18:
\[ p_{\text{CR-C}} = p_g + 0.4 \ (1 - p_g) \]  
\[ p'_{\text{CR-C}} = p_{\text{CR-C}} + 0.4 \ (1 - p_{\text{CR-C}}) \]

When the lesson response is incorrect, since \( t_0 = 1 \), increase the activation level of the correct response per the delta rule:

\[ p'_{\text{CR-I}} = p_g + 0.4 \ (1 - p_g) \]

Proportional combination of Equations 18 and 19:

\[ p'_{\text{CR}} = (p_g \cdot p'_{\text{CR-C}}) + [(1 - p_g) \cdot p'_{\text{CR-I}}] \]

In Equations 17 and 18, we again follow the assumption that delayed feedback gains two delta increases due to the double exposure to the items. When the lesson response is incorrect, \( t_0 = 1 \) rather than \( t_0 = 0 \) (see discussion above).

\[ \text{Predicted Test Values} \]

\[ \text{Lesson Item Difficulty } p_g \]

**Figure 5.** Graph of the Predicted CR Posttest Values for Delayed Feedback.

**Implications**

The different predicted effects for immediate and delayed feedback for posttest memory of ILRs and CRs are displayed below together (see Figure 6). These calculations suggest several hypotheses. First, the most noticeable difference is observed between the immediate and delayed
feedback predicted posttest values of ILR (Equations 5 & 10). Students receiving delayed feedback should remember their initial lesson responses (both when an error and when correct) much better than those students receiving immediate feedback. Next, for posttest CRs, both immediate and delayed feedback have the greatest effects for more difficult lesson items. Last, comparing posttest CRs, immediate feedback may be slightly better than delayed feedback (Equations 16 versus 20) with more difficult items, while delayed feedback is slightly better with the easier items.

![Graph showing the relationship between predicted test values and lesson item difficulty](image)

**Figure 6.** Composite graph of the Predicted Posttest Values for each feedback treatment (IF - Immediate feedback; DF - Delayed Feedback).

These three hypotheses should be examined experimentally. Further, these findings suggest that previous investigations comparing immediate and delayed feedback may be confounded by the difficulty of the lesson materials. Previous studies of feedback timing should be reexamined in terms of lesson difficulty.

Can a connectionist approach account for feedback effects on memory and learning? The preceding descriptions are likely neither completely right or wrong. In several equations, it was
necessary to make a decision that the association would work one way or another way. Hopefully, we guessed right most of the time. Either way, this paper places a stake in the ground that can be adjusted with additional thought.

References


